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Exact analytic solutions for an elliptic hole with asymmetric collinear cracks in a one-dimensional hexagonal quasi-crystal*

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Using the complex variable function method and the technique of conformal mapping, the anti-plane shear problem of an elliptic hole with asymmetric collinear cracks in a one-dimensional hexagonal quasi-crystal is solved, and the exact analytic solutions of the stress intensity factors (SIFs) for mode III problem are obtained. Under the limiting conditions, the present results reduce to the Griffith crack and many new results obtained as well, such as the circular hole with asymmetric collinear cracks, the elliptic hole with a straight crack, the mode T crack, the cross crack and so on. As far as the phonon field is concerned, these results, which play an important role in many practical and theoretical applications, are shown to be in good agreement with the classical results.

Keywords: one-dimensional hexagonal quasi-crystals, elliptic hole with asymmetric collinear cracks, stress intensity factor, complex variable function method

PACC: 6140M, 6220M, 4630N

1. Introduction

Quasi-crystals (QCs) are characterized by their own unusual structures: they are orientationally ordered, but noncrystallographic, i.e. they have five-, eight-, ten-, or twelvefold symmetry axes.[1] Their translational order is not so periodic as that in conventional crystals, but it is quasi-periodic. Since their discovery, they have attracted the extensive attention of researchers engaged in experimental and theoretical work. The mechanical behaviour of the new solid phase, in which elasticity and defects play a central role, is of fundamental importance. Meanwhile, many important achievements of QCs have been obtained.[2−14]

It is well known that defects (holes, cracks, dislocations, etc.) greatly influence physical and mechanical properties of solid materials including QCs. Experiments have shown that QCs are quite brittle, and brittle materials are generally sensitive to the defects. At present, there are some research papers about single defect problem of QCs. A moving screw dislocation in one-dimensional (1D) hexagonal QCs was investigated;[15] a semi-infinite crack in a strip of 1D hexagonal QCs[16] and two semi-infinite collinear cracks in a strip of 1D hexagonal QCs[17] were investigated;a straight dislocation in 1D hexagonal QCs[18] and the interaction between dislocations and cracks[19] were also investigated, etc. However, for more complicated defect problems in QCs, just a few research papers have been published so far. In fact, the defects are quite complicated in QCs, some of them can be regarded as holes with cracks proximately. What is more, the exact analytic solutions for the defect problems are significant in the research of QCs. The present paper is devoted to investigating the elastic problem of an elliptic hole with asymmetric collinear cracks in a 1D hexagonal QC by means of complex variable function method with conformal mapping. The results derived can be applied to stimulating many models under the limiting conditions.

2. Basic theory

Let $[\varepsilon_{ij}, w_j] = [\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, 2\varepsilon_{23}, 2\varepsilon_{31}, 2\varepsilon_{12}, w_3, w_1, w_2]$ denote the phonon strain ε_{ij} and phason strain w_j

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and $[\sigma_{ij}, H_j] = [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{31}, \sigma_{12}, H_3, H_1, H_2]$ denote the phonon stress σ_{ij} and phason stress H_j, respectively. According to quasi-crystal elasticity theory,[16] the generalized Hooke's law of elasticity problem in a 1D hexagonal QC is given by

$$\begin{aligned}
&\sigma_{11} = C_{11}\varepsilon_{11} + C_{12}\varepsilon_{22} + C_{13}\varepsilon_{33} + R_1 w_3,\\
&\sigma_{22} = C_{12}\varepsilon_{11} + C_{11}\varepsilon_{22} + C_{13}\varepsilon_{33} + R_1 w_3,\\
&\sigma_{33} = C_{13}\varepsilon_{11} + C_{13}\varepsilon_{22} + C_{33}\varepsilon_{33} + R_2 w_3,\\
&\sigma_{23} = \sigma_{32} = 2C_{44}\varepsilon_{23} + R_3 w_2,\\
&\sigma_{31} = \sigma_{13} = 2C_{44}\varepsilon_{31} + R_3 w_1, \quad \sigma_{12} = \sigma_{21} = 2C_{66}\varepsilon_{12},\\
&H_1 = 2R_3\varepsilon_{31} + K_2 w_1,\\
&H_2 = 2R_3\varepsilon_{23} + K_2 w_2,\\
&H_3 = R_1(\varepsilon_{11} + \varepsilon_{22}) + R_2\varepsilon_{33} + K_1 w_3, \qquad (1)
\end{aligned}$$

where C_{ij} denotes the independent elastic constant of phonon field, K_i denotes the independent elastic constant of phason field, and R_i denotes the coupling elastic constant of phonon field and phason field.

Besides, the geometry equations are given by

$$\varepsilon_{ij} = \frac{1}{2}(\partial_j u_i + \partial_i u_j), w_j = \partial_j v, \text{ and } i, j = 1, 2, 3, \quad (2)$$

where u_i and v denote the displacements of phonon field and phason field, respectively. Here we have used the tensor notation and $\partial_j u_i = \partial u_i / \partial x_j$, the same hereafter.

The equilibrium equations (if the body force is neglected) are given by

$$\begin{aligned}
&\partial_1\sigma_{11} + \partial_2\sigma_{12} + \partial_3\sigma_{13} = 0,\\
&\partial_1\sigma_{21} + \partial_2\sigma_{22} + \partial_3\sigma_{23} = 0,\\
&\partial_1\sigma_{31} + \partial_2\sigma_{32} + \partial_3\sigma_{33} = 0,
\end{aligned}$$

and

$$\partial_1 H_1 + \partial_2 H_2 + \partial_3 H_3 = 0. \qquad (3)$$

Equations (1)–(3) are the basic equations of elasticity theory of 1D hexagonal QCs.

When defects such as elliptic holes with asymmetric collinear cracks parallel to the quasi-periodic axis of 1D hexagonal QCs exist, the geometric properties of the materials will not change in the quasi-periodic direction. If we take quasi-periodic axis of 1D hexagonal QCs as the x_3 axis, then

$$\partial_3 u_i = 0, \quad \partial_3 v = 0$$

and

$$\partial_3\sigma_{ij} = 0\,, \quad \partial_3 H_j = 0\,, \quad i, j = 1, 2, 3. \qquad (4)$$

Substitution of Eq.(4) into Eqs.(1)–(3) leads to two independent problems. One is a plane elasticity problem of general crystals, which can be solved by the route of linear elastic theory.[20] We do not discuss it here. And the other is an anti-plane elasticity problem of coupling phonon and phason fields as follows:

$$\begin{aligned}
&\sigma_{23} = \sigma_{32} = 2C_{44}\varepsilon_{23} + R_3 w_2,\\
&\sigma_{31} = \sigma_{13} = 2C_{44}\varepsilon_{31} + R_3 w_1, \qquad (5)\\
&H_1 = 2R_3\varepsilon_{31} + K_2 w_1\,, \quad H_2 = 2R_3\varepsilon_{23} + K_2 w_2, \quad (6)\\
&\partial_1\sigma_{31} + \partial_2\sigma_{23} = 0\,, \quad \partial_1 H_1 + \partial_2 H_2 = 0, \qquad (7)\\
&\varepsilon_{3j} = \varepsilon_{j\,3} = \frac{1}{2}\partial_j u_3\,, \text{ and } w_j = \partial_j v\,, \quad j = 1, 2. \quad (8)
\end{aligned}$$

Substituting Eq.(8) into Eqs.(5) and (6), then substituting the obtained results into Eq.(7), the equilibrium equations in terms of displacements are obtained as follows:

$$C_{44}\nabla^2 u_3 + R_3\nabla^2 v = 0,$$

and

$$R_3\nabla^2 u_3 + K_2\nabla^2 v = 0, \qquad (9)$$

where the harmonic operator $\nabla^2 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$. Moreover, under the condition of $C_{44}K_2 - R_3^2 \neq 0$, one has

$$\nabla^2 u_3 = 0, \text{ and } \quad \nabla^2 v = 0. \qquad (10)$$

Equation (10) indicates that the terminal governing equations of the anti-plane elasticity problem for coupling phonon and phason fields finally turn into two harmonic equations.

According to the theory of complex variable function, u_3 and v can be represented as the real part and the imaginary part of one analytic function, respectively. We assume that

$$u_3 = \mathrm{Re}\varphi_1(z), \quad \text{and} \quad v = \mathrm{Re}\psi_1(z), \qquad (11)$$

where $z = x_1 + \mathrm{i}x_2$, Re denotes the real part of the function, $\varphi_1(z)$ and $\psi_1(z)$ are two analytic functions.

3. Solution of elastic field and computations of stress intensity factors (SIFs)

Considering a problem of an elliptic hole with asymmetric collinear cracks in a 1D hexagonal QC,

a and b denote the long semi-axis and the short semi-axis respectively, $c_1 - a$ and $c_2 - a$ the crack lengths. Supposing that there is an elliptic hole with asymmetric collinear cracks parallel to the quasi-periodic direction in the body of an infinite 1D hexagonal QC, using the symmetry of body is enough to investigate any plane perpendicular to the quasi-periodic direction of 1D hexagonal QCs. We set the appropriate coordinate system as shown in Fig.1. From Eqs.(3) and (4), the equilibrium equations are simplified as follows:

$$\partial_1\sigma_{11} + \partial_2\sigma_{12} = 0, \quad \partial_2\sigma_{21} + \partial_2\sigma_{22} = 0,$$

$$\partial_1\sigma_{31} + \partial_2\sigma_{32} = 0,$$

and

$$\partial_1 H_1 + \partial_2 H_2 = 0. \tag{12}$$

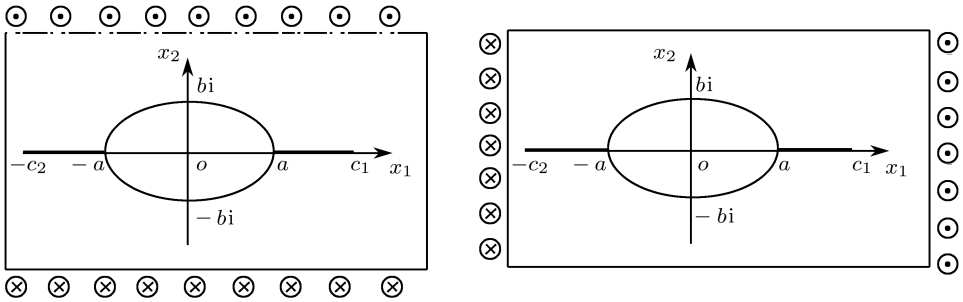


Fig.1. An elliptic hole with asymmetric collinear cracks in a one-dimensional hexagonal quasi-crystal.

We solve the anti-plane problem under different shear loading conditions at infinity as follows:

Case I As shown in Fig.1(a), we have the following boundary conditions:

$$\begin{aligned}&\sqrt{x_1^2 + x_2^2} \to \infty : \sigma_{32} = p, \sigma_{31} = 0,\\ &H_2 = q, \quad \text{and} \quad H_1 = 0,\\ &(x_1, x_2) \in L : \sigma_{32} = \sigma_{31} = H_2 = H_1 = 0,\end{aligned} \tag{13}$$

where L denotes the boundary of the elliptic hole with asymmetric collinear cracks.

The analytic result of linear elasticity theory shows that for stress concentration only, the above problem can be turned into the problem of which the state of stress at infinity is free from shear stress and the edge of the elliptic hole with asymmetric collinear cracks is subjected to shear stress. Therefore, in order to calculate the SIFs, we consider the latter problem which has the following boundary conditions:

$$\begin{aligned}&\sqrt{x_1^2 + x_2^2} \to \infty : \sigma_{32} = \sigma_{31} = H_2 = H_1 = 0,\\ &(x_1, x_2) \in L : \sigma_{31} = 0\,, \sigma_{32} = -p,\\ &H_1 = 0, \quad \text{and} \quad H_2 = -q.\end{aligned} \tag{14}$$

Case II Similarly, for the state of stress $\sigma_{31} = p_0$, and $H_1 = q_0$ at infinity as shown in Fig.1(b), we consider the boundary conditions as follows:

$$\begin{aligned}&\sqrt{x_1^2 + x_2^2} \to \infty : \sigma_{32} = \sigma_{31} = H_2 = H_1 = 0,\\ &(x_1, x_2) \in L : \sigma_{31} = -p_0, \sigma_{32} = 0,\\ &H_1 = -q_0, \quad \text{and} \quad H_2 = 0.\end{aligned} \tag{15}$$

For case I, we solve the boundary value problems (10) and (14) by using the complex variable function method as follows:

Denoting $z = x_1 + \mathrm{i}x_2$, where $\mathrm{i} = \sqrt{-1}$, and letting $f(z)$ be an analytic function, we have

$$\frac{\partial f}{\partial x_1} = \frac{\mathrm{d}f}{\mathrm{d}z}, \quad \text{and} \quad \frac{\partial f}{\partial x_2} = \mathrm{i}\frac{\mathrm{d}f}{\mathrm{d}z}. \tag{16}$$

And let

$$\begin{aligned}f(z) &= P(x_1, x_2) + \mathrm{i}Q(x_1, x_2)\\ &= \mathrm{Re}f(z) + \mathrm{i}\mathrm{Im}f(z),\end{aligned} \tag{17}$$

where Im denotes the imaginary part of the function, then by using the Cauchy–Riemann relation, we will have

$$\frac{\partial P}{\partial x_1} = \frac{\partial Q}{\partial x_2}, \quad \text{and} \quad \frac{\partial P}{\partial x_2} = -\frac{\partial Q}{\partial x_1}. \tag{18}$$

Substituting Eqs.(11) and (16) into Eqs.(5), (6) and (8), through some manipulations, the complex representations of stresses are obtained as follows:

$$\sigma_{23}=\sigma_{32}=C_{44}\frac{\partial}{\partial x_2}\mathrm{Re}\varphi_1+R_3\frac{\partial}{\partial x_2}\mathrm{Re}\psi_1,$$
$$\sigma_{31}=\sigma_{13}=C_{44}\frac{\partial}{\partial x_1}\mathrm{Re}\varphi_1+R_3\frac{\partial}{\partial x_1}\mathrm{Re}\psi_1,$$
$$H_1=K_2\frac{\partial}{\partial x_1}\mathrm{Re}\psi_1+R_3\frac{\partial}{\partial x_1}\mathrm{Re}\varphi_1,$$

and

$$H_2=K_2\frac{\partial}{\partial x_2}\mathrm{Re}\psi_1+R_3\frac{\partial}{\partial x_2}\mathrm{Re}\varphi_1. \tag{19}$$

From Eqs.(18) and (19), we have

$$\sigma_{31}-\mathrm{i}\sigma_{32}=C_{44}\varphi_1{}'+R_3\psi_1{}',$$

and

$$H_1-\mathrm{i}H_2=K_2\psi_1{}'+R_3\varphi_1{}', \tag{20}$$

where $\varphi_1{}'=\mathrm{d}\varphi_1/\mathrm{d}z$, and $\psi_1{}'=\mathrm{d}\psi_1/\mathrm{d}z$. From Eq.(20), we obtain $\sigma_{23}=\sigma_{32}=-\mathrm{Im}(C_{44}\varphi_1{}'+R_3\psi_1{}')$, and $H_2=-\mathrm{Im}(K_2\psi_1{}'+R_3\varphi_1{}')$. For an arbitrary complex function $f(z)$, we have $\mathrm{Im}f(z)=\dfrac{1}{2\mathrm{i}}(f-\overline{f})$, further

$$\sigma_{23}=\sigma_{32}=-\frac{1}{2\mathrm{i}}\left[C_{44}(\varphi_1{}'-\overline{\varphi_1{}'})+R_3(\psi_1{}'-\overline{\psi_1{}'})\right],$$

and

$$H_2=-\frac{1}{2\mathrm{i}}\left[K_2(\psi_1{}'-\overline{\psi_1{}'})+R_3(\varphi_1{}'-\overline{\varphi_1{}'})\right]. \tag{21}$$

From Eqs.(14) and (21), we have

$$C_{44}(\varphi_1{}'-\overline{\varphi_1{}'})+R_3(\psi_1{}'-\overline{\psi_1{}'})=2p\mathrm{i},$$

and

$$K_2(\psi_1{}'-\overline{\psi_1{}'})+R_3(\varphi_1{}'-\overline{\varphi_1{}'})=2q\mathrm{i},\quad z\in L. \tag{22}$$

We find that the function

$$\begin{aligned}z&=\omega(\zeta)\\&=\frac{a+b}{2}\frac{\alpha(1+\zeta)^2+\beta(1-\zeta)^2+\sqrt{(\alpha^2-1)(1+\zeta)^4+2(\alpha\beta+1)(1-\zeta^2)^2+(\beta^2-1)(1-\zeta)^4}}{4\zeta}\\&\quad+\frac{a-b}{2}\frac{4\zeta}{\alpha(1+\zeta)^2+\beta(1-\zeta)^2+\sqrt{(\alpha^2-1)(1+\zeta)^4+2(\alpha\beta+1)(1-\zeta^2)^2+(\beta^2-1)(1-\zeta)^4}}\end{aligned} \tag{23}$$

provides a conformal map from the unit disc in the ζ plane to the exterior region of an elliptic hole with asymmetric collinear cracks in the z plane, and a unit circle is transformed into an elliptic hole with asymmetric collinear cracks L, where we take

$$\alpha=\frac{d_1^2+1}{2d_1},\quad \beta=\frac{d_2^2+1}{2d_2},$$

$$d_1=\frac{c_1+\sqrt{c_1^2-a^2+b^2}}{a+b},$$

and

$$d_2=\frac{c_2+\sqrt{c_2^2-a^2+b^2}}{a+b}. \tag{24}$$

The above conformal mapping maps the upper edge of a onto A, the lower edge of a onto A_1, the upper edge of $-a$ onto C, the lower edge of $-a$ onto C_1, and $\omega^{-1}(c_1)\to 1$, $\omega^{-1}(b\mathrm{i})\to B$, $\omega^{-1}(-c_2)\to -1$, and $\omega^{-1}(-b\mathrm{i})\to B_1$ as shown in Fig.2.

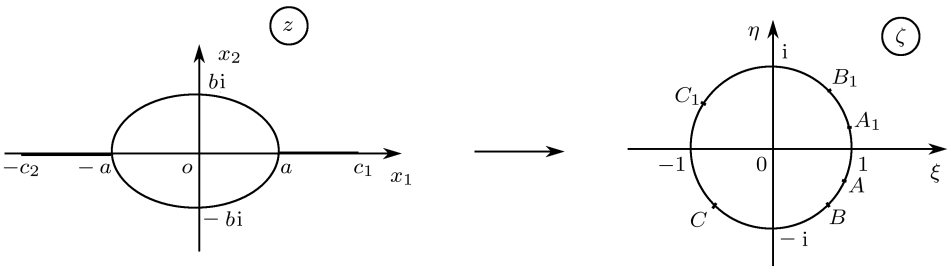


Fig.2. Conformal mapping from the exterior of the elliptic hole with asymmetric collinear cracks onto the unit disc.

We introduce the notations

$$\varphi(\zeta) = \varphi_1(z) = \varphi_1(\omega(\zeta)),$$

and

$$\psi(\zeta) = \psi_1(z) = \psi_1(\omega(\zeta)). \tag{25}$$

By using the chain rule of derivative, we obtain

$$\varphi_1{}'(z) = \varphi'(\zeta)/\omega'(\zeta),$$

and

$$\psi_1{}'(z) = \psi'(\zeta)/\omega'(\zeta). \tag{26}$$

Substituting Eq.(26) into Eq.(22), noting that $\zeta \equiv \sigma = \mathrm{e}^{\mathrm{i}\theta}$, we have

$$\varphi'(\sigma) - \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\varphi'(\sigma)} + \frac{R_3}{C_{44}}\left[\psi'(\sigma) - \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\psi'(\sigma)}\right] = \frac{2p\mathrm{i}}{C_{44}}\omega'(\sigma),$$

and

$$\psi'(\sigma) - \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\psi'(\sigma)} + \frac{R_3}{K_2}\left[\varphi'(\sigma) - \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\varphi'(\sigma)}\right] = \frac{2q\mathrm{i}}{K_2}\omega'(\sigma). \tag{27}$$

Multiplying both sides of two equations of Eq.(27) by $\frac{1}{2\pi\mathrm{i}}\frac{\mathrm{d}\sigma}{\sigma-\zeta}$, where ζ is an arbitrary point in $|\zeta| < 1$, and integrating around the unit circle τ, one has

$$\frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\varphi'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma - \frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\frac{\overline{\varphi'(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma + \frac{R_3}{C_{44}}\frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\psi'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma - \frac{R_3}{C_{44}}\frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\frac{\overline{\psi'(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma = \frac{2p\mathrm{i}}{C_{44}}\frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma, \tag{28}$$

and

$$\frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\psi'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma - \frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\frac{\overline{\psi'(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma + \frac{R_3}{K_2}\frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\varphi'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma - \frac{R_3}{K_2}\frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\frac{\overline{\varphi'(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma = \frac{2q\mathrm{i}}{K_2}\frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma. \tag{29}$$

We notice that $\varphi'(\zeta)$ and $\psi'(\zeta)$ are analytic in $|\zeta| < 1$ and continuous in $|\zeta| \le 1$. It follows from the Cauchy's integral formula in complex functions that

$$\frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\varphi'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma = \varphi'(\zeta), \quad \text{and} \quad \frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\psi'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma = \psi'(\zeta), \quad |\zeta| < 1. \tag{30}$$

Through derivation of Eq.(23), we have

$$\omega'(\zeta) = (\zeta^2-1)\left[\alpha+\beta+\frac{(\alpha^2-1)(1+\zeta)^2+2(\alpha\beta+1)(\zeta^2+1)+(\beta^2-1)(1-\zeta)^2}{\sqrt{(\alpha^2-1)(1+\zeta)^4+2(\alpha\beta+1)(1-\zeta^2)^2+(\beta^2-1)(1-\zeta)^4}}\right] \times \left\{\frac{(a+b)}{8\zeta^2} - \frac{2(a-b)}{[\alpha(1+\zeta)^2+\beta(1-\zeta)^2+\sqrt{(\alpha^2-1)(1+\zeta)^4+2(\alpha\beta+1)(1-\zeta^2)^2+(\beta^2-1)(1-\zeta)^4}]^2}\right\}. \tag{31}$$

From Eq.(31), noting that $\overline{\sigma} = 1/\sigma$, we have $\omega'(\sigma)/\overline{\omega'(\sigma)} = -1/\sigma^2$.

Since $\varphi'(\zeta)$ and $\psi'(\zeta)$ are analytic in $|\zeta| < 1$, we can assume that $\varphi'(\zeta)$ and $\psi'(\zeta)$ have Taylor series as follows:

$$\varphi'(\zeta) = \sum_{n=0}^{\infty} a_n\zeta^n,$$

and

$$\psi'(\zeta) = \sum_{n=0}^{\infty} b_n\zeta^n, |\zeta| < 1. \tag{32}$$

We notice that $\overline{\varphi'(\sigma)}$ and $\overline{\psi'(\sigma)}$ are regarded as the boundary values (the values on the unit circle τ) of $\overline{\varphi'}\left(\frac{1}{\zeta}\right) = \sum\limits_{n=0}^{\infty}\overline{a_n}\left(\frac{1}{\zeta}\right)^n$ and $\overline{\psi'}\left(\frac{1}{\zeta}\right) = \sum\limits_{n=0}^{\infty}\overline{b_n}\left(\frac{1}{\zeta}\right)^n$ respectively. Thus, $\frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\varphi'(\sigma)}$ and $\frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\overline{\psi'(\sigma)}$ in Eqs.(28) and (29) are regarded as the boundary values of $-\frac{1}{\zeta^2}\overline{\varphi'}\left(\frac{1}{\zeta}\right)$ and $-\frac{1}{\zeta^2}\overline{\psi'}\left(\frac{1}{\zeta}\right)$ respectively, which are analytic functions in $|\zeta| > 1$. According to the Cauchy's integral formula of infinite region, for $|\zeta| < 1$, we have

$$\begin{aligned}&\frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\frac{\overline{\varphi'(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma\\ &= \frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\frac{\overline{\psi'(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma = 0.\end{aligned} \tag{33}$$

Substituting Eqs.(33) and (30) into Eqs.(28) and (29), yields

$$\varphi'(\zeta) + \frac{R_3}{C_{44}}\psi'(\zeta) = \frac{2p\mathrm{i}}{C_{44}}\frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma,$$

and

$$\psi'(\zeta) + \frac{R_3}{K_2}\varphi'(\zeta) = \frac{2q\mathrm{i}}{K_2}\frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma. \tag{34}$$

From Eq.(31), it is not difficult to find that $\omega'(\zeta)$ is analytic in $|\zeta| > 1$ and continuous in $|\zeta| \geq 1$. According to the Cauchy's integral formula of infinite region, one finds

$$\begin{aligned}&\frac{1}{2\pi\mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma = \omega'(\infty)\\ &= \frac{(a+b)(\alpha+\beta)}{4} \overset{\text{def}}{=} F(\zeta).\end{aligned} \tag{35}$$

Substituting Eq.(35) into Eq.(34), one has

$$\varphi'(\zeta) + \frac{R_3}{C_{44}}\psi'(\zeta) = \frac{2p\mathrm{i}}{C_{44}}F(\zeta),$$

and

$$\psi'(\zeta) + \frac{R_3}{K_2}\varphi'(\zeta) = \frac{2q\mathrm{i}}{K_2}F(\zeta). \tag{36}$$

From Eq.(36), one obtains

$$\varphi'(\zeta) = 2\mathrm{i}\frac{pK_2 - qR_3}{C_{44}K_2 - R_3^2}F(\zeta),$$

and

$$\psi'(\zeta) = 2\mathrm{i}\frac{qC_{44} - pR_3}{C_{44}K_2 - R_3^2}F(\zeta), \tag{37}$$

and from Eq.(26), one has

$${\varphi_1}'(z) = \frac{\varphi'(\zeta)}{\omega'(\zeta)} = \frac{2\mathrm{i}(pK_2 - qR_3)}{C_{44}K_2 - R_3^2}\frac{F(\zeta)}{\omega'(\zeta)},$$

and

$${\psi_1}'(z) = \frac{\psi'(\zeta)}{\omega'(\zeta)} = \frac{2\mathrm{i}(qC_{44} - pR_3)}{C_{44}K_2 - R_3^2}\frac{F(\zeta)}{\omega'(\zeta)}. \tag{38}$$

Substituting Eq.(38) into Eq.(20), we have

$$\sigma_{31} - \mathrm{i}\sigma_{32} = 2p\,\mathrm{i}\frac{F(\zeta)}{\omega'(\zeta)},$$

and

$$H_1 - \mathrm{i}H_2 = 2q\mathrm{i}\frac{F(\zeta)}{\omega'(\zeta)}. \tag{39}$$

Equation (39) indicates that the stress distribution has nothing to do with the material constants, which is identical to the classical elasticity theory (linear elasticity fracture mechanics). From Eq.(39), one can obtain $\sigma_{31}, \sigma_{32}, H_1$, and H_2. Because of the diffusive calculation, we neglect them here. Substituting $\zeta = \omega^{-1}(z)$ which is given by Eq.(23) into Eq.(39), we can obtain the stress potential functions in the z plane.

As is known, the crack tip always causes the destruction of materials. The SIFs at the crack tip can reflect the stress intensity around the area of the crack tip. Therefore, computation and determination of the SIFs are very important. According to the quasi-crystal elasticity theory,[16] around the area of $r/a \ll 1$, the definitions of the SIFs are

$$\begin{aligned}\sigma_{31} - \mathrm{i}\sigma_{32} &= -\frac{K_{\mathrm{III}}^{\parallel}}{\sqrt{2\pi r}}\left(\sin\frac{\theta}{2} + \mathrm{i}\cos\frac{\theta}{2}\right)\\ &= -\mathrm{i}\frac{K_{\mathrm{III}}^{\parallel}}{\sqrt{2\pi z}},\end{aligned} \tag{40}$$

and

$$\begin{aligned}H_1 - \mathrm{i}H_2 &= -\frac{K_{\mathrm{III}}^{\perp}}{\sqrt{2\pi r}}\left(\sin\frac{\theta}{2} + \mathrm{i}\cos\frac{\theta}{2}\right)\\ &= -\mathrm{i}\frac{K_{\mathrm{III}}^{\perp}}{\sqrt{2\pi z}},\end{aligned} \tag{41}$$

where $z = r\mathrm{e}^{\mathrm{i}\theta}$.

Utilizing the conformal mapping $z = \omega(\zeta)$, we have $z_1 = \omega(\zeta_1)$, where $\zeta = \zeta_1$ is the corresponding point of the crack tip z_1. From Eqs.(39)–(41), the computational formulae of the SIFs in ζ plane are obtained as follows:

$$K_{\mathrm{III}}^{\parallel} = \lim_{\zeta\to\zeta_1} 2\sqrt{\pi}\,p\frac{F(\zeta)}{\sqrt{\omega''(\zeta)}},$$

and

$$K_{\mathrm{III}}^{\perp} = \lim_{\zeta\to\zeta_1} 2\sqrt{\pi}q\frac{F(\zeta)}{\sqrt{\omega''(\zeta)}}. \tag{42}$$

From Eq.(31), we have

$$\sqrt{\omega''(\zeta)} \to \frac{\sqrt{(\alpha+\beta)(\sqrt{\alpha^2-1}+\alpha)[(a+b)(\alpha+\sqrt{\alpha^2-1})^2-(a-b)]}}{2(\alpha^2-1)^{\frac{1}{4}}(\alpha+\sqrt{\alpha^2-1})},\ \zeta \to 1, \tag{43}$$

and from Eq.(35), we have

$$F(\zeta) \to \frac{(a+b)(\alpha+\beta)}{2},\quad \zeta \to 1. \tag{44}$$

Substituting Eqs.(43) and (44) into Eq.(42), we obtain the SIFs at the crack tip $(c_1, 0, 0)$ for mode III problem as follows:

$$K_{\mathrm{III}}^{\parallel} = \frac{p(a+b)\sqrt{\pi(\alpha+\beta)}(\alpha^2-1)^{\frac{1}{4}}(\alpha+\sqrt{\alpha^2-1})}{\sqrt{(\sqrt{\alpha^2-1}+\alpha)[(a+b)(\alpha+\sqrt{\alpha^2-1})^2-(a-b)]}},$$

and

$$K_{\mathrm{III}}^{\perp} = \frac{q(a+b)\sqrt{\pi(\alpha+\beta)}(\alpha^2-1)^{\frac{1}{4}}(\alpha+\sqrt{\alpha^2-1})}{\sqrt{(\sqrt{\alpha^2-1}+\alpha)[(a+b)(\alpha+\sqrt{\alpha^2-1})^2-(a-b)]}}. \tag{45}$$

Expression (45) is the exact analytic solutions of the SIFs for case I.

Similarly, for case II, we solve the boundary value problems (10) and (15). From Eq.(20), one has

$$C_{44}(\varphi_1{}' + \overline{\varphi_1{}'}) + R_3(\psi_1{}' + \overline{\psi_1{}'}) = -2p_0,$$

and

$$K_2(\psi_1{}' + \overline{\psi_1{}'}) + R_3(\varphi_1{}' + \overline{\varphi_1{}'}) = -2q_0, z \in L. \tag{46}$$

Substituting Eq.(26) into Eq.(46), multiplying both sides of Eq.(46) by $\frac{1}{2\pi \mathrm{i}}\frac{\mathrm{d}\sigma}{\sigma-\zeta}$ and integrating them around the unit circle τ yield

$$\begin{aligned}&\frac{1}{2\pi \mathrm{i}}\int_\tau \frac{\varphi'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma + \frac{1}{2\pi \mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\frac{\overline{\varphi'(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma\\ &+\frac{R_3}{C_{44}}\frac{1}{2\pi \mathrm{i}}\int_\tau \frac{\psi'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma + \frac{R_3}{C_{44}}\frac{1}{2\pi \mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\frac{\overline{\psi'(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma\\ =&-\frac{2p_0}{C_{44}}\frac{1}{2\pi \mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma,\end{aligned} \tag{47}$$

and

$$\begin{aligned}&\frac{1}{2\pi \mathrm{i}}\int_\tau \frac{\psi'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma + \frac{1}{2\pi \mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\frac{\overline{\psi'(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma\\ &+\frac{R_3}{K_2}\frac{1}{2\pi \mathrm{i}}\int_\tau \frac{\varphi'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma + \frac{R_3}{K_2}\frac{1}{2\pi \mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\overline{\omega'(\sigma)}}\frac{\overline{\varphi'(\sigma)}}{\sigma-\zeta}\mathrm{d}\sigma\\ =&-\frac{2q_0}{K_2}\frac{1}{2\pi \mathrm{i}}\int_\tau \frac{\omega'(\sigma)}{\sigma-\zeta}\mathrm{d}\sigma.\end{aligned} \tag{48}$$

From Eqs.(33), (35) and (26), one obtains

$$\varphi_1{}'(z) = \frac{\varphi'(\zeta)}{\omega'(\zeta)} = -\frac{2(p_0K_2 - q_0R_3)}{C_{44}K_2 - R_3^2}\frac{F(\zeta)}{\omega'(\zeta)},$$

and

$$\psi_1{}'(z) = \frac{\psi'(\zeta)}{\omega'(\zeta)} = -\frac{2(q_0C_{44} - p_0R_3)}{C_{44}K_2 - R_3^2}\frac{F(\zeta)}{\omega'(\zeta)}. \tag{49}$$

Substituting Eq.(49) into Eq.(20), one has

$$\sigma_{31} - \mathrm{i}\sigma_{32} = -2p_0\frac{F(\zeta)}{\omega'(\zeta)},$$

and

$$H_1 - \mathrm{i}H_2 = -2q_0\frac{F(\zeta)}{\omega'(\zeta)}. \tag{50}$$

From Eqs.(40), (41) and (50), it is easily found that the SIFs at the crack tip $(c_1, 0, 0)$ are not affected by shear stress σ_{31}, which is in good agreement with the results of Refs.[21, 22].

4. Discussion and results

We can obtain many new results from expression (45) by changing the long semi-axis and the short semi-axis of ellipse and the crack length. These results are discussed in detail as follows.

Case 1 When $b \to 0$, from expression (24), one has $d_1 \to (c_1 + \sqrt{c_1^2 - a^2})/a$, and $d_2 \to (c_2 + \sqrt{c_2^2 - a^2})/a$, and from expression (45), we have

$$K_{\mathrm{III}}^{\parallel} = p\sqrt{\frac{\pi(c_1+c_2)}{2}},$$

and

$$K_{\mathrm{III}}^{\perp}=q\sqrt{\frac{\pi(c_1+c_2)}{2}}. \tag{51}$$

This is the state of the Griffith crack in a 1D hexagonal QC, which is identical to the result given in Ref.[16].

Case 2 When $c_2\to c_1$, from expression (24), we have $d_2\to d_1$, and $\beta\to\alpha$. Then from expression (45), one finds

$$K_{\mathrm{III}}^{\parallel}=\frac{p(a+b)\sqrt{\pi(d_1^4-1)}}{\sqrt{2d_1[(d_1^2-1)a+(d_1^2+1)b]}},$$

and

$$K_{\mathrm{III}}^{\perp}=\frac{q(a+b)\sqrt{\pi(d_1^4-1)}}{\sqrt{2d_1[(d_1^2-1)a+(d_1^2+1)b]}}. \tag{52}$$

These are the exact analytic solutions of the SIFs at the crack tip of an elliptic hole with symmetric cracks in a 1D hexagonal QC, which are related to the long semi-axis a, the short semi-axis b of the ellipse and the crack length c_1-a as well (shown in Fig.3).

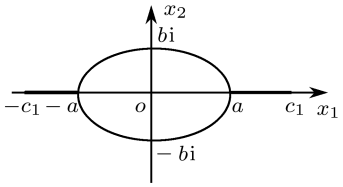


Fig.3. Elliptic hole with symmetric cracks.

(1) If $b\to 0$, expression (52) reduces to the result of the Griffith crack given in Ref.[16].

(2) If $a\to 0$, from expression (52), one has

$$K_{\mathrm{III}}^{\parallel}=p\sqrt{\pi c_1},\quad\text{and}\quad K_{\mathrm{III}}^{\perp}=q\sqrt{\pi c_1}. \tag{53}$$

These are the SIFs of a symmetric cross crack in a 1D hexagonal QC. we can find that if the solid is shear loading, the SIFs at the crack tip of the crack parallel to the x_1 axis are not affected by the crack parallel to the x_2 axis. Because of its symmetry, the results are the same as those of Griffith crack as shown in Fig.4.

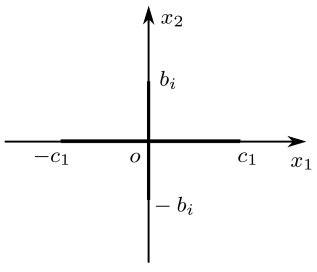


Fig.4. Symmetric cross crack.

(3) If $a\to b$, from expression (52), one obtains

$$K_{\mathrm{III}}^{\parallel}=p\sqrt{\pi c_1\left(1-\frac{b^4}{c_1^4}\right)},$$

and

$$K_{\mathrm{III}}^{\perp}=q\sqrt{\pi c_1\left(1-\frac{b^4}{c_1^4}\right)}. \tag{54}$$

These are the exact analytic solutions of the SIFs of a circular hole with symmetric cracks in a 1D hexagonal QCs as shown in Fig.5. As far as the phonon field is concerned, expression (54) is shown to be in good agreement with the exact analytic solution of mode III problem given in Refs.[21–23]. Compared with the corresponding expression in Ref.[21], let $s=(c_1-b)/c_1$, expression (54) can be rewritten as

$$K_{\mathrm{III}}^{\parallel}=p\sqrt{\pi(c_1-b)}F(s), \tag{55}$$

where

$$F(s)=\sqrt{1-(1-s)^4}/\sqrt{s}. \tag{56}$$

In particular, if $b\to 0$, the above results reduce to those for the Griffith crack.

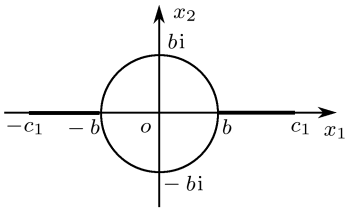


Fig.5. Circular hole with symmetric cracks.

Case 3 When $a\to b$, from expression (24), one has $d_1\to c_1/b$, and $d_2\to c_2/b$. Then from expression (45), we have

$$K_{\mathrm{III}}^{\parallel}=\frac{p\sqrt{2\pi\left(\dfrac{b^2+c_1^2}{c_1}+\dfrac{b^2+c_2^2}{c_2}\right)(c_1^2-b^2)}}{2c_1},$$

and

$$K_{\mathrm{III}}^{\perp}=\frac{q\sqrt{2\pi\left(\dfrac{b^2+c_1^2}{c_1}+\dfrac{b^2+c_2^2}{c_2}\right)(c_1^2-b^2)}}{2c_1}. \tag{57}$$

These are the exact analytic solutions of the SIFs of a circular hole with asymmetric collinear cracks in a 1D hexagonal QC, which are related to not only the radius of the circle, but also the crack length (shown in Fig.6).

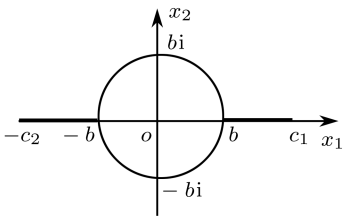


Fig.6. Circular hole with asymmetric collinear cracks.

(1) If $c_2 \to c_1$, expression (57) reduces to expression (54) which is the state of circular hole with symmetric cracks.

(2) If $b \to 0$, expression (57) reduces to the state of the Griffith crack.

(3) If $c_2 \to b$, from expression (57), one has

$$K_{\rm III}^{\parallel} = \frac{p\sqrt{2\pi}(b+c_1)\sqrt{c_1^2-b^2}}{2c_1\sqrt{c_1}},$$

and

$$K_{\rm III}^{\perp} = \frac{q\sqrt{2\pi}(b+c_1)\sqrt{c_1^2-b^2}}{2c_1\sqrt{c_1}}. \tag{58}$$

These are the exact analytic solutions of the SIFs of a circular hole with a straight crack in a 1D hexagonal QC, as shown in Fig.7. In particular, if $b \to 0$, the above result reduces to the state of the Griffith crack.

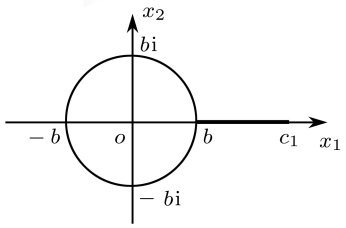


Fig.7. Circular hole with a straight crack.

Case 4 When $c_2 \to a$, from expression (24), we have $d_2 \to 1$, and $\beta \to 1$. Then expression (45) is simplified into

$$K_{\rm III}^{\parallel} = \frac{p(a+b)(d_1+1)\sqrt{\pi(d_1^2-1)}}{2\sqrt{d_1[(d_1^2-1)a+(d_1^2+1)b]}},$$

and

$$K_{\rm III}^{\perp} = \frac{q(a+b)(d_1+1)\sqrt{\pi(d_1^2-1)}}{2\sqrt{d_1[(d_1^2-1)a+(d_1^2+1)b]}}. \tag{59}$$

These are the exact analytic solutions of the SIFs of an elliptic hole with a straight crack in a 1D hexagonal QC, which are related to not only the long semi-axis and the short semi-axis of ellipse, but also the crack length (shown in Fig.8). For the phonon field side, expression (59) accords with those in Ref.[22].

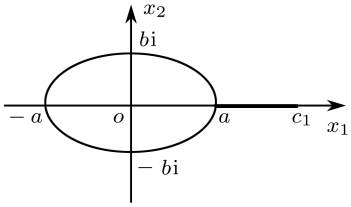


Fig.8. Elliptic hole with a straight crack.

(1) If $b \to 0$, expression (59) yields the result of the Griffith crack given in Ref.[16].

(2) If $a \to b$, expression (59) yields expression (58), which is the state of the circular hole with a straight crack.

(3) If $a \to 0$, from expression (24), $d_1 \to (c_1+\sqrt{c_1^2+b^2})/b$, then from expression (59), we obtain

$$K_{\rm III}^{\parallel} = \frac{p\sqrt{\pi c_1}(b+c_1+\sqrt{c_1^2+b^2})}{2\sqrt{c_1^2+b^2+c_1\sqrt{c_1^2+b^2}}},$$

and

$$K_{\rm III}^{\perp} = \frac{q\sqrt{\pi c_1}(b+c_1+\sqrt{c_1^2+b^2})}{2\sqrt{c_1^2+b^2+c_1\sqrt{c_1^2+b^2}}}. \tag{60}$$

These are the exact analytic solutions of the SIFs of a mode T crack in a 1D hexagonal QC, which are related to not only the length of crack parallel to the x_1 axis, but also the length of crack parallel to the x_2 axis (shown in Fig.9). In particular, if $b \to \infty$, from expression (60), one finds

$$K_{\rm III}^{\parallel} = p\sqrt{\pi c_1}, \quad \text{and} \quad K_{\rm III}^{\perp} = q\sqrt{\pi c_1}. \tag{61}$$

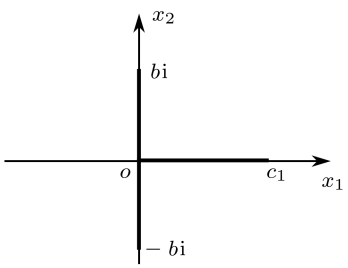


Fig.9. Mode T crack.

These are the exact solutions of the SIFs for a half-plane with an edge crack as shown in Fig.10. As far as the phonon field is concerned, expression (61) is identical to the result given in Refs.[21, 22].

Case 5 When $a \to 0$, from expression (24), we have $d_1 \to \dfrac{c_1+\sqrt{c_1^2+b^2}}{b}$, and $d_2 \to \dfrac{c_2+\sqrt{c_2^2+b^2}}{b}$, and from expression (45), we have

$$K_{\mathrm{III}}^{\parallel}=\frac{p\sqrt{2\pi c_1}}{2}\sqrt{1+\frac{c_1+\sqrt{c_1^2+b^2}}{c_2+\sqrt{c_2^2+b^2}}\cdot\frac{c_2^2+b^2+c_2\sqrt{c_2^2+b^2}}{c_1^2+b^2+c_1\sqrt{c_1^2+b^2}}},$$

and

$$K_{\mathrm{III}}^{\perp}=\frac{q\sqrt{2\pi c_1}}{2}\sqrt{1+\frac{c_1+\sqrt{c_1^2+b^2}}{c_2+\sqrt{c_2^2+b^2}}\cdot\frac{c_2^2+b^2+c_2\sqrt{c_2^2+b^2}}{c_1^2+b^2+c_1\sqrt{c_1^2+b^2}}}. \tag{62}$$

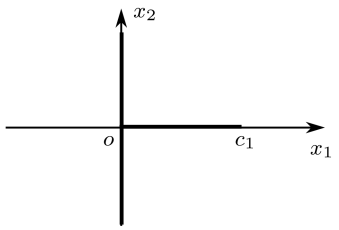


Fig.10. Half-plane with an edge crack.

These are the exact analytic solutions of the SIFs for asymmetric cross crack in a 1D hexagonal QC, which are related to not only the length of crack parallel to the x_1 axis, but also the length of crack parallel to the x_2 axis (shown in Fig.11).

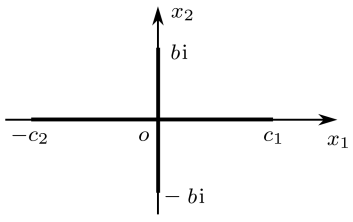


Fig.11. Asymmetric cross crack.

(1) If $c_2 \to c_1$, expression (62) reduces to the state of symmetric cross crack.

(2) If $c_2 \to 0$, expression (62) reduces to expression (60) which is the state of mode T crack.

(3) If $b \to 0$, expression (62) reduces to the Griffith crack.

5. Conclusion

The stress intensity factors expressions (45), (52)–(54) and (57)–(62) as exact solutions in closed form, are found for the first time in 1D hexagonal QCs. These solutions have significance to the theory of brittle fracture of solids including QCs. First of all, some of the complicated defects in QCs can be regarded as the elliptic holes with asymmetric collinear cracks approximately. Besides, the results derived can also be applied to stimulating many practical problems, such as circular holes with asymmetric collinear cracks, elliptic holes with a straight crack, circular holes with a straight crack, mode T crack, cross crack and half-plane with an edge crack, which are new and useful to theoretical researchers, in particular for the study of QCs. Finally, by using the conformal mapping technique developed in this paper and the complex variable function method, the complicated defect problem is solved effectively, which can analyse similar fracture problems of this kind.

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