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Rotating soliton clusters in nonlocal nonlinear media*

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From the study of the dynamics for the ring-like soliton clusters, we find that there exists a critical value of the ring radius, $d_{\rm cr}$, for the stationary rotation of the clusters with respect to the beam centre even in the presence of the relatively strong noise, and that the soliton clusters will not rotate but only undergo periodic collisions in the form of simple harmonic oscillator if the ring radius is large enough. We also show that the direction of the rotation can be opposite to the direction of phase gradient when the relative phase difference is within the domain $0 < |\theta| < \pi$, while along the direction of phase gradient when the relative phase difference is within the domain $\pi < |\theta| < 2\pi$.

Keywords: nonlocal nonlinear Schrödinger equation, spatial optical soliton, soliton clusters
PACC: 4265, 4265S, 4270D

1. Introduction

Self-trapping of optical beam occurs when the nonlinear change in the properties of the medium induced by the beam itself is balanced by the diffraction that is associated with a finite size of the beam.[1] In 1997, Snyder and Mitchell[2] simplified the nonlocal nonlinear Schrödinger equation (NNLSE) into a linear harmonic oscillator equation and obtained a Gaussian-shape spatial optical soliton solution. And since then, more properties of the fundamental nonlocal solitons have been found theoretically.[3−11] Subsequently, Assanto's team[12,13] has observed that a nematic liquid crystal (NLC) is one of the excellent strongly nonlocal nonlinear materials and the strongly nonlocal solitons (SNSs) in the NLCs. And because of their fundamental interest and potential applications in optical configurations, basic properties and interactions of the SNSs have been studied in NLCs.[14−20]

Up to now, many researches have concentrated on the lowest order bright single soliton. However, in recent years, the properties of the soliton clusters (or soliton trains) have received a great interest. Intuitively, a soliton cluster is a self-localized state which can be viewed as a nonlinear superposition of several fundamental solitons with appropriately engineered phases.[21−23] As it is of a composite structure, it can behave in a more complicated way than a conventional single fundamental soliton. It has been found that there exists a stationary bound state of the soliton cluster in both the Kerr-type and saturable Kerr-type nonlinear media with a periodic transverse modulation of the refractive index,[21,22] which is in sharp contrast to the case for a local Kerr-type nonlinearity, in which stable in-phase (out-of-phase) solitons do not exist.[24] The solitons may even possess a transverse energy flow if they are appropriately constructed, like spiraling of interacting spatial solitons,[25,26] and rotating propeller solitons.[27] However, by using the parameters for a single soliton, when the solitons with appropriate relative phases were launched in parallel along a circular ring of radius d, the minimum value of effective potential for the system corresponded to the stationary rotation of the bound state and the dynamics of such clusters was studied in detail in the saturable Kerr media.[28,29] The quasi-stable soliton clusters in a medium with cubic-quintic nonlinearity have also been studied in Ref.[30]. Recently, Rotschild *et al*[31] also experimentally observed the scalar multipole solitons in highly nonlocal nonlinear media. However, the important issue of the stationary bound state of bright soliton clusters in the strongly nonlocal nonlinear media has not been solved theoretically. In particular, the dynamics of the soliton clusters composed of three or more solitons has not been addressed so far. And an open question is whether there exists a stationary

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rotation of the cluster. In this paper, we report the dynamics of such soliton clusters in strongly nonlocal nonlinear media.

The main purpose of this paper is to search for stable soliton clusters in the 1+2D NNLSE model. Ring-like 1+2D clusters, composed of several individual solitons, are constructed in Section 2. The values of the parameters for a single soliton are calculated analytically, and by using these parameters, the results of the numerical simulations for the propagation of the soliton clusters are given in Section 3. Detailed analyses for the dynamics of the soliton clusters in Section 4 show that they are indeed in good agreements with our simulations. The results are presented in Section 5.

2. Model

We consider the propagation of paraxial optical beams in a nonlocal self-focusing medium, described by the dimensionless NNLSE[4,6,7]

$$\mathrm{i}\frac{\partial\psi}{\partial z}+\frac{1}{2}\nabla_{\perp}^{2}\psi +\psi\int\!\!\int R(\boldsymbol{r}-\boldsymbol{r}')|\psi(\boldsymbol{r}')|^{2}\mathrm{d}^{2}\boldsymbol{r}'=0, \quad (1)$$

where the longitudinal (propagation direction) coordinate z is scaled to the diffraction length, $\boldsymbol{r}=(x,y)$ denotes the transverse coordinate vector and is normalized by the input beam width, $\nabla_{\perp}^{2}=\partial_{x}^{2}+\partial_{y}^{2}$ is the transverse Laplacian operator, $r^{2}=x^{2}+y^{2}$, $\iint \mathrm{d}^{2}\boldsymbol{r}=\iint \mathrm{d}x\mathrm{d}y$, $\psi=\psi(\boldsymbol{r},z)$ is the complex scalar function describing the distribution of the field amplitude, the nonlocal term $\Delta n=\iint R(\boldsymbol{r}-\boldsymbol{r}')|\psi(\boldsymbol{r}')|^{2}\mathrm{d}^{2}\boldsymbol{r}'$ is the refractive-index change of the medium, and R is the nonlinear response of the medium which is normalized, real and symmetric such that $\iint R(\boldsymbol{r})\mathrm{d}^{2}\boldsymbol{r}=1$.

What follows is the construction of the soliton clusters. Consider a simple configuration in which the beams are arranged in a ring-shaped array and launched in parallel with a fixed difference between adjacent solitons. Therefore, the stationary solutions for the soliton clusters have the form

$$\psi=\sum\psi_{n}(\boldsymbol{r},z), n=1,2,\cdots,N \quad (2)$$

with the analytically obtained Gaussian-type stationary solution for each single soliton, $\psi_{n}(\boldsymbol{r},z)$, being of the form

$$\psi_{n}(\boldsymbol{r},z)=A\exp\left(-\frac{|\boldsymbol{r}-\boldsymbol{r}_{n}|^{2}}{2a^{2}}+\mathrm{i}\alpha_{n}+\mathrm{i}\lambda z\right), \quad (3)$$

where $\boldsymbol{r}_{n}=(d\cos\varphi_{n},d\sin\varphi_{n})$ is the position of the soliton centre with $\varphi_{n}=2n\pi/N$, the soliton phase at the position is $\alpha_{n}=2n\pi m/N$, where m can be considered as the topological charge which may be interpreted simply as axial rotation of a soliton, A, a and λ are the results of the variational approach which describe the amplitude, beam width and the propagation constant for the single soliton,[17,28,29] respectively.

3. Numerical results

In this paper we define the degree of non-locality as $\alpha=a/a_{m}$,[6] where a_{m} is the characteristic length of the nonlocal nonlinear response to the medium, the smaller the value of α is, the stronger the non-locality will be, and as no stationary bound state for the soliton cluster exists in the medium with local nonlinearity, we need to study only the strongly nonlocal case. Provided that the response function is twice differentiable at its centre, $r=0$, then the response function R will be expanded into Taylor's series, and the nonlocal term can be simplified[6–8] into

$$\Delta n=\int\!\!\int\left[R_{0}-\frac{1}{2}\gamma(\boldsymbol{r}-\boldsymbol{r}')^{2}\right]|\psi(\boldsymbol{r}')|^{2}\mathrm{d}^{2}\boldsymbol{r}', \quad (4)$$

where $R_{0}=R(\boldsymbol{0})$ and $\gamma=-R''(\boldsymbol{0})$.

In order to illustrate the basic properties of the soliton clusters, the parameters for a single soliton should be captured by a variational approach first. It is easily shown that expression (1) can be obtained from the Lagrangian density[9]

$$\mathcal{L}=\frac{\mathrm{i}}{2}\left(\psi^{*}\frac{\partial\psi}{\partial z}-\psi\frac{\partial\psi^{*}}{\partial z}\right)-\frac{1}{2}|\nabla\psi|^{2}+\frac{1}{2}|\psi|^{2}\Delta n. \quad (5)$$

Inserting the ground-state Gaussian trial solution of the form $\psi(\boldsymbol{r},z)=A\exp\left(-\dfrac{r^{2}}{2a^{2}}\right)\exp(\mathrm{i}\lambda z)$ and expression (4) into expression (5) and integrating the Lagrangian density with respect to x and y, the Lagrangian can be obtained as

$$L=-\pi\lambda a^{2}A^{2}-\frac{1}{2}\pi A^{2}+\frac{1}{2}\pi^{2}a^{4}A^{4}(-\gamma a^{2}+R_{0}). \quad (6)$$

The amplitude A and the beam width a can be found from the Euler–Lagrange equations $\dfrac{\partial L}{\partial A}=0$ and $\dfrac{\partial L}{\partial a}=0$ to be

$$A=\frac{\sqrt{\dfrac{27\gamma^2}{2}+18\gamma\lambda R_0+\sqrt{\gamma}\left(\dfrac{9}{2}\gamma^{+}2\lambda R_0\right)\sqrt{9\gamma+16\lambda R_0}}}{2\sqrt{2\pi}R_0^{\frac{3}{2}}}, \tag{7}$$

and

$$a=\sqrt{-\frac{3}{4\lambda}+\frac{\sqrt{9\gamma+16\lambda R_0}}{4\sqrt{\gamma}\lambda}}. \tag{8}$$

As the dimensionless form of Eq.(1) is obtained by scaling the transverse coordinate with the width of the input beam[7] and the results we obtained are soliton solutions, we set $a=1$, then the relation among λ, R_0 and γ follows

$$\lambda=\frac{-3\gamma+2R_0}{2\gamma}. \tag{9}$$

Given a concrete form of the response function, we can calculate the values of A, a and λ. Suppose that the response function has a Gaussian form

$$R(\boldsymbol{r})=\frac{1}{2\pi a_m^2}\exp\left(-\frac{r^2}{2a_m^2}\right), \tag{10}$$

then we will obtain $R_0=\dfrac{1}{2\pi a_m^2}$, and $\gamma=\dfrac{1}{2\pi a_m^4}$. It is easy to further obtain $A=141.421$, $a=1$ and $\lambda=98.5$ when $a_m=10$. From expression (9) we can see that the propagation constant λ depends on the material properties. Through the calculation of different values of a_m, we also find that the stronger the non-locality is, the larger the propagation constant will be.

With the parameters obtained above, now we can study the dynamics of the soliton clusters by numerically solving the original NNLSE, with the analytical solutions used as initial conditions. Note that in the numerical simulation, we keep the value of the nonlocality parameter $\alpha=0.1$ for all figures hereinafter.

For a particular example, here we mainly study the properties of the 1+2D soliton clusters composed of three fundamental solitons numerically, and their images for different cases are given in Fig.1. If the



Fig.1. Evolution of the soliton clusters with $N=3$. Panel (a) shows the solitons initially repulsing when $d=0.5$. Panel (b) indicates the exactly stable rotation of the soliton clusters with $d=0.7$. Panel (c) exhibits the solitons initially contracting when $d=1$. Panel (d) displays the solitons experiencing strong collisions when $d=3$. Panel (e) reveals the soliton cluster gradually fusing through oscillations when $m=0$ and $d=2$. $m=1$ is taken in the cases (a)–(d).

ring radius is smaller than 0.7, the repulsive centrifugal force plays a major role and the beams initially expand (see Fig.1(a)), whereas when $d > 0.7$, the reverse situation happens and the beams initially contract (see Fig.1(c)). From the propagation dynamics depicted in Fig.1(b), we know that a steady rotating bound state exists indeed for the cluster at about $d = 0.7$. However, when the ring radius is larger than 3, the solitons will not rotate any more, but only undergo strong periodic collisions around the beam centre in the form of simple harmonic oscillator as shown in Fig.1(d). Notice that when the difference in phase between adjacent solitons is 0, no stationary evolution for the cluster exists and the solitons will fuse through oscillations, Fig.1(e) shows a particular example.

To address the stability of the soliton cluster, we next simulate their propagation of our numerically computed solutions in the presence of relatively strong initial perturbations. The initial condition is assumed to be of the form $\psi(\boldsymbol{r}, 0) = f(\boldsymbol{r})\big[1+cg(\boldsymbol{r})\big]$,[22,32] where $f(\boldsymbol{r})$ is the multipole stationary solution, $g(\boldsymbol{r})$ is the normally distributed random function with a mean value of 0, variance $\sigma^2 = 1$, and standard deviation $\sigma = 1$. We simulate the evolution of the soliton cluster for the case of $c = 0.1$ and find that the soliton cluster turns out to be extremely stable at $z = 20$ and one cannot detect any substantial distortion of the cluster shape, and at the moment we could say that the evolution of the soliton cluster is stable. The images for $c = 0.1$ in the case $N = 3$ are presented in Fig.2.

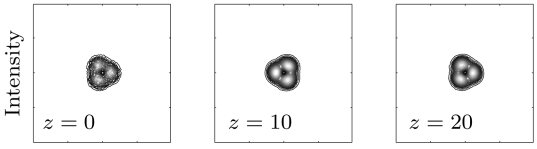


Fig.2. Evolution of the soliton clusters in the presence of strong initial random complex noise when $d = 0.7$.

4. Analytical results and discussion

By employing a variational approach, in 2006, Yakimenko *et al*[32] found that the minimum value of Lagrangian for the system implied the steady propagation of the bound-state dipole solitons, which was proved by their numerical simulations. However, in this paper, with the parameters calculated for the fundamental single soliton, we also find that the minimum of the Lagrangian corresponds to the stable propagation of the bound-state beams in the strongly nonlocal nonlinear media. In what follows are the detailed analyses of our method and results.

4.1. Analyses of the Lagrangian for the soliton clusters and comparison with the numerical simulations

In accordance with the simulations shown above, in this section we give only a detailed calculation for the case $N = 3$. Then substituting expressions (2), (3) and (4) into expression (5) and integrating with respect to the transverse dimensions, we arrive at the Lagrangian for an arbitrary response function in the case of $\pm\dfrac{2\pi}{3}$ phase intervals ($m = \pm 1$) as follows:

$$\begin{aligned} L \;=\; & 3a^2A^2\pi\lambda\left[-1+\exp\left(-\frac{3d^2}{4a^2}\right)\right] \\ & -\frac{1}{2}\left\{\frac{3A^2\pi}{4a^2}\left[4a^2+(-4a^2+3d^2)\exp\left(-\frac{3d^2}{4a^2}\right)\right]\right. \\ & +\frac{9}{4}a^4A^4\pi^2\exp\left(-\frac{3d^2}{2a^2}\right)\left[-1+\exp\left(\frac{3d^2}{4a^2}\right)\right] \\ & \times\left[-4a^2\gamma-d^2\gamma+4\exp\left(\frac{3d^2}{4a^2}\right)\right. \\ & \left.\left.\times(a^2\gamma+d^2\gamma-R_0)+4R_0\right]\right\}, \end{aligned} \tag{11}$$

and in the case of zero phase difference ($m = 0$),

$$\begin{aligned} L \;=\; & 3a^2A^2\pi\lambda\left[-1-2\exp\left(-\frac{3d^2}{4a^2}\right)\right] \\ & -\frac{1}{2}\left\{\frac{3A^2\pi}{2a^2}\left[2a^2+(4a^2-3d^2)\exp\left(-\frac{3d^2}{4a^2}\right)\right]\right. \\ & +\frac{9}{2}a^4A^4\pi^2\exp\left(-\frac{3d^2}{2a^2}\right)\left[2+\exp\left(\frac{3d^2}{4a^2}\right)\right] \\ & \times\left[4a^2\gamma+d^2\gamma+2\exp\left(\frac{3d^2}{4a^2}\right)\right. \\ & \left.\left.\times(a^2\gamma+d^2\gamma-R_0)-4R_0\right]\right\}, \end{aligned} \tag{12}$$

where A, a and λ have the same forms as expressions (7), (8) and (9), respectively.

The curves of the Lagrangian with a Gaussian form response function for the case $N = 3$ are shown in Fig.3. The results display a minimum at about $d_{\rm cr} = 0.74$ for the cases of $m = \pm 1$; while no extremum exists as the radius becomes larger for the

case $m = 0$, which strongly suggests what exists are the stable soliton clusters for $m = \pm 1$ but not for $m = 0$.[32] And as is known, the interaction between the solitons is always attractive in a strongly nonlocal nonlinear medium.[2] So, if the attraction is balanced by the repulsive centrifugal force introduced by the staircase-like phase distribution, the cluster will rotate steadily at a critical value of $d_{\rm cr}$.[28,29] Around the optimum value of $d_{\rm cr}$, the soliton cluster will either expand or shrink, and these phenomena are also confirmed in Figs.1(a) and 1(c), respectively. Comparing the simulations here with Fig.1, we arrive at the conclusion that the analytical results we obtained are in good agreements with our simulations of Eq.(1).

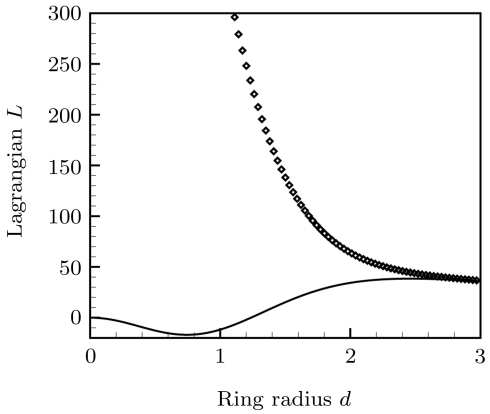


Fig.3. Profiles of the Lagrangian versus the ring radius, where $m = \pm 1$ are for the solid curves, and $m = 0$ is for the diamond curve.

4.2. Discussion about the angular speed for the soliton clusters

The results of our numerical simulations can also be illustrated through the analysis of angular speed that is given as

$$\omega = \frac{M}{I}, \tag{13}$$

where $I = \iint |\psi|^2 r^2 \mathrm{d}^2\boldsymbol{r} = Pd_{\rm eff}^2$ is the cluster's moment of inertia, and

$$M = \frac{\mathrm{i}}{2} \iint \boldsymbol{r} \times (\psi \nabla \psi^* - \psi^* \nabla \psi) \mathrm{d}^2\boldsymbol{r} \tag{14}$$

is the angular momentum.

Figure 4 shows the profile of the angular speed of three beams. It can clearly be seen that $\omega = 0$ if $d \geq 3$, from which we can learn that as the ring radius becomes larger, the solitons will not rotate any more but experience strong attraction irrespective of their phase differences. And our simulations also confirm this phenomenon just as shown in Fig.1(d). If the evolution of the cluster is stationary, Fig.1(b) shows that the numerically obtained value of the angular speed is $\omega = \pi/10 = 0.314$, while expression (13) gives a similar value $\omega = 0.357$. Since $M = I\omega$, the power P and the angular momentum M are conserved,[28,29] ω must go down with $d_{\rm eff}^2$,[33] and this analysis can also explain the fact that the soliton clusters only experience the periodic collisions, i.e. zero angular speed, when the ring radius is large enough, while the soliton clusters keep their stationary rotation at a short distance. Through the calculation of the angular momentum, we find that $M = 0$ for the case $m = 0$, then $\omega = 0$, so the profile is only for the case $m = \pm 1$.

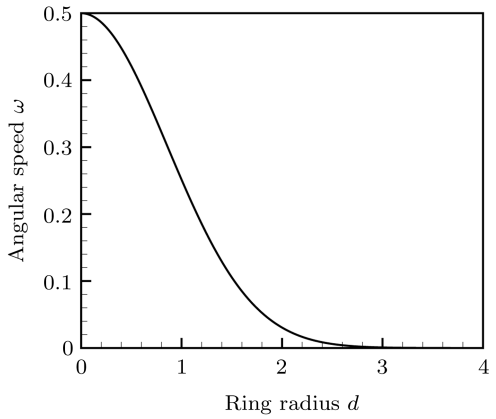


Fig.4. Profile of the angular speed versus the ring radius.

4.3. Proof of the direction of the rotation for the soliton clusters

We also find that the direction of the rotation is determined by the topological charges. This phenomenon can be understood through the analysis of the angular momentum. The result of the angular momentum can be obtained by substituting expressions (2) and (3) into expression (14) as follows:[28,29]

$$M = \pi A^2 \sum_{n,k=1}^{N} \exp(Y_{nk}) |\boldsymbol{r}_n \cdot \boldsymbol{r}_k| \sin\theta_{nk}, \tag{15}$$

where $Y_{nk} = -(d/a)^2 \sin^2\left[\pi(n-k)/N\right]$ and $\theta_{nk} = \alpha_n - \alpha_k$. From expression (15) we know that the sign of the angular momentum is determined by $\sin\theta_{nk}$, i.e. the directions of the rotation for the solitons are different when the difference in phase between adjacent solitons is within the domains: $0 < |\theta| < \pi$ and

$\pi < |\theta| < 2\pi$. Our following simulations also confirm these predictions.

As clearly depicted in Fig.5, their interaction results in a counterclockwise rotation with respect to the direction of phase gradient about the centre of the beams when $m = 1$ while clockwise with respect to the direction of phase gradient when $m = -1$. It should be emphasized that this rotating motion is due to the net angular momentum induced by the staircase-like phase distribution, and it can occur even though it is launched without any tangential velocity.

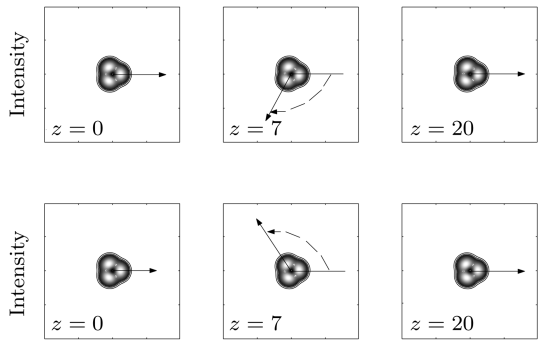


Fig.5. The soliton clusters rotate steadily against for $(m = 1)$ (a) and along the direction of the phase gradient for $(m = -1)$ (b), with parameters $N = 3$ and $d = 0.7$.

4.4. Extension of our analysis to the soliton clusters composed of more solitons

The following analysis will be valid for any number of solitons, N. Here, just take $N = 4$ for example. With a Gaussian-type response, Fig.6 shows the profiles of the Lagrangian of the clusters composed of four solitons with different phase intervals.

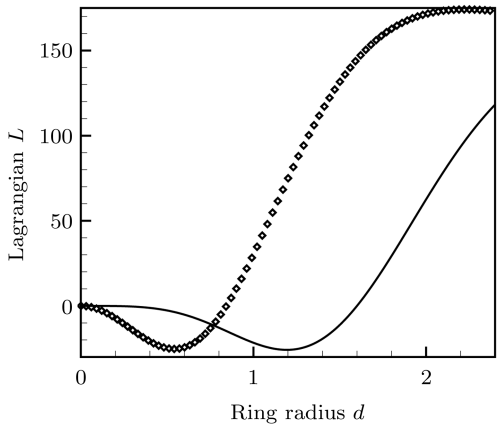
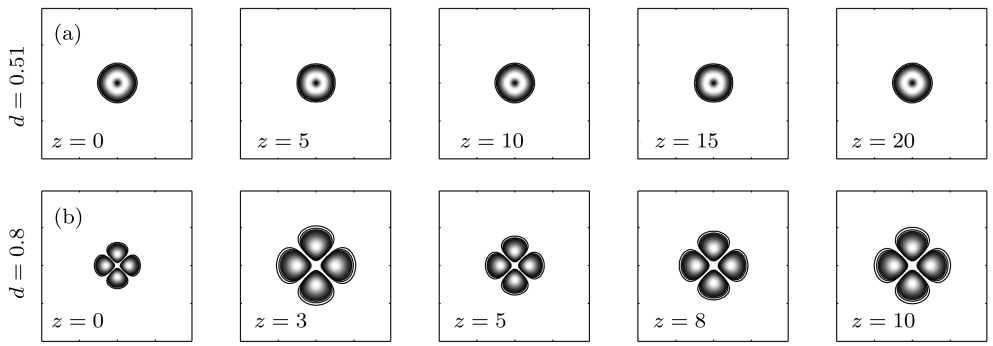


Fig.6. Profiles of the Lagrangian versus the ring radius for the soliton clusters composed of four fundamental solitons with the difference in phase between adjacent solitons being $\pi/2$ (diamond curve) and π (solid curve) separately.

For the case in which the difference in phase between neighbouring solitons is $\pi/2$, the analytical result shows that the stationary rotation happens at $d_{\rm cr} = 0.54$ (Fig.6), which is in very good agreement with our simulations in which a bound state of steady rotation exists at $d = 0.51$ (Fig.7(a)). From expression (15) we can also know that the soliton clusters will carry zero angular momentum in the special case of $m = kN$ $(k = 0, 1, 2, \cdots)$ for all solitons and the general case of $m = (2k + 1)N/2$ $(k = 0, 1, 2, \cdots)$ for even solitons, so for the case in which the difference in phase between adjacent solitons is π and the ring radius d has an appropriate value, the solitons do not possess a transverse energy flow and thus the evolution of the soliton cluster is absolutely stationary without any rotation. Figure 6 shows that the stationary evolution of the solitons with a π phase difference happens at $d_{\rm cr} = 1.19$, which is in good agreement with our numerical simulations in which a stationary non-rotating motion happens at $d = 1.05$ as shown in Fig.7(c). The solitons will either expand or shrink if $d < 1.05$ (see Fig.7(b)) or $d > 1.05$ (see Fig.7(d)) respectively.



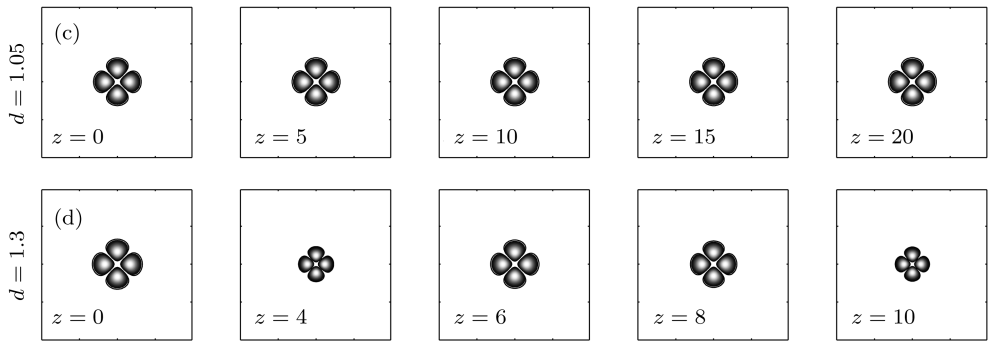


Fig.7. Evolution of the soliton clusters (without the noise added) with $N = 4$ at different input distances. Panel (a) shows the stationary evolution for the soliton cluster whose difference in phase between neighbouring solitons is $\pi/2$. Panel (b) indicates the solitons initially expanding when $d = 0.8$. Panel (c) exhibits the exact stationary bound state existing for the case $d = 1.05$. Panel (d) displays the solitons contracting when $d = 1.3$. For the non-rotating cases (b)–(d), the difference in phase between neighbouring solitons is π.

As mentioned above, our analytical results for the soliton clusters are in good agreement with all of our numerical simulations.

5. Conclusion

We have mainly investigated the dynamics of the ring-like soliton cluster composed of three fundamental solitons in a strongly nonlocal nonlinear medium. We find that there is a critical value of the ring radius $d_{\rm cr}$ which leads to the scenarios of expanding and shrinking solitons. And the soliton cluster whose phase and ring radius are appropriately constructed can rotate steadily even in the presence of relatively strong perturbations. We find that there exist no stationary bound states for in-phase soliton clusters. We also find that if the ring radius becomes large enough (usually several beam widths), the soliton clusters will not rotate any more but only undergo periodic collisions in the form of simple harmonic oscillator about centre of the beams, irrespective of their difference in phase. We show that when the choice of the topological charge makes the difference in phase between neighbouring solitons be somewhere between 0 and π, the soliton cluster will rotate against the direction of phase gradient, otherwise it will be along the direction of phase gradient if the phase difference is within the domain $\pi < |\theta| < 2\pi$. However, for the case in which the choice of the topological charge makes the difference in phase between adjacent solitons be tantamount to π, we find that an appropriate choice of ring radius can make the clusters composed of even solitons propagate stationarily without any rotation. These important properties of solitons imply their potential applications in all-optical signal processing, and all-optical switching and logic gating by using this kind of interaction.

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