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Distillation of bi-partite entanglement from W state with cavity QED*

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Following the theoretical protocol described by Fortescue and Lo [Fortescue B and Lo H K 2007 *Phys. Rev. Lett.* **98** 260501], we present a scheme in which one can distill maximally entangled bi-partite states from a tri-partite W state with cavity QED. Our scheme enables the concrete physical system to realize its protocol. In our scheme, the rate distillation also asymptotically approaches one. Based on the present cavity QED techniques, we discuss the experimental feasibility.

Keywords: entanglement distillation, W state, cavity QED
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In the field of quantum information, quantum entanglement[1−4] plays an important role. Its nonlocality can provide powerful tools for testing local hidden theory against quantum mechanics. In addition, quantum entanglement has been taken as an important resource and extensively used in some processes of quantum information processing, such as teleportation of quantum state,[5,6] remote state preparation,[7,8] quantum secrete communication[9,10] and so on. In quantum information processing, maximally entangled states, e.g. Einstein–Podolsky–Rosen (EPR) states or Greenberger–Horne–Zeilinger (GHZ) states, are required for the quantum channels. However, the decoherence effect, due to the environment or dissipation of system, changes the pure entangled state into a statistical mixture and degrades quantum entanglement in real word. In order to get maximally entangled states, some authors have proposed schemes on entanglement concentration or entanglement distillation.[11−13]

Nowadays, entanglement of bi-partite states is very well studied, but multi-partite entanglement is less understood. For multi-partite entangled states, GHZ state and W state are know. Considering the system of three-party, we know the GHZ state is a weak entanglement in the sense that if one party is lost, then all the entanglement disappears. However, W state retains entanglement even if one particle is lost.[14] Comparing W state with GHZ state, one can distill maximally entangled bi-partite states from a tripartite W state. Recently, some authors have proposed protocols of distillation from W state or W-like state. For the EPR state, the standard measure of entanglement is the Von Neumann entropy. For the tri-partite W state, if two parties want to distill EPR state with the help of a third party, the maximum distillation rate (that is, the maximum number of EPR distilled per W) they can get is 0.92,[15] which is obtained by calculating the Von Neumann entropy. The latest studies described the highly efficient protocol, where one can distill 1 EPR pair per W state.[16] If the highly efficient protocol in Ref.[16] can be realized in a concrete physical system, it provides experimental feasibility for distillation. In this paper, we follow the Fortescue and Lo's protocol and employ the cavity QED technique to realize the distillation of EPR state from the W state with the maximum rate 1.

In this paper, we choose ladder-type three-level atoms, whose states can be expressed as $|g\rangle$, $|e\rangle$ and $|i\rangle$, where the $|g\rangle$ is the ground state, $|e\rangle$ and $|i\rangle$ are two upper energy levels. In our scheme, we need to obtain a three-atom W state. Based on the scheme of preparation of W state,[17] we assume the three-atom W state to be expressed as

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$$|W\rangle_{123} = \frac{1}{\sqrt{3}}(|e\rangle_1|e\rangle_2|g\rangle_3 + |e\rangle_1|g\rangle_2|e\rangle_3 + |g\rangle_1|e\rangle_2|e\rangle_3). \quad (1)$$

The next problem is how to distill maximally entangled bi-partite state from this W state with cavity QED. Assuming each atom has an auxiliary level $|i\rangle$. $|g\rangle$, $|e\rangle$ and $|i\rangle$ constitute a ladder-level model. Through choosing suitable cavity field, we can obtain resonant atom-field interaction that occurs between $|e\rangle$ and $|i\rangle$. This kind of interaction[18,19] can be regarded as the Jaynes–Cummings (JCM) type. In the interaction picture, the atom-field interaction is described by the JCM Hamiltonian

$$H_{\rm I} = \lambda(a^-|i\rangle\langle e| + a^+|e\rangle\langle i|), \quad (2)$$

where a^- and a^+ are the annihilation and creation operators for the cavity mode, respectively, and λ is the atom-field coupling strength. Our process of distillation includes the following three steps:

Step 1 Let atom 1 cross classical field 1, atom 2 cross classical field 2, atom 3 cross classical field 3, simultaneously, where the classical field is tuned to the transition between the states $|e\rangle$ and $|i\rangle$. Each atom undergoes the transition

$$\begin{aligned} &|g\rangle \longrightarrow |g\rangle, \\ &|e\rangle \longrightarrow C_e(t)|e\rangle + C_i(t)|i\rangle, \end{aligned} \quad (3)$$

where $C_e(t)$ and $C_i(t)$ are the probability amplitudes.

Then, the state $|\Psi\rangle_{123}$ of atoms 1, 2 and 3 evolves to

$$\begin{aligned} &|W\rangle_{123} \\ \longrightarrow\ &|\Psi\rangle_{123} \\ =\ &C_e^2(t)|W\rangle_{123} + \frac{C_e(t)C_i(t)}{\sqrt{3}}(|e\rangle_1|i\rangle_2|g\rangle_3 \\ &+|i\rangle_1|e\rangle_2|g\rangle_3 + |e\rangle_1|g\rangle_2|i\rangle_3 \\ &+|i\rangle_1|g\rangle_2|e\rangle_3 + |g\rangle_1|e\rangle_2|i\rangle_3 \\ &+|g\rangle_1|i\rangle_2|e\rangle_3) + O(C_i^2(t)), \end{aligned} \quad (4)$$

where

$$O(C_i^2(t)) = \frac{C_i^2(t)}{\sqrt{3}}(|i\rangle_1|i\rangle_2|g\rangle_3 + |i\rangle_1|g\rangle_2|i\rangle_3 + |g\rangle_1|i\rangle_2|i\rangle_3). \quad (5)$$

Step 2 Send atom 1 through cavity field 1, atom 2 through cavity field 2, and atom 3 through cavity 3, simultaneously, where three cavity fields are initially in the vacuum state $|0\rangle_{c_j}$ $(j = 1, 2, 3)$, and let the three atoms undergo a π pulse, respectively. The state of the whole system can be expressed as

$$\begin{aligned} |\Psi\rangle_{123} =\ &C_e^2(t)|W\rangle_{123}|0\rangle_{c_1}|0\rangle_{c_2}|0\rangle_{c_3} \\ &+\frac{C_e(t)C_i(t)}{\sqrt{3}}(|e\rangle_1|e\rangle_2|g\rangle_3|0\rangle_{c_1}|1\rangle_{c_2}|0\rangle_{c_3} \\ &+|e\rangle_1|e\rangle_2|g\rangle_3|1\rangle_{c_1}|0\rangle_{c_2}|0\rangle_{c_3} \\ &+|e\rangle_1|g\rangle_2|e\rangle_3|0\rangle_{c_1}|0\rangle_{c_2}|1\rangle_{c_3} \\ &+|e\rangle_1|g\rangle_2|e\rangle_3|1\rangle_{c_1}|0\rangle_{c_2}|0\rangle_{c_3} \\ &+|g\rangle_1|e\rangle_2|e\rangle_3|0\rangle_{c_1}|0\rangle_{c_2}|1\rangle_{c_3} \\ &+|g\rangle_1|e\rangle_2|e\rangle_3|0\rangle_{c_1}|1\rangle_{c_2}|0\rangle_{c_3}. \end{aligned} \quad (6)$$

Step 3 The next step is to detect atom or cavity field. Firstly, we measure the states of the three cavity fields, and four main measurement outcomes are obtained. If the measurement outcome is $|0\rangle_{c_1}|0\rangle_{c_2}|0\rangle_{c_3}$, the state of three atoms is in $|W\rangle_{123}$ again, then step 1 is repeated.

If measurement outcome is $|0\rangle_{c_1}|1\rangle_{c_2}|0\rangle_{c_3}$, then three atoms are in

$$|e\rangle_1|e\rangle_2|g\rangle_3 + |g\rangle_1|e\rangle_2|e\rangle_3. \quad (7)$$

If measurement outcome is $|1\rangle_{c_1}|0\rangle_{c_2}|0\rangle_{c_3}$, then three atoms are in

$$|e\rangle_1|e\rangle_2|g\rangle_3 + |e\rangle_1|g\rangle_2|e\rangle_3. \quad (8)$$

If measurement outcome is $|0\rangle_{c_1}|0\rangle_{c_2}|1\rangle_{c_3}$, then three atoms are in

$$|e\rangle_1|g\rangle_2|e\rangle_3 + |g\rangle_1|e\rangle_2|e\rangle_3. \quad (9)$$

For Eq.(7), a single measurement on the state of atom 2 is performed in the basis $|e\rangle_2$, then the state of atoms 1 and 3 becomes

$$|\psi\rangle_{13} = \frac{1}{\sqrt{2}}(|e\rangle_1|g\rangle_3 + |g\rangle_1|e\rangle_3), \quad (10)$$

where $1/\sqrt{2}$ is the normalization coefficient. Obviously, Eq.(10) stands for a bi-partite entangle state. By symmetry, for Eq.(8) or (9), we can perform a measurement on atom 1 or 2 with basis $|e\rangle_1$ or $|e\rangle_3$, respectively. If the measurement is $|e\rangle_1$, then atoms 2 and 3 are in

$$|\psi\rangle_{23} = \frac{1}{\sqrt{2}}(|e\rangle_2|g\rangle_3 + |g\rangle_2|e\rangle_3). \quad (11)$$

If the measurement outcome is $|e\rangle_3$, then the state of atoms 1 and 2 is given by

$$|\psi\rangle_{12} = \frac{1}{\sqrt{2}}(|e\rangle_1|g\rangle_2 + |g\rangle_1|e\rangle_2). \quad (12)$$

So we distill an EPR state from the W state. Because there are the terms $O(C_i^2(t))$ in the Eq.(4), when the operations listed in step 2 on three atoms

are performed, three cavity fields maybe be in $|1\rangle_{c_1}|1\rangle_{c_2}|0\rangle_{c_3}$, $|1\rangle_{c_1}|0\rangle_{c_2}|1\rangle_{c_3}$ or $|0\rangle_{c_1}|1\rangle_{c_2}|1\rangle_{c_3}$ with probability $C_i^4(t)/3$. In this case, the state of three-atom is in a product state and distillation of EPR state from W state can not be realized. However, choosing the appropriate amplitude of the classical field and suitable interaction time, the probability amplitude $C_i(t)$ is arbitrarily small. Based on the protocol of distillation, steps 1, 2 and 3 are performed D rounds,[16] and distillation rate is asymptotically 1. In this scheme, the procedure of distillation requires a continual recycling. In order to improve the efficiency of performance, the W state needs to be repeatedly prepared.

In summary, by cavity QED technique, we realize the distillation of EPR from the three-atom W state. Our scheme follows the latest idea proposed by Fortescue and Lo. In this paper, we adopt the concrete physical system to realize the process of distillation. Based on this scheme, we can also distill an EPR state from a three-atom W-like state, and also extend to distill a N-party GHZ state from $(N+1)$-party W state. Our scheme opens the promising prospect for highly efficient distillation. Finally, we briefly discuss the feasibility of experiment of our scheme. Recently, the cavity QED technique is extensively applied in the field of quantum information and computation, and important development was achieved in experimental studies on cavity QED. For the Rydberg atoms with principal quantum numbers 50 and 51, atom in cavity combines a huge dipole coupling to a single photon with lifetime 3.0×10^{-2} s.[20] Thus the interaction time of the atom with cavity field is 4.0×10^{-5} s. Then the time required to complete the procedure can be assumed to be 4.0×10^{-4} s, much shorter than 3.0×10^{-2} s. Therefore, by the present or future cavity QED techniques, our scheme might be experimentally feasible.

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References

[1] Cirac J I and Zoller P 1994 *Phys. Rev.* A **50** R2799
Zeng K and Fang M F 2005 *Chin. Phys.* **14** 2009
[2] Wu Y, Payne M G, Hagley E W and Deng L 2004 *Phys. Rev.* A **69** 063803
Wu Y and Deng L 2004 *Opt. Lett.* **29** 1144
[3] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392
Yang W X, Zhan Z M and Li J H 2005 *Phys. Rev.* A **72** 062108
[4] Kempe J 1999 *Phys. Rev.* A **60** 910
Sun Y H and Kuang L M 2006 *Chin. Phys.* **15** 681
[5] Cirac J I and Parkins A S 1994 *Phys. Rev.* A **50** R4441
Zheng S B 2005 *Chin. Phys.* **14** 1825
[6] Moussa M H Y 1997 *Phys. Rev.* A **55** R3287
[7] Chen A X, Li J H and Jin L X 2005 *Commun. Theor. Phys.* (Beijing) **43** 631
Lin X, Li H C, Lin X M, Li X H and Yang R C 2007 *Chin. Phys.* **16** 1209
[8] Pati A K 2002 *Phys. Rev.* A **63** 014302
Chen A X and Li J H 2005 *Chin. Phys.* **14** 1507
[9] Cai Q Y and Li B W 2004 *Chin. Phys. Lett.* **21** 601
Deng F G, Li X H, Li C Y, Zhou P and Zhou H Y 2007 *Chin. Phys.* **16** 277
[10] Cai Q Y 2003 *Phys. Rev. Lett.* **91** 109801
[11] Groisman B, Linden N and Popescu S 2005 *Phys. Rev.* A **72** 062322
[12] Miyake A and Briegel H J 2005 *Phys. Rev. Lett.* **95** 220501
[13] Lo H Kand Popescu S 2001 *Phys. Rev.* A **63** 022301
[14] Cirac J I, Duan L and Zoller P 2004 quant-ph/0405030
[15] Smolin J A, Verstraete F and Winter A 2005 *Phys. Rev.* A **72** 052317
[16] Fortescue B and Lo H K 2007 *Phys. Rev. Lett.* **98** 260501
[17] Guo G P, Li C F, Li J and Guo G C 2002 *Phys. Rev.* A **65** 042102
[18] Wu Y 1996 *Phys. Rev.* A **54** 4534
[19] Peng J S and Li G X 1993 *Phys. Rev.* A **47** 3167
[20] Kaiser P, Westbrook C and David F 2001 *Coherent Atomic Matter Waves* (Berlin: Springer) Chap.9