Vol 17 No 7, July 2008
1674-1056/2008/17(07)/2509-05
Chinese Physics B
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The number of least degrees of freedom required for a polarization controller to transform any state of polarization to any other output covering the entire Poincaré sphere*

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(Received 12 October 2007; revised manuscript received 3 January 2008)

Using two typical types of polarization controller, this paper analyses theoretically and experimentally the fact that it is necessary to adjust at least three instead of two waveplates in order to transform any state of polarization to any other output covering the entire Poincaré sphere. The experimental results are exactly in accordance with the theory discussed in this paper. It has corrected the conventional and inaccurate point of view that two waveplates of a polarization controller are adequate to complete the transformation of state of polarization.

Keywords: polarization controller, degree of freedom, state of polarization
PACC: 4225, 4225J, 4281, 4281F

1. Introduction

Automatically controlled polarization controllers (PC) are of great interest for optical fibre communication systems to transform any state of polarization (SOP) to any other state. They are especially essential integral parts in an automatic compensator for polarization mode dispersion (PMD),[1−5] or PMD emulator.[6] It is widely believed that at least two degrees of freedom (DOF) for each PC are needed to complete the SOP transformation from any input state into any other state covering the entire Poincaré sphere.[1−3] Only for treating the reset-free problem, three DOFs are required.[7] As we know, the statement above is equivalent to the transform from any fixed input SOP into the output states covering the entire Poincaré sphere. In this paper, we will take two types of PC as examples to show theoretically and experimentally that except for the reset-free problem it is necessary to adjust at least three instead of two waveplates in order to transform a fixed SOP to output states covering the entire Poincaré sphere.

2. The theory

Two typical types of PC are shown in Fig.1. In Fig.1(a), a PC consists of a series of cascaded $\lambda/4$, $\lambda/2$, and $\lambda/4$ waveplate, which is a waveplate-fixed and angle-adjustable PC. Figure 1(b) shows a PC composed of four fibre-squeezer cells, which is an angle-fixed and retardation-adjustable PC (ϕ is proportional to the voltage applied to it).

The matrices for $\lambda/2$ plate, and $\lambda/4$ plate in the reference frame of laboratory are described as Eqs.(1) and (2) respectively[8]

$$\begin{pmatrix} \cos\theta_{\rm h} & \sin\theta_{\rm h} \\ -\sin\theta_{\rm h} & \cos\theta_{\rm h} \end{pmatrix} \begin{pmatrix} \exp\left(\mathrm{j}\frac{\pi}{2}\right) & 0 \\ 0 & \exp\left(-\mathrm{j}\frac{\pi}{2}\right) \end{pmatrix} \begin{pmatrix} \cos\theta_{\rm h} & -\sin\theta_{\rm h} \\ \sin\theta_{\rm h} & \cos\theta_{\rm h} \end{pmatrix}, \tag{1}$$

*Project supported by the National Natural Science Foundation of China (Grant No 60577046), the Corporative Building Project of Beijing Educational Committee of China (Grant No XK100130737) and Shandong High Technology Project of China (Grant No 2006GG2201002).
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$$\begin{pmatrix} \cos\theta_{\rm q} & \sin\theta_{\rm q} \\ -\sin\theta_{\rm q} & \cos\theta_{\rm q} \end{pmatrix} \begin{pmatrix} \exp\left(\mathrm{j}\frac{\pi}{4}\right) & 0 \\ 0 & \exp\left(-\mathrm{j}\frac{\pi}{4}\right) \end{pmatrix} \begin{pmatrix} \cos\theta_{\rm q} & -\sin\theta_{\rm q} \\ \sin\theta_{\rm q} & \cos\theta_{\rm q} \end{pmatrix}, \tag{2}$$

where $\theta_{\rm q}$ and $\theta_{\rm h}$ are the orientation angles of quarter-waveplate and half-waveplate respectively.

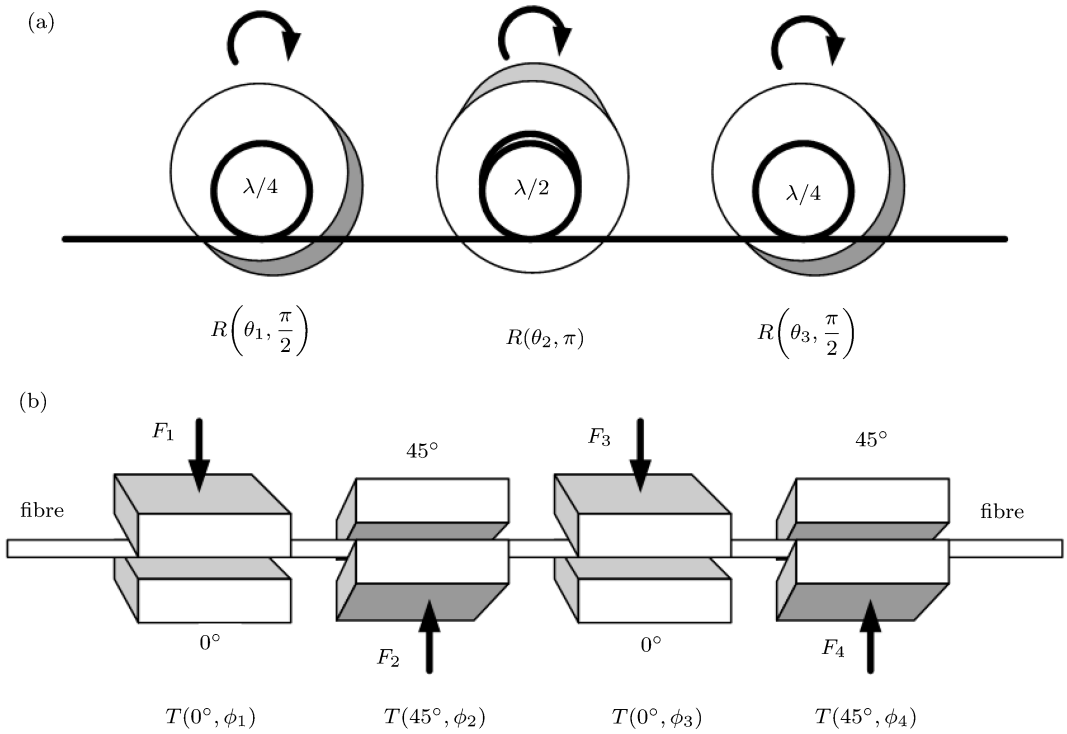


Fig.1. Two typical types of polarization controllers: (a) the waveplate-fixed and angle-adjustable PC, and (b) the angle-fixed and retardation-adjustable PC (for example PolaRITE$^{\rm TM}$ II, General Photonics Co).

If we only use a combination of a $\lambda/4$ plate and a $\lambda/2$ plate, the total transformation matrix is

$$T(\theta_1,\theta_2) = \frac{\sqrt{2}}{2} \begin{pmatrix} -\cos(2\theta_1 - 2\theta_2) + \mathrm{j}\cos(2\theta_2) & \sin(2\theta_1 - 2\theta_2) - \mathrm{j}\sin(2\theta_2) \\ -\sin(2\theta_1 - 2\theta_2) - \mathrm{j}\sin(2\theta_2) & -\cos(2\theta_1 - 2\theta_2) - \mathrm{j}\cos(2\theta_2) \end{pmatrix}. \tag{3}$$

The corresponding Mueller matrix in Stokes space is[9]

$$M(\theta_1,\theta_2) = \begin{pmatrix} \cos(2\theta_1)\cos(2\theta_1 - 4\theta_2) & -\sin(2\theta_1)\cos(2\theta_1 - 4\theta_2) & \sin(2\theta_1 - 4\theta_2) \\ \cos(2\theta_1)\sin(2\theta_1 - 4\theta_2) & -\sin(2\theta_1)\sin(2\theta_1 - 4\theta_2) & -\cos(2\theta_1 - 4\theta_2) \\ \sin(2\theta_1) & \cos(2\theta_1) & 0 \end{pmatrix}. \tag{4}$$

If the input light is circularly polarized, and its Stokes vector is $(0,\ 0,\ \pm 1)^{\rm t}$, the output SOP has the form of $(\pm\sin(2\theta_1 - 4\theta_2),\ \mp\cos(2\theta_1 - 4\theta_2),\ 0)^{\rm t}$, which will form a ring in $S_1 - S_2$ plane by adjusting θ_1 and θ_2.

Figure 2 shows this SOP transformation procedure. When passing through the first $\lambda/4$ plate whose orientation angle from the vertical direction in the frame of reference of laboratory is θ_1, a right-hand circularly polarized light at the north pole A rotates about axis $\overline{2\theta_1}$ along the quarter-circle ($\pi/2$) (a longitude line) to the equator point B. And then point B rotates about axis $\overline{2\theta_2}$ along the half-circle (π) (not necessarily a longitude line) to another equator point C as the result of passing through the second $\lambda/2$ plate. We can conclude from Fig.2 that whatever the orientations (θ_1 and θ_2) of the $\lambda/4$ plate and the $\lambda/2$ plate are, the output SOP C locates on the equator, forming a ring, leaving behind a large blind region no matter how θ_1 and θ_2 are adjusted.

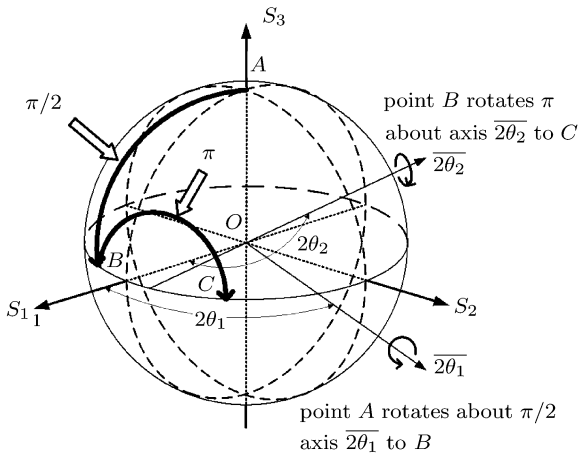


Fig.2. SOP transformations by two cascaded $\lambda/4$ and $\lambda/2$ plate.

If we use the entire $\lambda/4 - \lambda/2 - \lambda/4$ waveplate, the total transformation matrix is:

$$T(\theta_1, \theta_2, \theta_3) = \begin{pmatrix} -\cos\alpha\cos\beta - \mathrm{j}\sin\beta\sin\gamma & \sin\alpha\cos\beta - \mathrm{j}\sin\beta\cos\gamma \\ -\sin\alpha\cos\beta - \mathrm{j}\sin\beta\cos\gamma & -\cos\alpha\cos\beta + \mathrm{j}\sin\beta\sin\gamma \end{pmatrix}, \tag{5}$$

where $\alpha = \theta_1 - \theta_3$, $\beta = 2\theta_2 - (\theta_1 + \theta_3)$, $\gamma = \theta_1 + \theta_3$. The corresponding Mueller matrix in Stokes space is

$$M(\theta_1, \theta_2, \theta_3) = \begin{pmatrix} \cos(2\alpha)\cos^2\beta - \cos(2\gamma)\sin^2\beta & -\sin(2\alpha)\cos^2\beta + \sin(2\gamma)\sin^2\beta & -\sin(2\beta)\cos(\alpha-\gamma) \\ \sin(2\alpha)\cos^2\beta + \sin(2\gamma)\sin^2\beta & \cos(2\alpha)\cos^2\beta + \cos(2\gamma)\sin^2\beta & -\sin(2\beta)\sin(\alpha-\gamma) \\ \sin(2\beta)\cos(\alpha+\gamma) & -\sin(2\beta)\sin(\alpha+\gamma) & \cos(2\beta) \end{pmatrix}. \tag{6}$$

If we input an arbitrary SOP of $(\cos\chi\cos\varepsilon, \cos\chi\sin\varepsilon, \sin\varepsilon)^{\mathrm{t}}$ where χ and ε represent its azimuth and ellipticity respectively, the output SOP has the form of matrix with non-zero and angle-adjustable elements, as follows:

$$\begin{pmatrix} (\cos(2\alpha)\cos^2\beta - \cos(2\gamma)\sin^2\beta)\cos\chi\cos\varepsilon - (\sin(2\alpha)\cos^2\beta - \sin(2\gamma)\sin^2\beta)\cos\chi\sin\varepsilon - \sin(2\beta)\cos(\alpha-\gamma)\sin\varepsilon \\ (\sin(2\alpha)\cos^2\beta + \sin(2\gamma)\sin^2\beta)\cos\chi\cos\varepsilon + (\cos(2\alpha)\cos^2\beta + \cos(2\gamma)\sin^2\beta)\cos\chi\sin\varepsilon - \sin(2\beta)\sin(\alpha-\gamma)\sin\varepsilon \\ \sin(2\beta)\cos(\alpha+\gamma)\cos\chi\cos\varepsilon - \sin(2\beta)\sin(\alpha+\gamma)\cos\chi\sin\varepsilon + \cos(2\beta)\sin\varepsilon \end{pmatrix}. \tag{7}$$

It can be proved that all the output SOP points can cover the entire Poincaré sphere.

As a second example, we shall analyse the angle-fixed and retardation-adjustable PC in our laboratory (PolaRITE$^{\mathrm{TM}}$ II, General Photonics Co.) shown in Fig.1(b). The PC is composed of four retardation waveplates orientated in the directions of 0°, 45°, 0°, and 45° as is shown in Fig.1(b). Here we only deal with three out of four waveplates, because the fourth is set for reset-free problem. The three phase retardations are assumed as $2\phi_1$, $2\phi_2$, $2\phi_3$, any of which is linearly proportional to the control voltages applied to them. The Jones matrices of the three waveplates are

$$\begin{pmatrix} \exp(\mathrm{j}\phi_1) & 0 \\ 0 & \exp(-\mathrm{j}\phi_1) \end{pmatrix}, \tag{8}$$

$$\begin{pmatrix} \cos\phi_2 & -\mathrm{j}\sin\phi_2 \\ -\mathrm{j}\sin\phi_2 & \cos\phi_2 \end{pmatrix}, \tag{9}$$

$$\begin{pmatrix} \exp(\mathrm{j}\phi_3) & 0 \\ 0 & \exp(-\mathrm{j}\phi_3) \end{pmatrix}, \tag{10}$$

respectively.

If we only use the first two waveplates, the total transformation matrix is:

$$T(\phi_1,\phi_2)=\begin{pmatrix}\cos\phi_2\exp(\mathrm{j}\phi_1) & -\mathrm{i}\sin\phi_2\exp(-\mathrm{j}\phi_1)\\ -\mathrm{i}\sin\phi_2\exp(\mathrm{j}\phi_1) & \cos\phi_2\exp(-\mathrm{j}\phi_1)\end{pmatrix}. \tag{11}$$

The corresponding Mueller matrix in Stokes space is

$$M(\phi_1,\phi_2)=\begin{pmatrix}\cos(2\phi_2) & -\sin(2\phi_1)\sin(2\phi_2) & \cos(2\phi_1)\sin(2\phi_2)\\ 0 & \cos(2\phi_1) & \sin(2\phi_1)\\ -\sin(2\phi_2) & -\sin(2\phi_1)\cos(2\phi_2) & \cos(2\phi_1)\cos(2\phi_2)\end{pmatrix}. \tag{12}$$

If the input light is linearly polarized, having its Stokes vector $(1,0,0)^{\mathrm{t}}$, the output SOP has the form of $(\cos(2\phi_2),0,-\sin(2\phi_2))^{\mathrm{t}}$, which will form a ring in S_1-S_3 plane by adjusting ϕ_1 and ϕ_2.

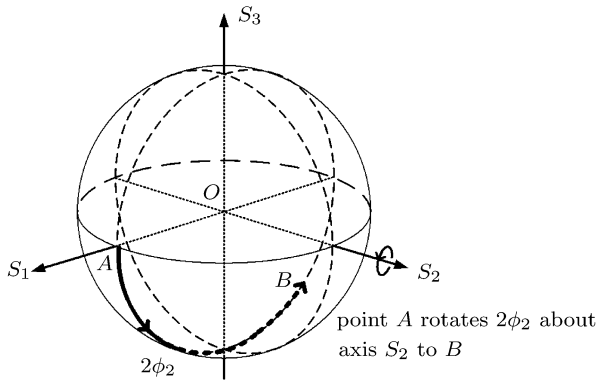


Fig.3. SOP transformation by two angle-fixed and retardation-adjustable waveplates.

In Fig.3, a linearly polarized state A lies on axis S_1. The first 0° waveplate cannot move it by adjusting $2\phi_1$, since the rotation of A about the axis S_1 makes no change. After passing through the second 45° waveplate, the SOP A rotates along a vertical arc of $2\phi_2$ to output B about the axis S_2. Therefore, a vertical ring will be formed by adjusting the control voltage relative to ϕ_2.

Using similar analysis, we can prove that the three-waveplate angle-fixed and retardation-adjustable PC can do complete transformation among any of the SOPs.

We can conclude from the analysis of the above two types of polarization controller that 3-waveplate PC can do complete transformation among any of the SOPs. Therefore, at least for the widely used types of PC, 2-waveplates are not adequate. The fourth waveplate in Fig.1(b) is designed for the reset-free problem, which is not the subject of this paper.

3. The experiment

The above conclusion has been confirmed by our designed experiment. The experimental set up is shown in Fig.4. A GS-DFB laser of wavelength 1563.8nm was modulated by a 2.5GHz microwave, generating a 2.5Gb/s optical pulse stream of pulsewidth 20ps. A fibre based polarizer was used to generate a linearly polarized optical signal. The SOP of optical signal was transformed to other output states through the fibre-squeezed polarization controller (PolaRITE$^{\mathrm{TM}}$ II), which was controlled by a computer. A set of random control voltages (V_1, V_2) or (V_1, V_2, V_3) were generated by the computer in order for the PC to transform a fixed input SOP into all other random output SOPs. The selection of control voltages (V_1, V_2) or (V_1, V_2, V_3) depended on whether 2-waveplate or 3-waveplate was used. The output SOPs were detected instantly by an in-line polarimeter (PolaDetect$^{\mathrm{TM}}$, General Photonics Co.), and collected by the computer through an A/D card.

The experimental results are shown in Fig.5. When we randomly adjusted the three waveplates of the PC, the output SOPs covered the entire Poincaré sphere uniformly (Fig.5(b)). On the contrary, when only the two waveplates of the PC were adjusted, a wheel-like ribbon was formed with S_2 as its central axis, which was very similar to the case in Fig.3. Moreover, the blind areas existing on the Poincaré sphere proved that the three-waveplate PC other than the two-waveplate one can make a complete SOP transformation.

These results are exactly in accordance with the theory discussed above. As for Fig.5(a), a wheel-like ribbon other than a ring was formed. The reason was that the input SOPs in the fibres have fluctuations around point A in Fig.3.

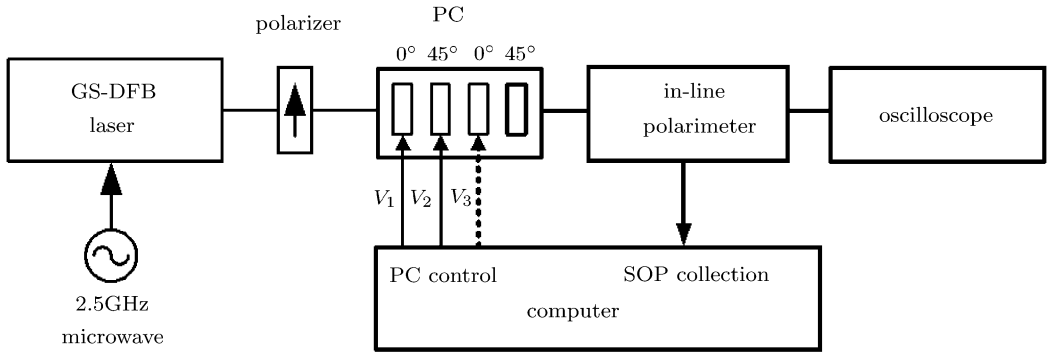


Fig.4. The experiment setup.

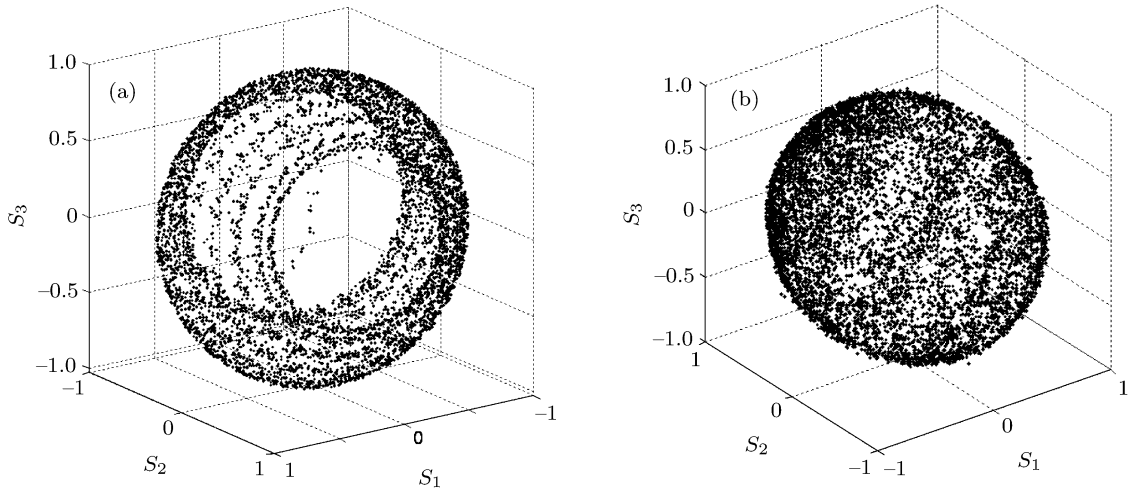


Fig.5. The output SOPs on the Poincaré sphere obtained by controlling (a) two waveplates and (b) three waveplates.

4. Conclusion

By analysing two types of generally used polarization controllers, we have proved analytically and experimentally that at least three instead of two waveplates are needed to achieve the goal of transformation from any input SOP into any other output SOP. We have corrected the conventional and inaccurate point of view that two waveplates of a PC are adequate to complete the SOP transformation from any input state into any other state covering the entire Poincaré sphere.

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