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Refraction of extraordinary rays and ordinary rays in the Savart polariscope*

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The refraction of rays in the Savart polariscope is different from the isotropic medium. We have analysed and discussed the refraction of rays in the Savart polariscope on the basis of the Snell law. The refraction formulae of the extraordinary rays and ordinary rays were derived. Results obtained may provide theoretical and practical guide lines for studying, developing and engineering of polarization interference imaging spectrometer.

Keywords: Savart polariscope, refraction, Snell law, extraordinary rays, ordinary rays
PACC: 4210Q, 4110H, 0765

1. Introduction

The Savart polariscope is the key component in the wide-field-of-view polarization interference imaging spectrometer (WPIIS).[1,2] The Savart polariscope consists of two identical uniaxial crystals that are cut so that their optical axes are aligned at 45° to the optical axis of the system.[3,4] Figure 1 shows the coordinate system of this polariscope. The optical axis of the second crystal is perpendicular to the first one (we use two identical plates with their principal section crossed). In the first plate the ordinary ray (o-ray) with its vibrations perpendicular to the principal section becomes an extraordinary ray (e-ray) in the second plate since its vibrations are in the principal section of the second plate.[5−7] Likewise, the e-ray in the first plate becomes an o-ray in the second plate. The output rays are parallel laterally sheared but not longitudinally sheared, and the Savart polariscope splits the source into two virtual sources.[8−10]

Anisotropic uniaxial crystals exhibit double-refraction effect; that is, when a light beam is incident upon the interface of a crystal, in the crystal or from the air, the o-ray and the e-ray will have different refractions and different reflections. The o-ray propagates in the direction of the wave vector, and refracts and reflects as in an isotropic medium. With the e-ray, the direction of the wave vector and the direction of rays are not the same in general.

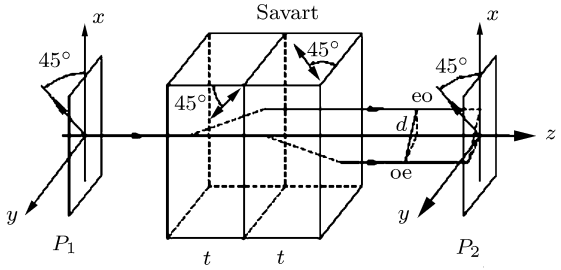


Fig.1. Savart polariscope.

2. The refraction of o-ray and e-ray in the Savart polariscope

An incident ray is split into an o-ray and an e-ray in the first plate, orthogonally polarized, then, the o-ray and e-ray go through the left plane and into the adhesive, which is isotropic medium. After the adhesive, the o-ray and e-ray enter the second uniaxial plate, splitting again, respectively.[11,12] So it may have four

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output rays: oo-ray, oe-ray, eo-ray and ee-ray when the rays propagate from right plate to the air.

We only discuss that condition for containing the principal section in the incident plane, which is $x-y$ plane, and the axis z is parallel to the normal of the crystal surface.

2.1. The refraction of o-ray and e-ray in the first crystal of Savart polariscope

In the first crystal of Savart polariscope, the direction of optical axis is expressed as follows:

$$\boldsymbol{c} = \frac{\sqrt{2}}{2}\boldsymbol{i} + \frac{\sqrt{2}}{2}\boldsymbol{k}. \tag{1}$$

Let θ_{i} be the angle between the incident wave and the interface normal, and θ_{r} be the angle between the refracted wave vector and the interface normal. These angles are limited to $-90^\circ < \theta_{\mathrm{i}} \leq 90^\circ$, $-90^\circ < \theta_{\mathrm{r}} \leq 90^\circ$ and measured positive toward the interface normal.[13,14] The subscript i refers to the incident wave while the subscript r refers to the refracted wave vector.

The law of refraction can be expressed as

$$n_1 \sin\theta_{\mathrm{i}} = n_2 \sin\theta_{\mathrm{r}}, \tag{2}$$

where n_1 denotes the absolute refractive index of the first medium, and n_2 the absolute refractive index of the second medium. The angle in the equation is between the wave vector and the normal of the medium, not the angle of refraction (between the refracted rays and the normal).

2.1.1. The refraction of the o-ray

The o-ray propagates in the direction of the wave vector and refracts and reflects as in an isotropic material, that is:

$$\theta_{\mathrm{o}} = \theta_{\mathrm{r}} = \arcsin(n_1 \sin\theta_{\mathrm{i}}/n_{\mathrm{o}}), \tag{3}$$

where n_{o} is the refractive index of the anisotropic uniaxial medium.

2.1.2. The refraction of the e-ray

(1) The wave vector of e-ray

For the e-ray, n_2 is expressed as

$$n_2 = \frac{n_{\mathrm{o}} n_{\mathrm{e}}}{\sqrt{n_{\mathrm{o}}^2 \sin^2\theta_{\mathrm{kp}} + n_{\mathrm{e}}^2 \cos^2\theta_{\mathrm{kp}}}}, \tag{4}$$

where n_{e} is the refractive index of the anisotropic uniaxial medium, θ_{kp} is the angle between the wave vector and the optical axis; Eq.(2) is then rewritten as

$$n_1 \sin\theta_{\mathrm{i}} = \frac{n_{\mathrm{o}} n_{\mathrm{e}}}{\sqrt{n_{\mathrm{o}}^2 \sin^2\theta_{\mathrm{kp}} + n_{\mathrm{e}}^2 \cos^2\theta_{\mathrm{kp}}}} \sin\theta_{\mathrm{r}}. \tag{5}$$

Here θ_{r} is the angle between the e-wave vector and the normal of the crystal. The direction of e-wave vector is expressed as

$$\boldsymbol{k}_{\mathrm{r}} = \sin\theta_{\mathrm{r}}\boldsymbol{i} + \cos\theta_{\mathrm{r}}\boldsymbol{k}, \tag{6}$$

thus we have

$$\cos\theta_{\mathrm{kp}} = \boldsymbol{k}_{\mathrm{r}} \cdot \boldsymbol{c} = \frac{\sqrt{2}}{2}\sin\theta_{\mathrm{r}} + \frac{\sqrt{2}}{2}\cos\theta_{\mathrm{r}}. \tag{7}$$

Substituting Eq.(7) into Eq.(5) leads to

$$\begin{aligned} &n_1 \sin\theta_{\mathrm{i}} \sqrt{n_{\mathrm{o}}^2 + \frac{1}{2}(n_{\mathrm{e}}^2 - n_{\mathrm{o}}^2)(\sin\theta_{\mathrm{r}} + \cos\theta_{\mathrm{r}})^2} \\ =&n_{\mathrm{o}} n_{\mathrm{e}} \sin\theta_{\mathrm{r}}. \end{aligned} \tag{8}$$

Using the relation:

$$\frac{1}{\cos^2\theta_{\mathrm{r}}} = 1 + \tan^2\theta_{\mathrm{r}}, \tag{9}$$

Eq.(8) is rewritten as

$$\begin{aligned} &(n_1 \sin\theta_{\mathrm{i}})^2 \Big[n_{\mathrm{o}}^2 (1 + \tan^2\theta_{\mathrm{r}}) + \frac{1}{2}(n_{\mathrm{e}}^2 - n_{\mathrm{o}}^2) \\ &\times (1 + \tan\theta_{\mathrm{r}})^2 \Big] = n_{\mathrm{o}}^2 n_{\mathrm{e}}^2 \tan^2\theta_{\mathrm{r}}. \end{aligned} \tag{10}$$

$$\begin{aligned} &\left[\frac{1}{2}(n_1 \sin\theta_{\mathrm{i}})^2 (n_{\mathrm{e}}^2 + n_{\mathrm{o}}^2) - n_{\mathrm{o}}^2 n_{\mathrm{e}}^2 \right] \tan^2\theta_{\mathrm{r}} \\ &+ (n_1 \sin\theta_{\mathrm{i}})^2 (n_{\mathrm{e}}^2 - n_{\mathrm{o}}^2) \tan\theta_{\mathrm{r}} \\ &+ \frac{1}{2}(n_1 \sin\theta_{\mathrm{i}})^2 (n_{\mathrm{e}}^2 + n_{\mathrm{o}}^2) = 0. \end{aligned} \tag{11}$$

Let

$$A = \frac{1}{2}(n_1 \sin\theta_{\mathrm{i}})^2 (n_{\mathrm{e}}^2 + n_{\mathrm{o}}^2) - n_{\mathrm{o}}^2 n_{\mathrm{e}}^2, \tag{12}$$

$$B = (n_1 \sin\theta_{\mathrm{i}})^2 (n_{\mathrm{e}}^2 - n_{\mathrm{o}}^2), \tag{13}$$

$$C = \frac{1}{2}(n_1 \sin\theta_{\mathrm{i}})^2 (n_{\mathrm{e}}^2 + n_{\mathrm{o}}^2). \tag{14}$$

Rewrite Eq.(11) as

$$A \tan^2\theta_{\mathrm{r}} + B \tan\theta_{\mathrm{r}} + C = 0, \tag{15}$$

$$\tan\theta_{\mathrm{r}} = \frac{1}{2A}[-B + \sqrt{B^2 - 4AC}]. \tag{16}$$

If $n_{\mathrm{e}} = n_{\mathrm{o}}$, Eq.(16) becomes $\tan\theta_{\mathrm{r}} = \dfrac{A}{\sqrt{n_{\mathrm{o}}^2 - A^2}}$, and $\theta_{\mathrm{r}} = \theta_{\mathrm{o}}$, i.e. the e-ray and o-ray are in the same direction.

(2) The direction of the e-ray

The angle θ_e between e-ray and optical axis is expressed as[15,16]

$$\tan\theta_e = (n_o^2/n_e^2)\tan\theta_{kp}. \tag{17}$$

Considering the e-ray, e-wave vector and optical axis in the same plane,[17,18] the relations can be expressed as

$$\boldsymbol{S} = \alpha\boldsymbol{k}_r + \beta\boldsymbol{c}. \tag{18}$$

and

$$\boldsymbol{S}\cdot\boldsymbol{c} = \cos\theta_e, \quad \boldsymbol{S}\cdot\boldsymbol{S} = 1. \tag{19}$$

So we have

$$\alpha\cos\theta_{kp} + \beta = \cos\theta_e, \tag{20}$$

$$\alpha^2 + \beta^2 + 2\alpha\beta\cos\theta_{kp} = 1. \tag{21}$$

The $\cos\theta_{kp}$ is obtained from Eq.(7). Substituting Eq.(20) into Eq.(21), we can express Eq.(21) as

$$\alpha^2(1-\cos^2\theta_{kp}) + \cos^2\theta_{kp} - 1 = 0. \tag{22}$$

So we obtain

$$\alpha = \sin\theta_e/\sin\theta_{kp} (\alpha > 0). \tag{23}$$

By using Eq.(17), Eq.(23) can be rewritten as

$$\begin{aligned}\alpha =& \frac{n_o^2\cos\theta_e}{n_e^2\cos\theta_{kp}}\\ =& \frac{n_o^2}{n_e^2\sqrt{\cos^2\theta_{kp} + (n_o^4/n_e^4)\sin^2\theta_{kp}}}.\end{aligned} \tag{24}$$

From Eq.(20), we have

$$\begin{aligned}\beta =& (1 - n_o^2/n_e^2)\cos\theta_e\\ =& \frac{(1-n_o^2/n_e^2)\cos\theta_{kp}}{\sqrt{\cos^2\theta_{kp} + (n_o^4/n_e^4)\sin^2\theta_{kp}}}.\end{aligned} \tag{25}$$

If $n_o = n_e$, Eq.(24) becomes $\alpha = 1$, Eq.(25) becomes $\beta = 0$. The e-ray and wave vector of e-ray are in the same direction, and the direction of e-ray can be obtained from Eqs.(1), (6) and (18)

$$\boldsymbol{S} = \left(\alpha\sin\theta_r + \frac{\sqrt{2}}{2}\beta\right)\boldsymbol{i} + \left(\alpha\cos\theta_r + \frac{\sqrt{2}}{2}\beta\right)\boldsymbol{k}. \tag{26}$$

2.2. The refraction of o-ray and e-ray in the adhesive of Savart polariscope

The adhesive is a homogeneous medium, thus the rays obey the law of refraction,[19] that is

$$n_1\sin\theta_i = n_2\sin\theta_r = n'\sin\theta_i', \tag{27}$$

where n' is the absolute refractive index of the adhesive, and θ_i' is the angle of refraction in the adhesive. We have

$$\theta_i' = \arcsin\left(\frac{n_1}{n'}\sin\theta_i\right). \tag{28}$$

So, in the adhesive, the wave vector of o-ray or e-ray strikes an interface of the adhesive and the second crystal of the Savart polariscope at an angle θ_i'.

2.3. The refraction of o-ray and e-ray in the second crystal of Savart polariscope

In this section symbols are the same as for the first crystal of Savart polariscope and use the prime $'$ to distinguish the quantities from those of the first crystal of Savart polariscope. In the second crystal of the Savart polariscope, the direction of the optical axis is expressed as follows:

$$\boldsymbol{c}' = \frac{\sqrt{2}}{2}\boldsymbol{j} + \frac{\sqrt{2}}{2}\boldsymbol{k}. \tag{29}$$

As is known, the law of refraction can be expressed as

$$n'\sin\theta_i' = n_2\sin\theta_r' \tag{30}$$

with n_2 being the absolute refractive index of the second crystal.

When the o-ray and the e-ray strike the interface between the adhesive and the second crystal of the Savart polariscope, the rays obey the double refraction law. There will be four rays in the Savart polariscope: namely oo-ray, oe-ray, eo-ray and ee-ray. Here θ_r' is the angle of refraction in the second crystal of Savart polariscope.[20−23]

2.3.1. The refraction of oo-ray and eo-ray

The oo-ray and eo-ray satisfy the same formula

$$\theta_{oo} = \theta_{eo} = \theta_r' = \arcsin(n'\sin\theta_i'/n_o). \tag{31}$$

From Eq.(27), we obtain the following result

$$\theta_{oo} = \theta_{eo} = \theta_r' = \arcsin(n_1\sin\theta_i/n_o), \tag{32}$$

where θ_{oo} is the refractive angle of oo-ray, θ_{eo} is the refractive angle of eo-ray.

2.3.2. The refraction of oe-ray and ee-ray

The laws for the oe-ray and ee-ray are the same in the second crystal of Savart polariscope.

(A) The oe-wave vector and ee-wave vector

For the oe-ray and ee-ray, n_2' is expressed as

$$n_2' = \frac{n_\mathrm{o} n_\mathrm{e}}{\sqrt{n_\mathrm{o}^2 \sin^2\theta_\mathrm{kp}' + n_\mathrm{e}^2 \cos^2\theta_\mathrm{kp}'}}. \tag{33}$$

Equation (30) is rewritten as

$$n' \sin\theta_\mathrm{i}' = \frac{n_\mathrm{o} n_\mathrm{e}}{\sqrt{n_\mathrm{o}^2 \sin^2\theta_\mathrm{kp}' + n_\mathrm{e}^2 \cos^2\theta_\mathrm{kp}'}} \sin\theta_\mathrm{r}'. \tag{34}$$

The direction of wave vector is expressed as

$$\boldsymbol{k}'_\mathrm{r} = \sin\theta_\mathrm{r}' \boldsymbol{i} + \cos\theta_\mathrm{r}' \boldsymbol{k}. \tag{35}$$

So we have

$$\cos\theta_\mathrm{kp}' = \boldsymbol{k}'_\mathrm{r} \cdot \boldsymbol{c}' = \frac{\sqrt{2}}{2} \cos\theta_\mathrm{r}'. \tag{36}$$

Substituting Eq.(36) into Eq.(34) yields

$$n' \sin\theta_\mathrm{i}' \sqrt{n_\mathrm{o}^2 + \frac{1}{2}(n_\mathrm{e}^2 - n_\mathrm{o}^2)\cos^2\theta_\mathrm{r}'} = n_\mathrm{o} n_\mathrm{e} \sin\theta_\mathrm{r}'. \tag{37}$$

Using the relation

$$\frac{1}{\cos^2\theta_\mathrm{r}'} = 1 + \tan^2\theta_\mathrm{r}'. \tag{38}$$

Equation (37) is rewritten as

$$\begin{aligned}&(n' \sin\theta_\mathrm{i}')^2 \left[n_\mathrm{o}^2(1 + \tan^2\theta_\mathrm{r}') + \frac{1}{2}(n_\mathrm{e}^2 - n_\mathrm{o}^2)\right] \\ =&n_\mathrm{o}^2 n_\mathrm{e}^2 \tan^2\theta_\mathrm{r}'\end{aligned} \tag{39}$$

and

$$\begin{aligned}&[n_\mathrm{o}^2 n_\mathrm{e}^2 - (n' \sin\theta_\mathrm{i}')^2 n_\mathrm{o}^2] \tan^2\theta_\mathrm{r}' \\ =&\frac{1}{2}(n_\mathrm{e}^2 + n_\mathrm{o}^2)(n' \sin\theta_\mathrm{i}')^2.\end{aligned} \tag{40}$$

We then have

$$\tan\theta_\mathrm{r}' = \sqrt{\frac{(n_\mathrm{e}^2 + n_\mathrm{o}^2)(n' \sin\theta_\mathrm{i}')^2}{2n_\mathrm{e}^2 n_\mathrm{o}^2 - 2(n' \sin\theta_\mathrm{i}')^2 n_\mathrm{o}^2}}. \tag{41}$$

Also we have

$$\theta_\mathrm{r}' = \arctan\sqrt{\frac{(n_\mathrm{e}^2 + n_\mathrm{o}^2)(n' \sin\theta_\mathrm{i}')^2}{2n_\mathrm{e}^2 n_\mathrm{o}^2 - 2(n' \sin\theta_\mathrm{i}')^2 n_\mathrm{o}^2}}. \tag{42}$$

By using Eq.(27), Eq.(42) is rewritten as

$$\theta_\mathrm{r}' = \arctan\sqrt{\frac{(n_\mathrm{e}^2 + n_\mathrm{o}^2)(n_1 \sin\theta_\mathrm{i})^2}{2n_\mathrm{e}^2 n_\mathrm{o}^2 - 2(n_1 \sin\theta_\mathrm{i})^2 n_\mathrm{o}^2}}. \tag{43}$$

The angles of the refraction obey the following equation:

$$\begin{aligned}\theta_\mathrm{oe} =&\theta_\mathrm{ee} = \theta_\mathrm{r}' \\ =&\arctan\sqrt{\frac{(n_\mathrm{e}^2 + n_\mathrm{o}^2)(n_1 \sin\theta_\mathrm{i})^2}{2n_\mathrm{e}^2 n_\mathrm{o}^2 - 2(n_1 \sin\theta_\mathrm{i})^2 n_\mathrm{o}^2}}.\end{aligned} \tag{44}$$

Here θ_oe is the angle between the oe-wave vector and the normal of crystal and θ_ee the angle between the ee-wave vector and the normal of the crystal.

(B) The directions of the oe-ray and ee-ray

The angle between the oe-ray (ee-ray) and optical axis θ_e' is expressed as

$$\tan\theta_\mathrm{e}' = (n_\mathrm{o}^2/n_\mathrm{e}^2) \tan\theta_\mathrm{kp}'. \tag{45}$$

From Eq.(24), Eq.(25) and Eq.(36), we have

$$\alpha = \frac{n_\mathrm{o}^2}{n_\mathrm{e}^2} \frac{\cos\theta_\mathrm{e}'}{\cos\theta_\mathrm{kp}'} = \frac{\sqrt{2} n_\mathrm{o}^2}{\sqrt{2n_\mathrm{o}^4 + (n_\mathrm{e}^4 - n_\mathrm{o}^4)\cos^2\theta_\mathrm{r}'}} \tag{46}$$

and

$$\begin{aligned}\beta =&(1 - n_\mathrm{o}^2/n_\mathrm{e}^2)\cos\theta_\mathrm{e}' \\ =&\frac{(n_\mathrm{e}^2 - n_\mathrm{o}^2)\cos\theta_\mathrm{r}'}{\sqrt{2n_\mathrm{o}^4 + (n_\mathrm{e}^4 - n_\mathrm{o}^4)\cos^2\theta_\mathrm{r}'}}.\end{aligned} \tag{47}$$

We can obtain the direction of oe-ray from $\boldsymbol{S} = \alpha \boldsymbol{k}'_\mathrm{r} + \beta \boldsymbol{c}'$

$$\boldsymbol{S} = (\alpha \sin\theta_\mathrm{r}')\boldsymbol{i} + \frac{\sqrt{2}}{2}\beta \boldsymbol{j} + (\frac{\sqrt{2}}{2}\beta + \alpha \cos\theta_\mathrm{r}')\boldsymbol{k}. \tag{48}$$

2.4. The refraction from the second crystal of Savart polariscope to the air

Four rays (oo-ray, oe-ray, ee-ray and eo-ray) propagate from the second crystal of the Savart polariscope into the air at an angle θ_i (the angle of incidence at the first crystal plane).

3. Conclusion

The refraction of rays in the Savart polariscope is analysed and discussed on the basis of the Snell law. The refraction and the refraction formulae of the e-ray and the o-ray are obtained. The formulae in the crystal are of more complicated forms than in isotropic medium. On the other hand, the wave vector and rays for e-ray are usually in different directions.

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