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Investigation on electromagnetic scattering from rough soil surface of layered medium using the small perturbation method*

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Electromagnetic scattering from a rough surface of layered medium is investigated, and the formulae of the scattering coefficients for different polarizations are derived using the small perturbation method. A rough surface with exponential correlation function is presented for describing a rough soil surface of layered medium, the formula of its scattering coefficient is derived by considering the spectrum of the rough surface with exponential correlation function; the curves of the bistatic scattering coefficient of HH polarization with variation of the scattering angle are obtained by numerical calculation. The influence of the permittivity of layered medium, the mean layer thickness of intermediate medium, the roughness surface parameters and the frequency of the incident wave on the bistatic scattering coefficient is discussed. Numerical results show that the influence of the permittivity of layered medium, the mean layer thickness of intermediate medium, the rms and the correlation length of the rough surface, and the frequency of the incident wave on the bistatic scattering coefficient is very complex.

Keywords: electromagnetic scattering, rough soil surface with exponential correlation function, layered medium, the small perturbation method

PACC: 4120, 4110H

1. Introduction

Electromagnetic wave scattering is an active interdisciplinary area of research with various practical applications in fields ranging from atomic physics to medical imaging to geoscience and remote sensing.[1−9] Since most naturally occurring surfaces are stratified, it is expected that electromagnetic scattering from rough boundary of a stratified medium will be of significance in the solution of many engineering, technical, and scientific problems, such as environmental remote sensing, ground-penetration-radar operation, radio-wave propagation and communication, microwave probing of composite materials and their surface roughness, thin-film physics and its applications in optics and electronics, etc. In recent decades, the theory of electromagnetic scattering by statistically rough surfaces has been enriched by the creation of new analytical methods. These methods include: the small-slope approximation,[10] the title-invariant theory of rough surface scattering,[11] the local perturbation method,[12] full-wave-method analysis,[13] the surface-field phase-perturbation technique,[14] two-scale models,[15−17] and the small perturbation method.[18,19]

In this paper we study the problem of electromagnetic scattering from a rough interface in an arbitrary, plane-layered medium, assuming a roughness small and gentle enough for the small perturbation method to be applicable. This method allows us to avoid cumbersome derivations and to obtain compact expressions for amplitudes of wave scattering not only in free space, but also in natural layered medium such as sea ice, topsoil, snow, and sand cover in arid regions, and also in remote-sensing problem solving. To obtain a small perturbation expansion for scattering amplitudes, we apply a procedure based on the Rayleigh hypothesis. This procedure, in fact, does not restrict the class of rough surfaces. We neglect the refraction in the thin layer adjacent to the interface, assuming that both media are homogeneous in the region occupied by roughness. At the same time, a rough surface

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with exponential correlation function is presented for describing rough soil surface of layered medium, and the electromagnetic scattering from this rough surface of layered medium is investigated using the method above.

2. Formulation

Consider the electromagnetic scattering from the rough surface S between two half-spaces, which is described by $z = f(\boldsymbol{r})$, where $\boldsymbol{r} = \{x, y\}$. Without loss of generality, we suppose that the upper homogeneous half-space $z > f(\boldsymbol{r})$ (medium 1) is characterized by a constant dielectric permittivity $\varepsilon_0 = 1$, $\mu_0 = 1$, and the lower stratified half-space $z < f(\boldsymbol{r})$ (medium 2) is characterized by an arbitrary "depth-profile" of a complex dielectric permittivity $\varepsilon(z)$. The random function $f(\boldsymbol{r})$ is assumed to be statistically spatially homogeneous with zero mean value $\langle f(\boldsymbol{r})\rangle = 0$, where $\langle\cdots\rangle$ denotes the statistical averaging over the ensemble of $f(\boldsymbol{r})$ realizations.

An incident plane monochromatic (a time-dependence $\sim \mathrm{e}^{-\mathrm{i}\omega t}$ is assumed) electromagnetic wave, with frequency ω and wave vector $\boldsymbol{k}_i = \{k_i \sin\theta_i, 0, -k_i\cos\theta_i\}$, $k_i = \omega/c$, illuminates the rough surface S from medium 1 at a given angle of incidence θ_i, as shown in Fig.1.

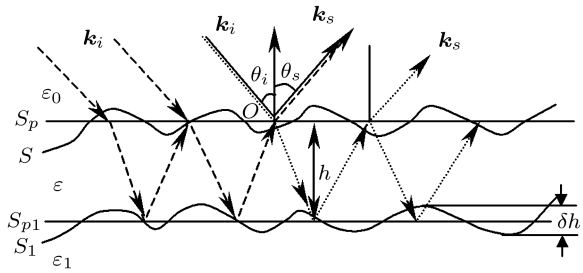


Fig.1. Geometry of scattering from the Gaussian rough surface of layered medium.

The boundary conditions for electric $\boldsymbol{E}$ and magnetic $\boldsymbol{H}$ fields mean continuity of their tangent components on the boundary S

$$\begin{aligned}&\boldsymbol{N}\times\left(\boldsymbol{H}^{(2)}-\boldsymbol{H}^{(1)}\right)_S\\ &=\boldsymbol{N}\times\left(\boldsymbol{E}^{(2)}-\boldsymbol{E}^{(1)}\right)_S=0.\end{aligned}\tag{1}$$

Here, $\boldsymbol{E}^{(2)}$ and $\boldsymbol{H}^{(2)}$ are the fields in the lower half-space $z < f(\boldsymbol{r})$, $\boldsymbol{E}^{(1)}$, $\boldsymbol{H}^{(1)}$ are the fields in the upper half-space $z > f(\boldsymbol{r})$, and $\boldsymbol{N}$ is a unit vector normal to the surface S and directed upward

$$\boldsymbol{N} = (\boldsymbol{n} - \boldsymbol{s})(1+s^2)^{-1/2},\tag{2}$$

where $\boldsymbol{n}$ is the unit vector of the normal to the horizontal plane $S_p(z=0)$, $\boldsymbol{s}(\boldsymbol{r}) = \nabla_r f(\boldsymbol{r})$ is a vector field of surface slopes, $\nabla_r = \{\partial/\partial x, \partial/\partial y\}$. Assuming that deviations and slopes of surface S with respect to S_p are small enough, we can expand the boundary conditions (1) in powers of $f(\boldsymbol{r})$ and $\boldsymbol{s}(\boldsymbol{r})$ and retain the first-order terms only[7]

$$\begin{aligned}&(\boldsymbol{n}\times\Delta\boldsymbol{H})_{S_P}\\ &=(\boldsymbol{s}\times\Delta\boldsymbol{H})_{S_P}-f\left(\boldsymbol{n}\times\frac{\partial\Delta\boldsymbol{H}}{\partial z}\right)_{S_P},\\ &(\boldsymbol{n}\times\Delta\boldsymbol{E})_{S_P}\\ &=(\boldsymbol{s}\times\Delta\boldsymbol{E})_{S_P}-f\left(\boldsymbol{n}\times\frac{\partial\Delta\boldsymbol{E}}{\partial z}\right)_{S_P}.\end{aligned}\tag{3}$$

Here, $\Delta\boldsymbol{H} = \boldsymbol{H}^{(2)} - \boldsymbol{H}^{(1)}$ and $\Delta\boldsymbol{E} = \boldsymbol{E}^{(2)} - \boldsymbol{E}^{(1)}$. We can express the solution of the diffraction problem in the form

$$\begin{aligned}\boldsymbol{H}^{(1,2)} &= \boldsymbol{H}_0^{(1,2)} + \boldsymbol{H}_1^{(1,2)},\\ \boldsymbol{E}^{(1,2)} &= \boldsymbol{E}_0^{(1,2)} + \boldsymbol{E}_1^{(1,2)},\end{aligned}\tag{4}$$

where $\boldsymbol{E}_0^{(1)}$, $\boldsymbol{H}_0^{(1)}$ $\left(\boldsymbol{E}_0^{(2)},\ \boldsymbol{H}_0^{(2)}\right)$ are the unperturbed wave fields in the upper $z > 0$ (lower $z < 0$) half-spaces corresponding to the specular reflection (refraction) at the plane boundary S_p. They satisfy the uniform boundary conditions

$$\begin{aligned}&\boldsymbol{n}\times\left(\boldsymbol{H}_0^{(2)}-\boldsymbol{H}_0^{(1)}\right)_{S_P}\\ &=\boldsymbol{N}\times\left(\boldsymbol{E}_0^{(2)}-\boldsymbol{E}_0^{(1)}\right)_{S_P}=0,\end{aligned}\tag{5}$$

and $\boldsymbol{E}_1^{(1)}$, $\boldsymbol{H}_1^{(1)}$ $\left(\boldsymbol{E}_1^{(2)},\ \boldsymbol{H}_1^{(2)}\right)$ are the corrections to the first order of $f(\boldsymbol{r})$ and $\boldsymbol{s}(\boldsymbol{r})$, i.e., the scattered fields in the upper and lower half-spaces, respectively. They satisfy the nonuniform boundary conditions

$$\begin{aligned}&(\boldsymbol{n}\times\Delta\boldsymbol{H}_1)_{S_P}\\ &=(\boldsymbol{s}\times\Delta\boldsymbol{H}_0)_{S_P}-f\left(\boldsymbol{n}\times\frac{\partial\Delta\boldsymbol{H}_0}{\partial z}\right)_{S_P}=\boldsymbol{J}^H,\\ &(\boldsymbol{n}\times\Delta\boldsymbol{E}_1)_{S_P}\\ &=(\boldsymbol{s}\times\Delta\boldsymbol{E}_0)_{S_P}-f\left(\boldsymbol{n}\times\frac{\partial\Delta\boldsymbol{E}_0}{\partial z}\right)_{S_P}=\boldsymbol{J}^E,\end{aligned}\tag{6}$$

where $\Delta\boldsymbol{H}_0 = \boldsymbol{H}_0^{(2)} - \boldsymbol{H}_0^{(1)}$, $\Delta\boldsymbol{H}_1 = \boldsymbol{H}_1^{(2)} - \boldsymbol{H}_1^{(1)}$, $\Delta\boldsymbol{E}_0 = \boldsymbol{E}_0^{(2)} - \boldsymbol{E}_0^{(1)}$ and $\Delta\boldsymbol{E}_1 = \boldsymbol{E}_1^{(2)} - \boldsymbol{E}_1^{(1)}$. It can be seen from Eq.(6) that surface roughness leads to the appearance of the effective electric and magnetic

surface currents $\boldsymbol{J}^H$ and $\boldsymbol{J}^E$ on the mean plane $z=0$, which generate the scattered fields. Currents $\boldsymbol{J}^H$ and $\boldsymbol{J}^E$ differ from the usually used electric and magnetic currents only by sign and a factor $4\pi/c$.

We consider first the incident monochromatic horizontally polarized (TE) plane electromagnetic wave, which propagates in medium 1, with its wave vector $\boldsymbol{k}_i$.

$$\boldsymbol{E}_{in}^{(1)} = \boldsymbol{P}_0\sqrt{\frac{\mu_0}{k_z}}\mathrm{e}^{\mathrm{i}\boldsymbol{k}_i\cdot\boldsymbol{R}} = \boldsymbol{P}_0\sqrt{\frac{\mu_0}{k_z}}\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}-\boldsymbol{k}_z z}. \tag{7}$$

Here, $\boldsymbol{R}=\{\boldsymbol{r},z\}$ is a three-dimensional spatial radius vector, $\boldsymbol{P}_0=\boldsymbol{e}_y=(0,1,0)$ is a unit polarization vector, $\boldsymbol{k}=\{k,0,0\}$ is a two-dimensional projection of incident wave vector $\boldsymbol{k}_i=\{\boldsymbol{k},-\boldsymbol{k}_z\}$ on the plane $z=0$, and $k_z=\sqrt{k_i^2\mu_0\varepsilon_0-k^2}=k_i\cos\theta_i$.

Let us express the scattered fields in the form of an expansion over plane waves, i.e.

$$\begin{aligned}\boldsymbol{E}_1^{(1)}(\boldsymbol{r},z) &= \iint\sqrt{\frac{\mu_0}{q_z}}\tilde{\boldsymbol{E}}_1^{(1)}(\boldsymbol{q})\mathrm{e}^{\mathrm{i}(\boldsymbol{q}\cdot\boldsymbol{r}+q_z z)}\mathrm{d}\boldsymbol{q},\\ \boldsymbol{H}_1^{(1)}(\boldsymbol{r},z) &= \iint\sqrt{\frac{\varepsilon_0}{q_z}}\tilde{\boldsymbol{H}}_1^{(1)}(\boldsymbol{q})\mathrm{e}^{\mathrm{i}(\boldsymbol{q}\cdot\boldsymbol{r}+q_z z)}\mathrm{d}\boldsymbol{q}.\end{aligned} \tag{8}$$

Here, $\tilde{\boldsymbol{E}}_1^{(1)}(\boldsymbol{q})$, $\tilde{\boldsymbol{H}}_1^{(1)}(\boldsymbol{q})$ are the amplitudes of scattered plane waves propagating in the upper half-space in the positive direction of z-axis, $q_z=\sqrt{k_i^2-q^2}=\sqrt{k_s^2-q^2}=k_i\cos\theta_s=k_s\cos\theta_s$ is the vertical component of the upward scattered field wave vector $\boldsymbol{k}_s=\{\boldsymbol{q},q_z\}$. A similar expansion over the plane waves can be used for upward and downward waves in the lower half-space $z<0$.

The solution of Maxwell's equations for scattered fields under boundary conditions (6) has the form [7]

$$\begin{aligned}\left(\boldsymbol{P}\tilde{\boldsymbol{E}}_1^{(1)}\right) &= \frac{1}{2}k_i\sqrt{\frac{\mu_0}{q_z}}(1+R_h^s)\left(\boldsymbol{P}\tilde{\boldsymbol{J}}^H\right)\\ &\quad+\frac{1}{2}\sqrt{\frac{q_z}{\mu_0}}(1-R_h^s)\left[\boldsymbol{P}\cdot(\boldsymbol{n}\times\tilde{\boldsymbol{J}}^E)\right],\\ \left(\boldsymbol{P}\tilde{\boldsymbol{H}}_1^{(1)}\right) &= -\frac{1}{2}k_i\sqrt{\frac{\varepsilon_0}{q_z}}(1+R_v^s)\left(\boldsymbol{P}\tilde{\boldsymbol{J}}^E\right)\\ &\quad+\frac{1}{2}\sqrt{\frac{q_z}{\varepsilon_0}}(1-R_v^s)\left[\boldsymbol{P}\cdot(\boldsymbol{n}\times\tilde{\boldsymbol{J}}^H)\right].\end{aligned} \tag{9}$$

Here, $\boldsymbol{P}$ is a horizontal unit vector orthogonal to the plane of scattering, R_h^s and R_v^s are the reflection coefficients of a horizontally (h) and a vertically (v) polarized plane waves with wave vector $k_s^{(1)}$ from a layered medium with a perfectly plane boundary $z=0$, and $\tilde{\boldsymbol{J}}^{E,H}$ is the Fourier transform of the surface current

$$\begin{aligned}\tilde{\boldsymbol{J}}^{E,H} &= \tilde{\boldsymbol{J}}^{E,H}(\boldsymbol{q})\\ &= \frac{1}{(2\pi)^2}\iint\boldsymbol{J}^{E,H}(\boldsymbol{r})\mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}\mathrm{d}\boldsymbol{r}.\end{aligned} \tag{10}$$

Using the definitions (6) of surface currents $\boldsymbol{J}^{E,H}$ and expressing the differences $\Delta\boldsymbol{H}_0$, $\Delta\boldsymbol{E}_0$ of zero-order fields through corresponding reflection coefficients R_h^i and R_v^i for an incident wave, we obtain the scattering amplitudes for a horizontally polarized incident wave

$$\begin{aligned}\boldsymbol{P}\tilde{\boldsymbol{E}}_1^{(1)} &= S_{hh}\\ &= \frac{\mathrm{i}k_i^2}{2\sqrt{q_z k_z}}\tilde{f}(\boldsymbol{q}-\boldsymbol{k})F_{hh}(\boldsymbol{q},\boldsymbol{k}),\\ \boldsymbol{P}\tilde{\boldsymbol{H}}_1^{(1)} &= S_{vh}\\ &= \frac{\mathrm{i}k_i^2}{2\sqrt{q_z k_z}}\tilde{f}(\boldsymbol{q}-\boldsymbol{k})F_{vh}(\boldsymbol{q},\boldsymbol{k}),\end{aligned} \tag{11}$$

where

$$\tilde{f}(\boldsymbol{q}-\boldsymbol{k}) = \frac{1}{(2\pi)^2}\iint f(\boldsymbol{r})\mathrm{e}^{-\mathrm{i}(\boldsymbol{q}-\boldsymbol{k})\cdot\boldsymbol{r}}\mathrm{d}\boldsymbol{r} \tag{12}$$

and

$$\begin{aligned}&F_{hh}\\ &= \mu_0(1+R_h^i)(1+R_h^s)\\ &\quad\times\left[\frac{\mu-\mu_0}{\mu}\varepsilon_0\sin\theta_s\sin\theta_i+(\varepsilon-\varepsilon_0)\cos\phi\right]\\ &\quad-\varepsilon_0(1+R_h^i)(1+R_h^s)(\mu-\mu_0)\cos\theta_s\cos\theta_i\cos\varphi,\\ &F_{vh}\\ &= \{\mu_0(1-R_v^s)(1+R_h^i)(\varepsilon-\varepsilon_0)\cos\theta_s\\ &\quad-\varepsilon_0(1-R_h^i)(1+R_v^s)(\mu-\mu_0)\cos\theta_i\}\sin\phi.\end{aligned} \tag{13}$$

In these equations ε and μ are the limiting values of $\varepsilon(z\to-0)$ and $\mu(z\to-0)$, and their variations on the vertical scale of f are neglected. The scattering amplitudes S_{vv}, S_{hv} for a vertically polarized incident wave can be obtained without any additional derivations, simply by changing notations in previous equations: $\boldsymbol{E}\to\boldsymbol{H}$, $\boldsymbol{H}\to-\boldsymbol{E}$, $h\to v$, $\mu\to\varepsilon$.

Specific incoherent scattering cross-sections into up-going waves are related with the corresponding scattering amplitudes by a simple equation[6,16]

$$\begin{aligned}&\sigma_{\alpha\beta}^{\circ}(k_s,k_i)\\ &= \lim_{S_P\to\infty}\frac{16\pi^3}{S_P}|q_z k_z|\left\langle|S_{\alpha\beta}|^2\right\rangle\\ &= \pi k_i^4|F_{\alpha\beta}|^2 W_{\mathrm{f}}(\boldsymbol{q}-\boldsymbol{k})\\ &= \pi k_i^4|\varepsilon-1|^2|f_{\alpha\beta}|^2 W_{\mathrm{f}}(\boldsymbol{q}-\boldsymbol{k}),\end{aligned} \tag{14}$$

where indices α and β characterize the polarization states of scattered and incident waves, and $W_{\mathrm{f}}(\boldsymbol{q}-\boldsymbol{k})$ is a spatial power spectrum of surface roughness which can be introduced as a Fourier transformation of roughness auto-correlation function, $\boldsymbol{q}-\boldsymbol{k}=k_s\sin\theta_s\cos\varphi-k_i\sin\theta_i$.

For the simplest case of scattering by a rough boundary of non-magnetic layered medium ($\mu = 1$) the full set of factors $f_{\alpha\beta}$ in an explicit form for different polarization states of incident and scattered waves are

$$\begin{aligned} f_{hh} &= [1 + R_h(\theta_i)]\,[1 + R_h(\theta_s)]\cos\phi, \\ f_{vh} &= -[1 + R_h(\theta_i)]\,[1 - R_v(\theta_s)]\cos\theta_s \sin\phi, \\ f_{vv} &= \frac{1}{\varepsilon}\,[1 + R_v(\theta_i)]\,[1 + R_v(\theta_s)]\sin\theta_i \sin\theta_s \\ &\quad -[1 - R_v(\theta_i)][1 - R_v(\theta_s)]\cos\theta_i \cos\theta_s \cos\phi, \\ f_{hv} &= [1 - R_v(\theta_i)]\,[1 + R_h(\theta_s)]\cos\theta_i \sin\phi. \end{aligned} \tag{15}$$

Here R_h and R_v are the specular reflection coefficients for horizontal and vertical polarization waves from the lower medium ($z < 0$) with a planar surface $S_p(z = 0)$ to the upper half-space respectively; θ_i is the incident angle, θ_s is the scattering angle, and ϕ is the azimuthal angle of scattering.

Equation (15) describes only the diffused part of scattered energy and does not include the specular reflected field, which is dominant at the specular direction ($\theta_s = \theta_i$, $\phi = 0$). In the general case of an arbitrarily stratified medium, the specular reflection coefficients R_h and R_v can be represented in the form

$$\begin{aligned} R_h &= \frac{R_{0h} + R'_h}{1 + R_{0h} R'_h}, \\ R_v &= \frac{R_{0v} + R'_v}{1 + R_{0v} R'_v}, \end{aligned} \tag{16}$$

where R_{0h} and R_{0v} are the Fresnel reflection coefficients from the planar surface between the two homogeneous media ($\varepsilon_0 = 1$ and ε),

$$\begin{aligned} R_{0h} &= \frac{\cos\theta - \sqrt{\varepsilon - \sin^2\theta}}{\cos\theta + \sqrt{\varepsilon - \sin^2\theta}}, \\ R_{0v} &= \frac{\varepsilon\cos\theta - \sqrt{\varepsilon - \sin^2\theta}}{\varepsilon\cos\theta + \sqrt{\varepsilon - \sin^2\theta}}, \end{aligned} \tag{17}$$

and R' is the reflection coefficient from the subsurface layer ($R' = 0$ for a uniform $\varepsilon(z) = \text{const}$ homogeneous half-space $z < 0$).

In what follows, we will consider the simplest case of a layered structure (Fig.1), namely a homogeneous layer of mean thickness h, with complex dielectric permittivity $\varepsilon = \varepsilon' + \mathrm{i}\varepsilon''$, overlying a homogeneous half-space (substrate) with a complex dielectric permittivity constant $\varepsilon_1 = \varepsilon'_1 + \mathrm{i}\varepsilon''_1$. The reflection coefficients R_h and R_v for this structure are given by the general equation (16), where R' can be written in the form

$$R'_h(\theta) = R_{1h}(\theta)\exp\left(2\mathrm{i}kh\sqrt{\varepsilon - \sin^2\theta}\right),$$

$$R'_v(\theta) = R_{1v}(\theta)\exp\left(2\mathrm{i}kh\sqrt{\varepsilon - \sin^2\theta}\right), \tag{18}$$

with the following expressions for the Fresnel reflection coefficients $R_1(\theta)$ from the surface $z = -H$ between two media with dielectric permittivities ε and ε_1,

$$\begin{aligned} R_{1h}(\theta) &= \frac{\sqrt{\varepsilon - \sin^2\theta} - \sqrt{\varepsilon_1 - \sin^2\theta}}{\sqrt{\varepsilon - \sin^2\theta} + \sqrt{\varepsilon_1 - \sin^2\theta}}, \\ R_{1v}(\theta) &= \frac{\varepsilon_1\sqrt{\varepsilon - \sin^2\theta} - \varepsilon\sqrt{\varepsilon_1 - \sin^2\theta}}{\varepsilon_1\sqrt{\varepsilon - \sin^2\theta} + \varepsilon\sqrt{\varepsilon_1 - \sin^2\theta}}. \end{aligned} \tag{19}$$

In Eq.(15), the oscillatory dependence of the scattering amplitudes $f_{\alpha\beta}$ on the incident and scattering angles is caused by the factor $\exp\left(2\mathrm{i}kh\sqrt{\varepsilon - \sin^2\theta}\right)$ in the reflection coefficients $R'_h(\theta)$ and $R'_v(\theta)$ that make contribution to the total specular reflection coefficients R_h and R_v. The physical cause of these oscillations is quite clear: they are the product of interference of three kinds of waves (see Fig.1). The first one is the wave propagating in the direction $\boldsymbol{k}_i$ and scattered in the direction $\boldsymbol{k}_s$ by the surface roughness (in the vicinity of point O) without penetrating into the layer. This wave is schematically depicted in Fig.1 by solid arrows. The second wave (the dashed lines in Fig.1) is refracted at the upper plane (without scattering), penetrates into the layer, then is specularly reflected from the substrate, and finally is scattered by the roughness into the upper half-space in wave propagating in $\boldsymbol{k}_s$ direction. The third wave (dotted lines in Fig.1) is similar to the previous one, but passes its way in the opposite order: at first it is scattered by roughness into the layer, then is specularly reflected from the substrate and goes out in $\boldsymbol{k}_s$ direction after refracting at the upper plane (this time without scattering by its roughness).

The interference between the first and the second waves does not affect the angular distribution of scattered intensity, but leads only to the oscillation of total 'brightness' of interference pattern, depending on the incident angle θ_i with a period determined by the factor $\exp\left(2\mathrm{i}kh\sqrt{\varepsilon - \sin^2\theta_i}\right)$. The interference between the first and the third waves (the solid and dotted lines in Fig.1) leads to the oscillation of scattered power, depending on the scattered angle θ_s with the same period caused by the factor $\exp\left(2\mathrm{i}kh\sqrt{\varepsilon - \sin^2\theta_s}\right)$. In the general case, the period of these oscillations does not depend on the wave polarization and the permittivity ε_1 of the substrate. Finally, the interference between the second and the third waves leads to the oscillation of scattered power angular dependence, whose max-

ima are given by

$$kh\left(\sqrt{\varepsilon'-\sin^2\theta_i}-\sqrt{\varepsilon'-\sin^2\theta_s}\right)=n\pi,\quad n=0,1,2\cdots. \tag{20}$$

3. Rough soil surface in layered medium model

Here, a rough surface with exponential correlation function is selected as the rough soil surface in layered medium model and for a rough surface with exponential correlation function, $W_{\rm f}$ is given by

$$W_{\rm f}(K)=\sqrt{2\pi}\delta^2l^2(1+K^2l^2)^{-\frac{3}{2}}. \tag{21}$$

Substituting Eq.(21) in Eq.(14), specific incoherent scattering cross sections into up-going waves from rough soil surface of a layered medium are obtained, according to the small perturbation method above,

$$\sigma^{\circ}_{\alpha\beta}(k_s,k_i)=\pi\sqrt{2\pi}k_i^4\left|\varepsilon-1\right|^2\left|f_{\alpha\beta}\right|^2\delta^2l^2\times(1+K^2l^2)^{-\frac{3}{2}}. \tag{22}$$

In this way, the scattering coefficient of the rough soil surface with exponential correlation function of a layered medium is

$$\sigma=10\log_{10}\sigma^{\circ}_{\alpha\beta}(k_s,k_i). \tag{23}$$

We use Eq.(23) to investigate the electromagnetic scattering from a rough soil surface with exponential correlation function of layered medium.

4. Numerical results and discussion

The small perturbation method requires the surface standard deviation to be less than about 5 percent of the electromagnetic wavelength. It should be emphasized that the surface standard deviation here refers to that computed from frequency components of the surface responsible for scattering at a given electromagnetic wavelength. This point is important because when all frequency components of the surface are included in the standard-deviation computation, a substantially different value may be obtained. In addition to the standard-deviation requirement, the average slope of the surface should be of the same order of magnitude as the wave number times the surface standard deviation. Mathematically, the above two conditions are[1]

$$k\delta<0.3, \tag{24}$$

$$\sqrt{2}\delta/l<0.3. \tag{25}$$

The sampling of δ and l is based on Eqs.(24) and (25) in numerical calculation concerned below, and the sampling of the incident frequency is $f=10\,\mathrm{GHz}$, the sampling of the incident angle is $\theta_i=30^\circ$.

For details, HH polarization of bistatic scattering is numerically calculated only in this paper.

In this case, when $\phi=0$, $k_i=k_s$, K in Eq.(22) is $K=k_i\sin\theta_s-k_i\sin\theta_i$, we consider the influence of ε, ε_1, h, δ, l and f on the bistatic scattering coefficient under the condition that other parameters are stationary.

4.1. The influence of ε_1 on the bistatic scattering coefficient

Figure 2 depicts the variation of σ with θ_s in hh and vv polarizations for different ε_1 with $\delta=0.2/k$, $l=10/k$, $\varepsilon=1.6+0.01\mathrm{i}$ and $h=10\lambda$ (λ is incident wavelength and $\lambda=c/f$).

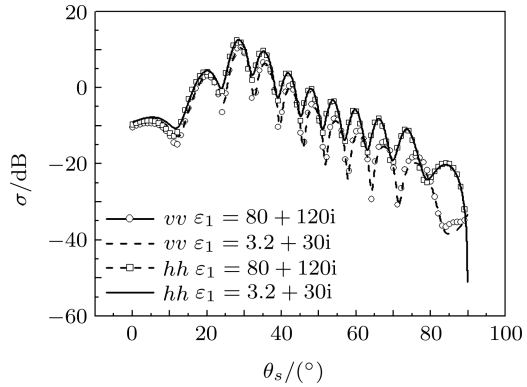


Fig.2. Variation of σ with θ_s for different ε_1.

All the curves corresponding to hh polarization in Fig.2 almost coincide with each other, and it is obvious that the influence of ε_1 on the bistatic scattering coefficient of hh polarization is small and obscure. Similarly, all the curves corresponding to vv polarization in Fig.2 almost coincide with each other, and the influence of ε_1 on the bistatic scattering coefficient of vv polarization is also small and obscure.

When $\theta_s < \theta_i = 30°$, the amplitude of σ corresponding to hh polarization is almost equal to that corresponding to vv polarization, but when $\theta_s > \theta_i = 30°$, the amplitude of σ corresponding to hh polarization is appreciably bigger than that corresponding to vv polarization.

4.2. The influence of ε on the bistatic scattering coefficient

The variation of σ with θ_s for different ε with $\delta = 0.2/k$, $l = 10/k$, $\varepsilon_1 = 80 + 30\mathrm{i}$, and $h = 10\lambda$ is depicted in Fig.3.

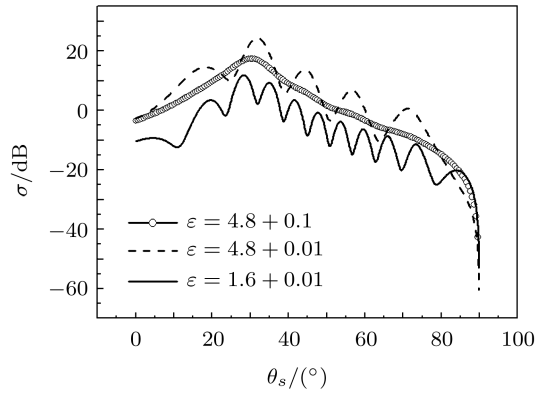


Fig.3. Variation of σ with θ_s for different ε.

It is obvious that the influence of ε on the bistatic scattering coefficient is remarkable; the bigger the real part of ε, the bigger the σ, and the smaller the frequency of the curve oscillation; the bigger the imaginary part of ε, the smaller the σ, and the smaller the frequency of the curve oscillation.

4.3. The influence of h on the bistatic scattering coefficient

Figure 4 depicts the variation of σ with θ_s for different h with $\varepsilon = 1.6 + 0.01\mathrm{i}$, $\varepsilon_1 = 80 + 30\mathrm{i}$, $\delta = 0.2/k$, $l = 10/k$.

There is a regularity in the variation of σ with θ_s for different h from Fig.4, that is, the σ corresponding to $h = 0$ is maximal, and it is a result of homogeneous medium. The bigger the h, the smaller the σ, the bigger of the frequency of the curve oscillation; and over a certain scope of h, when $30° < \theta_s < 70°$, h increases to two times of its initial value, and the frequency of the curve oscillation reduces to one-half of its primary value, evidenced by majority of numerical calculations. This point is very important to inverse problem of electromagnetic scattering from rough surface of layered medium.

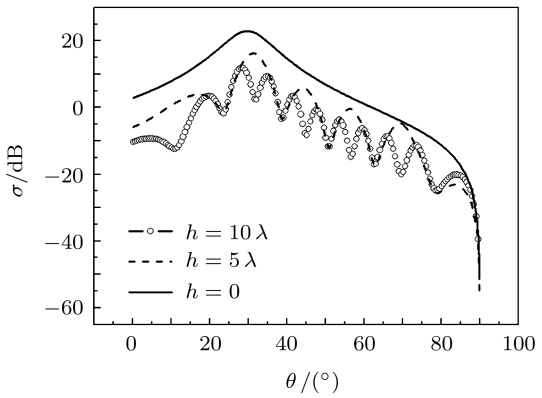


Fig.4. Variation of σ with θ_s for different h.

4.4. The influence of δ on the bistatic scattering coefficient

Variation of σ with θ_s for different δ with $\varepsilon = 1.6 + 0.01\mathrm{i}$, $\varepsilon_1 = 80 + 30\mathrm{i}$, $h = 10\lambda$, $l = 10/k$ is depicted in Fig.5.

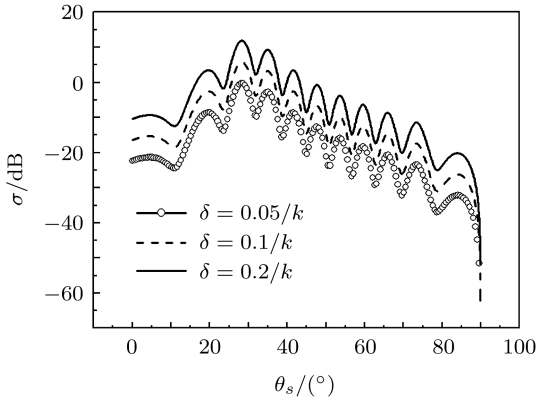


Fig.5. Variation of σ with θ_s for different δ.

It is obvious that the influence of δ on the backscattering coefficient is remarkable, that is, the bigger the δ, the bigger the σ, while δ does not affect the frequency of curve oscillation.

4.5. The influence of l on the bistatic scattering coefficient

Figure 6 depicts the distribution of σ with θ_s for different l with $\varepsilon = 1.6 + 0.01\mathrm{i}$, $\varepsilon_1 = 80 + 30\mathrm{i}$, $h = 10\lambda$, $\delta = 0.2/k$.

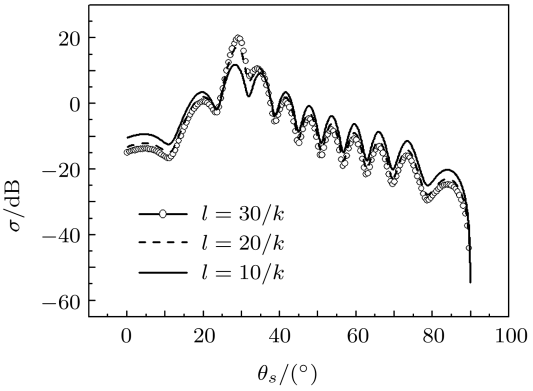


Fig.6. Variation of σ with θ_s for different l.

It is obvious that the influence of l on the backscattering coefficient is remarkable, that is, in the vicinity of $\theta_s = \theta_i = 30°$, the bigger the l, the bigger the σ, but for other θ_s, the bigger the l, the smaller the σ, while l does not affect the frequency of curve oscillation, similar to the influence of δ.

From Fig.2 to Fig.6, the curve of the variation of σ with θ_s is a oscillatory, but there is a regularity, that is, when $\theta_s < \theta_i$, the bigger the θ_s, the bigger the σ; when $\theta_s > \theta_i$, the bigger the θ_s, the smaller the σ, as a whole.

4.6. The influence of incident frequency f on the bistatic scattering coefficient

The variation of σ with f under the condition that $\varepsilon = 1.6 + 0.01\mathrm{i}$, $h = 10\lambda$, $\varepsilon_1 = 80 + 30\mathrm{i}$, $\delta = 0.001\mathrm{m}$, $l = 0.1\,\mathrm{m}$ (i.e. a certain rough surface), $\theta_s = 30°$, $60°$ is depicted in Fig.7.

The curve of the variation of σ with f is oscillatory, there is a regularity as a whole, that is, when $\theta_s = 30° = \theta_i$, the bigger the f, the bigger the σ, the oscillation of the curve is continuous approximatively; but, when $\theta_s = 60° \neq \theta_i$, the oscillation of the curve is not continuous, and when $f < 2\,\mathrm{GHz}$, the bigger the f, the bigger the σ; when $f > 2\,\mathrm{GHz}$, σ does not change on the whole, but the value of σ for $\theta_s = 60°$ is smaller than that for $\theta_s = 30°$.

In the above, we have studied the case of hh polarization in Figs.3–6, the results obtained are for the case of vv polarization by numerical calculation, only one point is worth to note, that is, under the condition that other parameters are kept unchanged, when $\theta_s > \theta_i = 30°$, the amplitude of σ for hh polarization is appreciably bigger than that for vv polarization, same as the result in Fig.2.

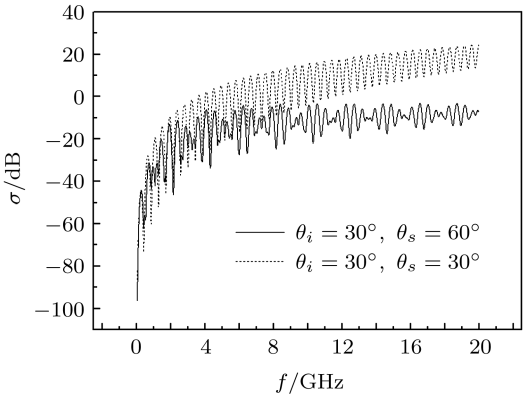


Fig.7. Variation of σ with f.

5. Conclusions

In this paper, the electromagnetic scattering from the rough surface of a layered medium has been investigated, the formulae of the scattering coefficient for different polarizations are derived using the small perturbation method. A rough surface with exponential correlation function is presented for describing the rough soil surface of the layered medium, the formulae of its scattering coefficient is derived by considering the spectrum of the rough surface with exponential correlation function. The influence of the dielectric permittivity of the layered medium, the mean thickness of intermediate medium, the roughness surface parameters and the frequency of the incident wave, on the bistatic scattering coefficient for hh polarization is discussed. These results will be applicable to solving many engineering, technical, and scientific problems, such as environmental remote sensing, ground-penetration-radar operation, radio-wave propagation and communication, microwave probing of composite materials and their surface roughness, thin-film physics and its applications in optics and electronics, the inverse problem of reconstructing rough surface.

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