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Spectral anomalies of diffracted pulsed Hermite–Gaussian beams in dispersive media*

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This paper derives and uses the recurrence expressions for the power spectra of diffracted pulsed Hermite–Gaussian (HG) beams in dispersive media to study the spectral anomalies of pulsed HG beams in the far field. Numerical results are given to illustrate the dependence of spectral switches on the pulse parameters, truncation parameter and dispersive property of the medium. The potential application of spectral anomalies of ultrashort pulsed beams in information encoding and transmission is discussed.

Keywords: singular optics, spectral anomaly, pulsed Hermite–Gaussian beam, dispersive medium

PACC: 0765, 4225F, 4280W

1. Introduction

Considerable attention has been paid to singular optics, which has been recently extended to polychromatic wavefields.[1] Specifically, Foley and Wolf pointed out that the phenomenon of spectral switches is an anomalous spectral behaviour and belongs to a new effect in singular optics with polychromatic light.[2] The spectral switches and spectral shifts of fully and partially coherent polychromatic steady-state beams have been extensively studied both theoretically and experimentally,[1−10] only few publications have dealt with the anomalous spectral behaviour of ultrashort pulsed beams.[11] On the other hand, a typical example of multi-mode lasers, i.e. Hermite–Gaussian (HG) beams including partially coherent HG beams and elliptical HG beams etc. has been studied extensively in the literature.[12−14] This paper aims at studying the spectral anomalies of ultrashort multi-mode pulsed beams in dispersive media. The pulsed HG beam is taken as an example, the recurrence analytical expressions for the power spectra of diffracted pulsed HG beams in dispersive media are derived in Section 2. Numerical calculation results are presented in Section 3 to illustrate the anomalous spectral behaviour in diffraction of pulsed HG beams and its potential application in information encoding and transmission. Finally, Section 4 summarizes the main results obtained in this paper.

2. Theoretical formulation

Assume that the spatiotemporal profile of an ultrashort pulsed beam at the plane z=0 takes a HG form

$$E_m^0(x_0, z=0, t) = H_m\left(\frac{\sqrt{2}x_0}{w_0}\right)\exp\left(\frac{x_0^2}{w_0^2}\right) f(t), \quad (1)$$

where $H_m(\bullet)$ denotes the mth order Hermite polynomial, w_0 is the waist width which is assumed to be independent of the frequency ω,[15] and $f(t)$ is the temporal profile of the pulsed beam.

Performing the Fourier transform of Eq.(1), we obtain the initial field in the frequency domain:

$$\begin{aligned} &E_m^0(x_0, z=0, \omega) \\ =&\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} E_m^0(x_0, z=0, t)\exp(\mathrm{i}\omega t)\mathrm{d}t \\ =&H_m\left(\frac{\sqrt{2}x_0}{w_0}\right)\exp\left(\frac{x_0^2}{w_0^2}\right) f(\omega), \end{aligned} \quad (2)$$

where

$$f(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\exp(\mathrm{i}\omega t)\mathrm{d}t \quad (3)$$

is the Fourier spectrum at the source plane z=0.

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If there is a slit with full width $2a$ at the plane z=0, within the framework of the paraxial approximation the field of the pulsed HG beam after passing through the slit in linear dispersive media obeys the Fresnel diffraction integral[16] and is given by

$$E_m(x,z,\omega)=\sqrt{\frac{\mathrm{i}k(\omega)}{2\pi z}}\exp\left[-\mathrm{i}k(\omega)z\right]f(\omega)\times\int_{-a}^{a}E_m^0(x_0,0,\omega)\times\exp\left[-\frac{\mathrm{i}k(\omega)}{2z}(x-x_0)^2\right]\mathrm{d}x_0, \tag{4}$$

where $k(\omega)$=$n(\omega)\omega/c$ denotes the wave number, $n(\omega)$ is the frequency-dependent refractive index and c is the velocity of light in vacuum.

Substituting Eq.(2) into Eq.(4) and performing the integral by parts,[17] we obtain the recurrence propagation equations:

$$E_0(x,z,\omega)=\frac{\sqrt{\pi}PF}{2A}f(\omega), \tag{5}$$

$$E_1(x,z,\omega)=\frac{\sqrt{2}P}{A^2w_0}\{\exp[-(A\delta w_0+Bx)^2]-\exp[-(A\delta w_0-Bx)^2]+\sqrt{\pi}BxF\}f(\omega), \tag{6}$$

$$E_m(x,z,\omega)=\frac{2\sqrt{2}Bx}{Aw_0}E_{m-1}(x,z,\omega)+2(m-1)\left(\frac{2}{A^2w_0^2}-1\right)E_{m-2}(x,z,\omega)-\frac{\sqrt{2}P}{A^2w_0}H_{m-1}(\sqrt{2}\delta)\times\{\exp[-(A\delta w_0-Bx)^2]+(-1)^m\exp[-(A\delta w_0+Bx)^2]\}f(\omega), \tag{7}$$

and

$$A=\sqrt{\frac{1}{w_0^2}+\frac{\mathrm{i}k(\omega)}{2z}}, \tag{8}$$

$$B=\frac{\mathrm{i}k(\omega)}{2Az}, \tag{9}$$

$$P=\sqrt{\frac{\mathrm{i}k(\omega)}{2\pi z}}\exp[-\mathrm{i}k(\omega)z]\times\exp\left[-\left(\frac{\mathrm{i}k(\omega)}{2z}-B^2\right)x^2\right], \tag{10}$$

$$F=\mathrm{erf}[A\delta w_0+Bx]+\mathrm{erf}[A\delta w_0-Bx], \tag{11}$$

where erf(•) stands for the error function, δ=a/w_0 denotes the truncation parameter.

In the far field Eqs.(5)–(7) become

$$E_{0f}(x,z,\omega)=\frac{1}{2}\sqrt{\frac{\mathrm{i}k(\omega)}{2z}}w_0\exp(\eta^2)[\mathrm{erf}(\delta+\eta)+\mathrm{erf}(\delta-\eta)]\times\exp[-\mathrm{i}k(\omega)z(1+\tan^2\theta/2)]f(\omega), \tag{12}$$

$$E_{1f}(x,z,\omega)=\sqrt{\frac{\mathrm{i}k(\omega)}{\pi z}}w_0\exp(\eta^2)\{\exp[-(\delta+\eta)^2]-\exp[-(\delta-\eta)^2]+\sqrt{\pi}\eta[\mathrm{erf}(\delta+\eta)+\mathrm{erf}(\delta-\eta)]\}\exp[-\mathrm{i}k(\omega)z(1+\tan^2\theta/2)]f(\omega), \tag{13}$$

$$E_{mf}(x,z,\omega)=2\sqrt{2}\eta E_{(m-1)f}(x,z,\omega)+2(m-1)E_{(m-2)f}(x,z,\omega)-\sqrt{\frac{\mathrm{i}k(\omega)}{\pi z}}w_0\cos\theta\exp(\eta^2)\times H_{m-1}(\sqrt{2}\delta)\{\exp[-(\delta-\eta)^2]+(-1)^m\exp[-(\delta+\eta)^2]\}\times\exp[-\mathrm{i}k(\omega)z(1+\tan^2\theta/2)]f(\omega), \tag{14}$$

respectively, where $\eta=\mathrm{i}k(\omega)w_0\mathrm{tg}\,(\theta/2)$, and θ=x/z is the diffraction angle.

Assume that the initial pulse takes a chirped Gaussian form[18]

$$f(t)=\exp\left[-\frac{(1+\mathrm{i}C)t^2}{2T^2}\right]\exp(-\mathrm{i}\omega_0 t), \tag{15}$$

where C denotes the chirp parameter, ω_0 is the carrier frequency and T is the pulse duration.

Substituting Eq.(15) into Eq.(3) we obtain

$$f(\omega)=\sqrt{\frac{(1-\mathrm{i}C)T^2}{1+C^2}}\exp\left[-\frac{(1-\mathrm{i}C)T^2}{2(1+C^2)}(\omega-\omega_0)^2\right]. \tag{16}$$

Thus, the original spectrum at the source plane z=0 is

$$S^{(0)}(\omega)=|f(\omega)|^2=\frac{T^2}{\sqrt{1+C^2}}\exp\left[-\frac{T^2}{1+C^2}(\omega-\omega_0)^2\right]. \tag{17}$$

From Eqs.(12)–(14) the power spectrum in the far field turns out to be

$$S_m(x,z,\omega)=|E_{mf}(x,z,\omega)|^2=S^{(0)}(\omega)M_m(x,z,\omega), \tag{18}$$

where

$$M_0(x,z,\omega)=\frac{k(\omega)w_0^2}{8z}\exp(2\eta^2)[\mathrm{erf}(\delta+\eta)+\mathrm{erf}(\delta-\eta)]^2, \tag{19}$$

$$\begin{aligned}M_1(x,z,\omega)=&\frac{k(\omega)w_0^2}{\pi z}\exp(2\eta^2)|\exp[-(\delta+\eta)^2]-\exp[-(\delta-\eta)^2]\\&+\sqrt{\pi}\eta[\mathrm{erf}(\delta+\eta)+\mathrm{erf}(\delta-\eta)]|^2,\end{aligned} \tag{20}$$

$$\begin{aligned}M_m(x,z,\omega)=&\left|2\sqrt{2}\eta E_{(m-2)f}(x,z,\omega)/f(\omega)+2(m-1)E_{(m-2)f}(x,z,\omega)/f(\omega)-\sqrt{\frac{\mathrm{i}k(\omega)}{\pi z}}w_0\right.\\&\left.\times\exp(\eta^2)H_{m-1}(\sqrt{2}\delta)\{\exp[-(\delta-\eta)^2]+(-1)^m\exp[-(\delta+\eta)^2]\}\right|^2,\end{aligned} \tag{21}$$

where $M_0(x,\ z,\ \omega)$, $M_1(x,\ z,\ \omega)$ and $M_m(x,\ z,\ \omega)$ denote the spectral modifiers of pulsed HG beams of the zeroth, first and mth orders respectively. Equations (18)–(21) indicate that the far-field power spectrum of diffracted pulsed HG beams is a product of the original spectrum and spectral modifier. The original spectrum $S^{(0)}(\omega)$ is dependent on the pulse duration T and chirp parameter C. The spectral modifier $M_m(x,\ z,\ \omega)$ depends on the truncation parameter δ, diffraction angle θ and beam order m.

Performing the inverse Fourier transform of Eq.(14), we express the far-field spatiotemporal distribution of diffracted pulsed HG beams as

$$\begin{aligned}E_{mf}(x,z,t)=&\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}E_{mf}(x,z,\omega)\\&\times\exp(-\mathrm{i}\omega t)\mathrm{d}\omega.\end{aligned} \tag{22}$$

3. Anomalous spectral behaviour of diffracted pulsed Hermite–Gaussian beams in dispersive media

Consider the propagation of diffracted pulsed HG beams in a fused-silica glass medium in the space $z>0$, whose refractive index $n(\lambda)$ is given by the Sellmeier relation[15]

$$n^2(\lambda)=1+\sum_{j=1}^{3}\frac{B_j}{1-\lambda_j^2/\lambda^2}, \tag{23}$$

where B_1=0.6961663, B_2=0.4079426, B_3=0.8974794, λ_1=0.0684043μm, λ_2=0.1162414μm, λ_3=9.8961661μm, $\lambda=2\pi c/\omega$ is the wavelength in vacuum. In the following numerical calculations we take m=1, w_0=0.05mm, ω_0=1.9fs^{-1}, z=1000mm.

Figure 1 represents the spectral modifier $M_1(x,\ z,\ \omega_0)$ versus diffraction angle θ for different values of the truncation parameter δ=0.8, 1.0 and 1.1. The positions $\theta_{\max}$, $\theta_{\min}$ of the maximum and minimum of the spectral modifier $M_1(x,\ z,\ \omega_0)$ in the range $0 < \theta < 1.5°$ are listed in Table 1. As can be seen, the positions $\theta_{\max}$ and $\theta_{\min}$ depend on the truncation parameter δ. From Eq.(18) it follows that the positions $\theta_{\min}$ of the spectral modifier $M_m(x,\ z,\ \omega_0)$ coincide with those of the power spectrum $S_m(x,\ z,\ \omega_0)$.

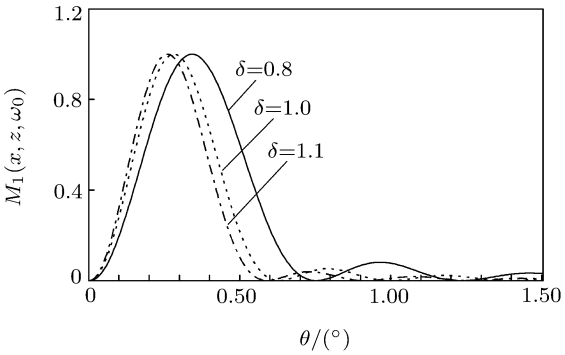


Fig.1. The spectral modifier $M_1(x,\ z,\ \omega_0)$ versus diffraction angle θ for different values of the truncation parameter δ=0.8, 1.0 and 1.1.

Table 1. The positions $\theta_{\max}$, $\theta_{\min}$ of the maximum and minimum of the spectral modifier $M_1(x,\ z,\ \omega_0)$.

δ	$\theta_{\min}/(°)$			$\theta_{\max}/(°)$		
0.8	0.7512	1.2314		0.3395	0.9617	1.4575
1.0	0.6289	0.9975	1.3836	0.2792	0.7886	1.1765
1.1	0.5884	0.9133	1.2624	0.2581	0.7283	1.0751

The normalized power spectra of a pulsed TEM_1-HG beam near $\theta_{\min}$=0.5884° are given in Fig.2 for different values of the diffraction angle θ=0.5940°, 0.6089° and 0.6250°, where δ=1.1, T=5 fs and C=1. The dotted curves denote the original spectrum. It is seen that at θ_1=0.5940° and θ_2=0.6250° the spectrum is red- and blue-shifted, respectively, whereas a remarkable spectral change takes place at the critical angle θ_c=0.6089° which is called the spectral switch.[3]

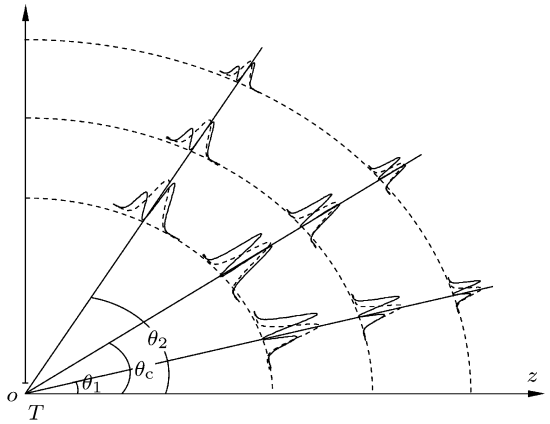


Fig.2. A schematic illustration of the anomalous spectral behaviour near $\theta_{\min}$=0.5884°.

The spectral switches of a pulsed TEM_1-HG beam are plotted in Fig.3 for different values of the pulse duration T=3, 5 and 8 fs, where C=1 and δ=1.1. The spectral parameters relating to Fig.3 are compiled in Table 2, where $\omega_{\max,i}$ (i=1, 2) denotes the frequency at which the power spectrum reaches its two equal maxima and $\omega_{\min}$ is the frequency at which the power spectrum is zero. From Fig.3 and Table 2 we see that the spectral switches appear at the critical angle θ_c=0.6571°, 0.6089° and 0.5960° for T=3, 5 and 8 fs, respectively, which are located in the vicinity of $\theta_{\min}$=0.5884° of the spectral modifier, and the spectral components $\omega_{\min}$=0.8964ω_0, 0.9667ω_0 and 0.9874ω_0 for T=3, 5 and 8 fs, respectively, at which the power spectrum has a zero value, are near the carrier frequency ω_0. At such points the phase of the spectral field component becomes singular. This is the basic property of spectral anomalies of diffracted pulsed HG beams in the far field. Additionally, Fig.3 indicates that the critical angle θ_c and relative spectral width $\Delta\omega_{\max} = |\omega_{\max,2} - \omega_{\max,1}|/\omega_0$ decrease with increasing pulse duration T.

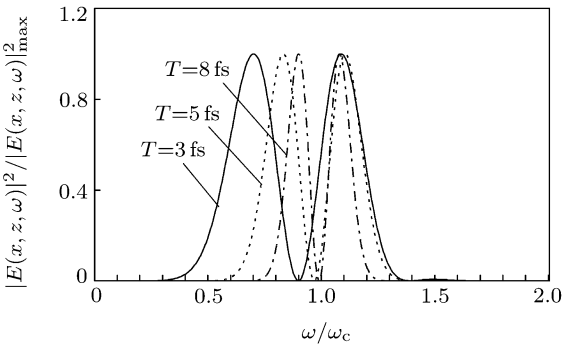


Fig.3. The spectral switches of a pulsed TEM_1 HG beam for different values of the pulse duration T=3, 5 and 8 fs.

Table 2. The spectral parameters relating to Fig.3.

T/fs	3	5	8
$\theta_{\min}/(^\circ)$	0.5884	0.5884	0.5884
$\theta_c/(^\circ)$	0.6571	0.6089	0.5960
$\omega_{\min}/\omega_0$	0.8964	0.9667	0.9874
$\omega_{\max1}/\omega_0$	0.6989	0.8296	0.8974
$\omega_{\max2}/\omega_0$	1.089	1.1023	1.0773
$\Delta\omega_{\max}/\omega_0$	0.3861	0.2727	0.1799

The critical angle θ_{ci} and relative spectral width $\Delta\omega_{\max,i}$ (i=1, 2) versus truncation parameter δ for the first- and second-order spectral switches[5] are plotted in Figs.4(a) and 4(b), respectively, where T=5 fs and C=1. It is seen that θ_c and $\Delta\omega_{\max}$ decrease as δ increases. However, the first- and second-order spectral switches disappear when $\delta > 1.58$. It means that the spectral switch of spatially fully coherent ultrashort pulsed beams is a diffraction-induced phenomenon.

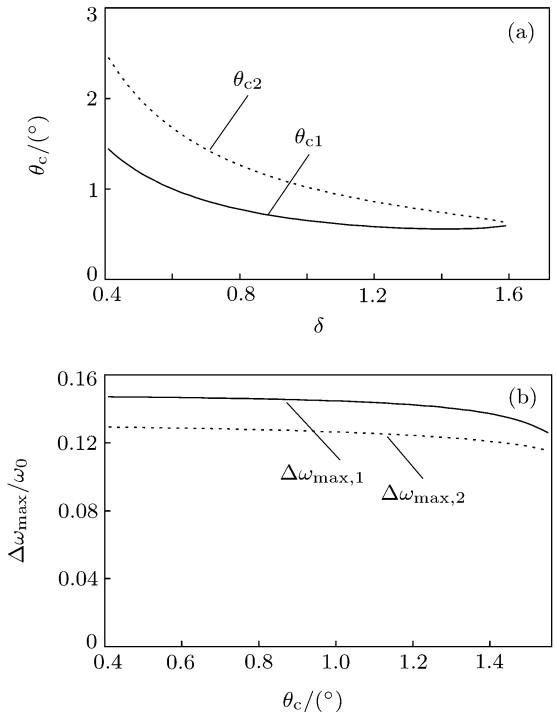


Fig.4. (a) The critical angle θ_c, and (b) relative spectral width $\Delta\omega_{\max}/\omega_0$ versus truncation parameter δ for the first- and second-order spectral switches.

The relative spectral shift is defined as

$$\frac{\delta\omega}{\omega_0} = \frac{\omega_{\max} - \omega_0}{\omega_0}, \tag{24}$$

where $\omega_{\max}$ denotes the frequency at which the power spectrum takes the maximum value. Figure 5 gives the relative spectral shift $\delta\omega/\omega_0$ versus diffraction angle θ for different values of the carrier frequency ω_0, where T=5 fs, δ=1.1, and C=1. It is known that at ω_0=1.208 fs^{-1}, $k''(\omega_0) = \mathrm{d}^2k(\omega)/\mathrm{d}\omega^2|_{\omega=\omega0} < 0$, the fused silica is an anomalous dispersive medium, and at ω_0=1.90, 2.36fs^{-1}, $k''(\omega_0) = \mathrm{d}^2k(\omega)/\mathrm{d}\omega^2|_{\omega=\omega0} > 0$, the silica is a normal dispersive medium. Figure 5 indicates that the relative spectral shift and number of spectral switches depend on the carrier frequency. In a certain range of the diffraction angle, e.g., $0 < \theta < 1.5^\circ$, the number of spectral switch increases and the relative spectral shift decreases with an increase of the carrier frequency.

The relative spectral shift $\delta\omega/\omega_0$ as a function of the diffraction angle θ in free space and fused silica is plotted in Fig.6, where T=5 fs, δ=1.1, C=1 and ω_0=1.90 fs^{-1}. As can be seen, in the range of $0 < \theta < 1.5^\circ$ the number of spectral switches in free space is less than that in the normal dispersive medium.

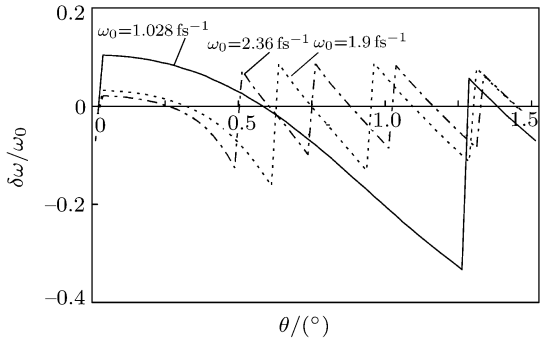


Fig.5. The relative spectral shift $\delta\omega/\omega_0$ versus diffraction angle θ for different values of the carrier frequency ω_0.

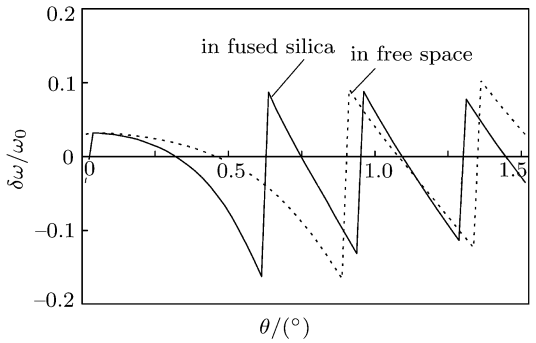


Fig.6. The relative spectral shift $\delta\omega/\omega_0$ as a function of the diffraction angle θ in free space and fused silica.

In Refs.[7–9] the potential application of spectral anomalies in optical information encoding and transmission was discussed. For such a type of applications the spectral anomalies of ultrashort pulsed beams are more attractive because there are more adjustable physical parameters than those for steady-state beams. Figures 7(a)–7(c) give the relative mean

frequency $(\varpi\text{-}\omega_0)/\,\omega_0$ versus diffraction angle θ, truncation parameter δ, pulse duration T and chirp parameter C, where the mean frequency is defined as

$$\varpi(x,z)=\frac{\int_{-\infty}^{\infty}\omega|E(x,z,\omega)|^2\mathrm{d}\omega}{\int_{-\infty}^{\infty}|E(x,z,\omega)|^2\mathrm{d}\omega}, \tag{25}$$

and T=5 fs, C=1 in (a), δ=1.1, C=1 in (b), T=5 fs, δ=1.1 in (c). It is seen that by a suitable choice of the truncation parameter δ, pulse duration T and chirp parameter C at a fixed point the mean frequency ϖ can be blue- or red-shifted. For example, as shown in Fig.7(a), at θ=0.7°, δ=1.0, the mean frequency is blue-shifted, whereas at θ=0.7°, δ=0.8, ϖ is red-shifted. If the blue- and red-shift are associated with a bit of information '0' and '1', respectively, by varying truncation parameter δ, the information encoding and transmission are realizable at θ=0.7°. In addition, Figs.7(a)–7(b) indicate that the information encoding and transmission can be also achieved by varying diffraction angle θ, e.g., at δ=1.1 the mean frequency is changed from red shift to blue shift with an increase of θ from 0.5° to 0.7° in Fig.7(a).

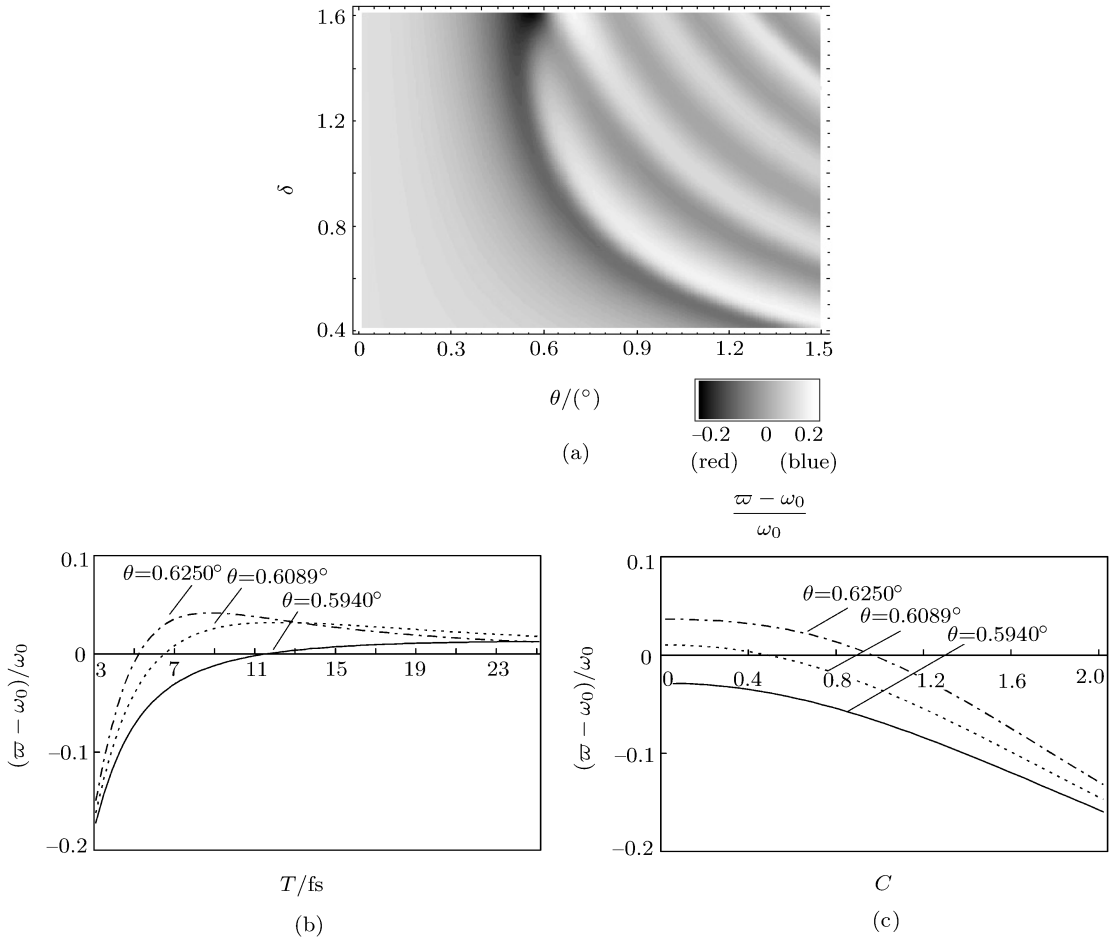


Fig.7. (a) The relative mean frequency $(\varpi\text{–}\omega_0)/\,\omega_0$ as a function of the diffraction angle θ and truncation parameter δ. The plot is more black or white as the spectrum is more red-shifted or blue-shifted, respectively. (b) The relative mean frequency versus pulse duration T for different values of the diffraction angle θ=0.5940°, 0.6089° and 0.6250°. (c) The relative mean frequency versus chirp parameter C for different values of the diffraction angle θ=0.5940°, 0.6089° and 0.6250°.

4. Concluding remarks

This paper derives the recurrence analytical expressions for the power spectra of diffracted pulsed HG beams in dispersive media and studies the anomalous spectral behaviour of pulsed HG beams in the far field. It shows that the spectral switch takes place at the critical position θ_{c} in the vicinity of the position $\theta_{\min}$ of the minimum of the spectral modifier (i.e., the power spectrum), and the power spectrum takes a zero value at the frequency $\omega_{\min}$ which is near the carrier frequency ω_0. The spectral switch of pulsed HG beams

is diffraction-induced because the spectral switch disappears as the truncation parameter is larger than a certain value. The anomalous spectral behaviour depends on the pulse parameters, truncation parameter, and dispersive property of the medium. The information encoding and transmission could be realizable by a suitable variation of the truncation parameter, pulse duration, chirp parameter and diffraction angle. For the sake of simplicity, our mathematical manipulation is presented in the two-dimensional case, the extension to the three-dimensional pulsed HG beams is straightforward because the x and y variables in the paraxial approximation are separable.

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