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Dynamical behaviour of a controlled vibro-impact system*

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In this paper, we give a controlled two-degree-of-freedom (TDOF) vibro-impact system based on the damping control law, and then investigate the dynamical behaviour of this system. According to numerical simulation, we find that this control scheme can suppress chaos to periodic orbit successfully. Furthermore, the feasibility and the robustness of the controller are confirmed, separately. We also find that this scheme cannot only suppress chaos, but also generate chaos in this system.

Keywords: damping control law, chaos, two-degree-of-freedom (TDOF), vibro-impact system
PACC: 0547

1. Introduction

Vibro-impact systems, a kind of non-smooth systems, which arise when the components collide with each other or with rigid obstacles, are often encountered in engineering fields. So the systems have been studied by many investigators these years. Many interesting phenomena in these systems, such as grazing bifurcation,[1] and torus bifurcation,[2] have been found in the past several years. Some unique methods have been proposed to investigate the behaviour of these systems. For example, mean Poincaré map and superposition theorem were presented to explore a kind of non-smooth stochastic system in Ref.[3]. In Refs.[4, 5] a certain transformation was applied to researching the behaviours of a series of nonlinear non-smooth systems, which can change a system with impact to one without impact. As is known, chaotic behaviour, which exist widely in smooth systems, are also exhibited in these vibro-impact systems.[2] So, the study of these systems with certain controllers become quite important and should be paid more attention to.

Since the Ott–Grebogi–Yorke (OGY) method was proposed to control chaos in physical systems,[6] a lot of effective methods have been presented successively.[7−15] For instance, Pyragas proposed an important chaos control strategy by using a delayed feedback signal in Ref.[11], and he also considered the dynamical properties of a chaotic attractor to stabilize unstable periodic orbits. Huang[12] established a feedback scheme to stabilize nonlinearly finite-dimensional chaotic systems based on the invariance principle of differential equations. However, most of these results were proposed for smooth systems, and the conclusions for non-smooth systems were very limited. References [13, 14] applied some methods to studying the behaviour of the controlled vibro-impact systems successfully, but the results were limited to single-degree-of-freedom systems. It is well known that the multi-degree-of-freedom and high dimension are the characteristics of vibro-impact systems, so it is necessary to develop some methods to study the motions of the controlled multiple freedom system with impact.

In this paper, the damping control law[13] is adopted to implement the controlling in a two-degree-of-freedom TDOF vibro-impact system. Two piecewise-linear absolute value functions are used to describe the control signals. The rest of this paper is organized as follows: in Section 2 a TDOF vibro-impact system is introduced, with the control functions considered. Section 3 shows the motions of the controlled system by numerical simulation. The conclusions are presented in Section 4.

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2. Theoretical model

The impact oscillator with TDOF that we consider in the paper can be depicted as Fig.1,[2] where x_1 and x_2 are the displacements of mass 1 and mass 2, respectively, and both $f_1(t)$ and $f_2(t)$ are the external excitations. The impact occurs just when the displacement x_1 is equal to b. In the following discussion, the non-dimensional equations of the motion are introduced,[2] which are

$$\begin{aligned} &\ddot{x}_1 + 2a\dot{x}_1 - 2a\dot{x}_2 + x_1 - x_2 = (1-F)\sin(\omega t), \\ &m\ddot{x}_2 - 2a\dot{x}_1 + 2ac\dot{x}_2 - x_1 + kx_2 = F\sin(\omega t), \end{aligned} \quad x_1 < b, \tag{1}$$

$$\dot{x}_{1+} = -r\dot{x}_{1-}, \qquad x_1 = b, \tag{2}$$

where a, m, c, k, F and ω are the non-dimensional quantities, r is the restitution coefficient. Equation (2) shows the mutation of the speed of mass 1 when the impact happens. Thus $\dot{x}_{1+}$ and $\dot{x}_{1-}$ are actually the rebound and the impact speeds, respectively.

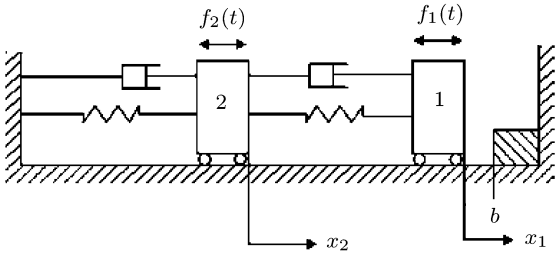


Fig.1. Schematic of the TDOF vibro-impact system.

Considering the damping control law, the controllers could be chosen as $u_1 = \mu|\dot{x}_1|$ and $u_2 = \mu|\dot{x}_2|$, and thus the resulting equations of the motion between two successive impacts can be given by

$$\begin{aligned} &\ddot{x}_1 + 2a_1\dot{x}_1 - 2a_2\dot{x}_2 + x_1 - x_2 = (1-F)\sin(\omega t), \\ &m\ddot{x}_2 - 2a_1\dot{x}_1 + 2a_2c\dot{x}_2 - x_1 + kx_2 = F\sin(\omega t), \end{aligned} \tag{3}$$

where

$$a_i = \begin{cases} a-\mu, & \dot{x}_i \geq 0, \\ a+\mu, & \dot{x}_i < 0, \end{cases} \quad (i = 1, 2) \tag{4}$$

and μ is the coefficient of the controllers.

According to the above, a system with the control functions is generated. The numerical simulation will be performed in the following.

3. Numerical results

Two Poincaré sections will be applied to exhibiting the motions of the system, they are $\Sigma_1 = \{(x_1, \dot{x}_1, x_2, \dot{x}_2, t)\,|t = \mathrm{mod}\,(2\pi/\omega)\}$ and $\Sigma_2 = \{(x_1, \dot{x}_1,\ x_2,\ \dot{x}_2, t)\ |x_1 = b,\ \dot{x}_1 = \dot{x}_{1-}\}$, respectively. The system with parameter values $a = 0.2$, $m = 2.0$, $c = k = 6.0$, $F = 0$, $\omega = 2.6$, $b = 0$, and $r = 0.8$ has been chosen for analysis. Under this condition, the phase portrait and the Poincaré section of the system without control are shown in Fig.2, and we can see that the chaotic behaviour comes into being.

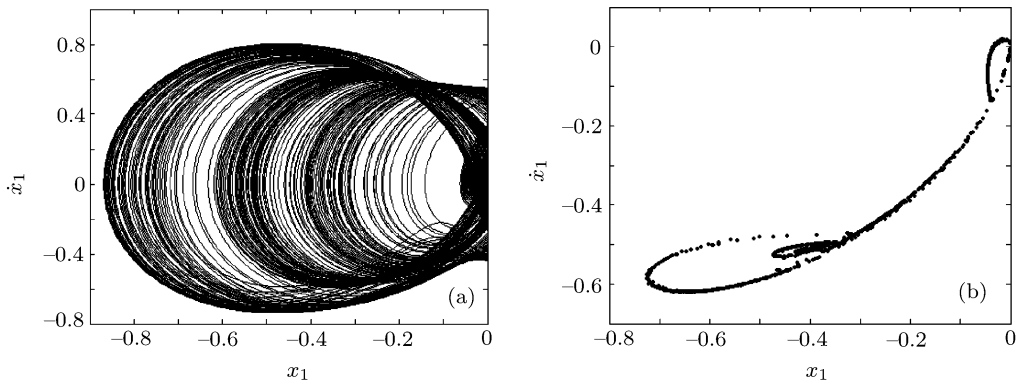


Fig.2. Chaotic behaviour of mass 1: (a) phase portrait; (b) Poincaré section Σ_1.

Then we plot the bifurcation diagrams of the system with control functions, i.e $\dot{x}_1$ versus the coefficient of the controllers μ as shown in Fig.3(a) to investigate the influence of the control inputs on the chaotic attractor. It is found that the behaviour of the system changes from chaos to period-1 orbit through a reverse period-doubling bifurcation as the increase of μ. In Fig.3(b) we show the phase portraits of a period-1 orbit (for $\mu = 0.13$), a period-2 orbit (for $\mu = 0.1$) and a period-4 orbit (for $\mu = 0.06$) as samples of periodic motion controlled in the diagrams. Thus, the chaotic behaviour of the system is controlled to periodic orbits by the damping control law successfully.

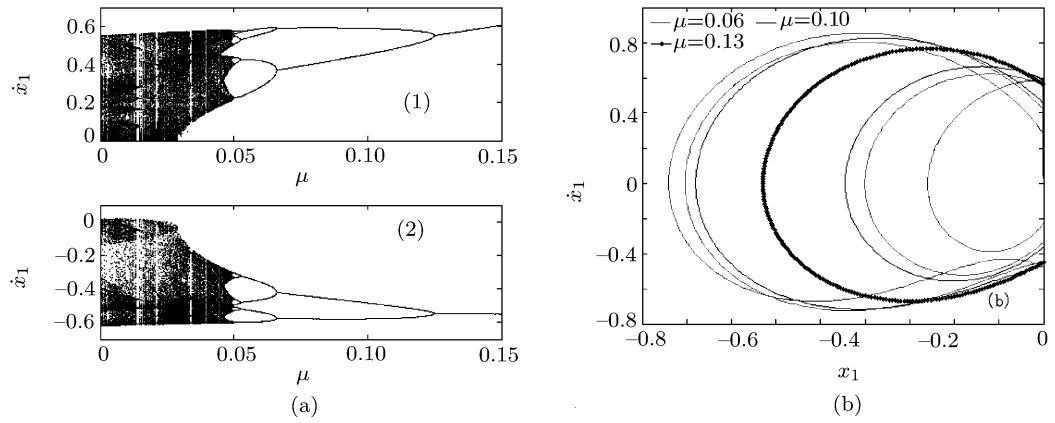


Fig.3. (a) Bifurcation diagrams on Σ_2 (1) and Σ_1 (2), and (b) phase portraits of periodic motions.

The evolution of u_1 for the controlled period-1 orbit ($\mu = 0.13$) is displayed in Fig.4(a) to compare the acceleration of mass 1 $\ddot{x}_1$ between with and without control. Figure 4(b) shows similar comparison between u_2 and $\ddot{x}_2$. Then we can find that the control inputs, i.e. u_1 and u_2, are both small-amplitude signals and much smaller than the maximum amplitudes of the corresponding accelerations. It is implied that the implementation of the control needs only a small quantity of energy. In other words, the method is feasible.

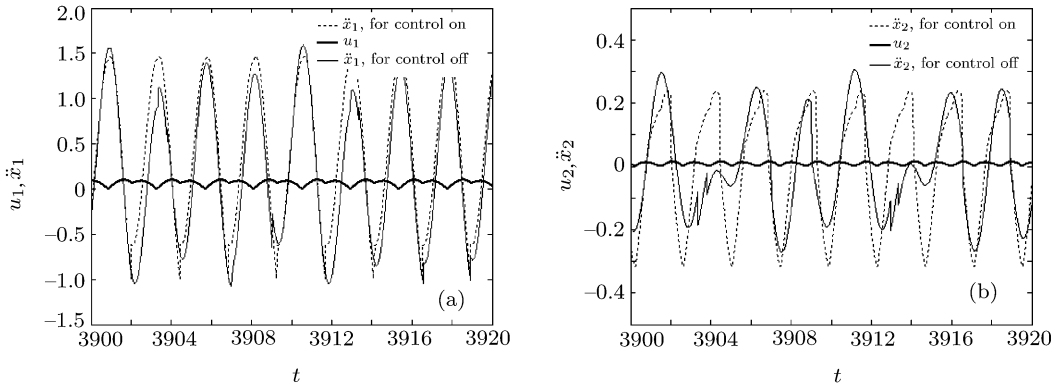


Fig.4. Comparison between the evolutions of control input and acceleration with time: (a) u_1 and $\ddot{x}_1$ versus t; (b) u_2 and $\ddot{x}_2$ versus t.

It is necessary to examine the robustness of the controllers for this system. So we may suppose that there are random perturbations exerted on the external excitations as shown below:

$$(1-F)\sin(\omega t) \to (1-F)(\sin(\omega t) + \sigma N),$$
$$F\sin(\omega t) \to F(\sin(\omega t) + \sigma N), \qquad (5)$$

where N is a noise to which the normal distribution is subjected, and σ represents the intensity. In Figs.5(a) and 5(b) we show the evolutions of $\dot{x}_1$ without and with noise, collected just after every period $T(T = 2\pi/\omega)$, as a function of the period number n, respectively. The controllers are switched on only at times $n = 1000$ (for $\mu = 0.06$), $n = 3000$ (for $\mu = 0.1$) and $n = 5000$ (for $\mu = 0.13$), while at times $n = 2000$ and $n = 4000$ the controllers are turned off. We can find that the method is also effective even for quite a strong noise (for $\sigma = 0.1$). It is implied that the method adopted here is robust.

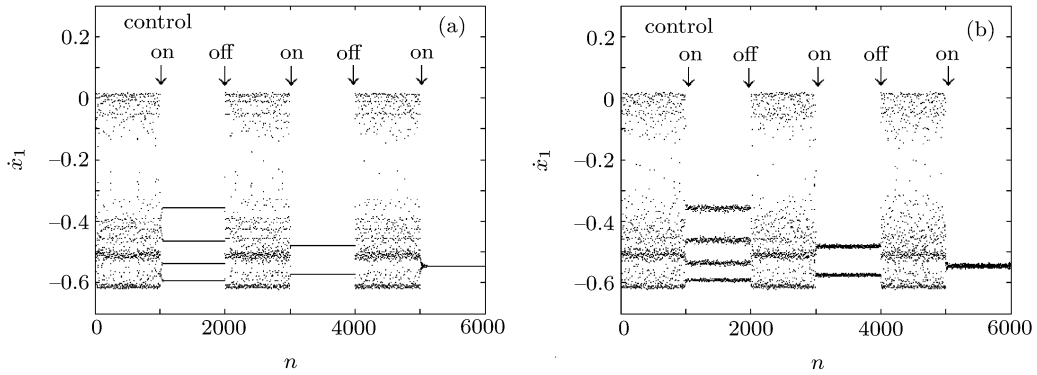


Fig.5. Variations of $\dot{x}_1$ (just after every period T) with period number n without noise (a) and with noise ($\sigma = 0.1$) (b).

In addition, we also find that this control scheme cannot only suppress chaos in the system, but also generate chaos. Fix the parameters at $a = 0.5$, $m = 2.0$, $c = k = 6.0$, $F = 0$, $\omega = 2.3$, $b = 0$, and $r = 0.8$, we show the bifurcation diagrams of $\dot{x}_1$ against the controller coefficient μ in Fig.6.

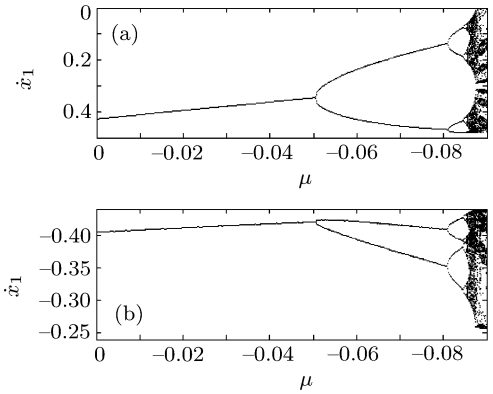


Fig.6. Bifurcation diagrams on Σ_2 (a) and Σ_1 (b) separately.

It is shown that the periodic motion of the system changes to chaotic motion through a period-doubling bifurcation as μ decreases from 0 to –0.09. Figure 7 exhibits the comparisons of u_1 (for $\mu = -0.09$) and $\ddot{x}_1$, illustrating the availability of the result as done in Fig.4.

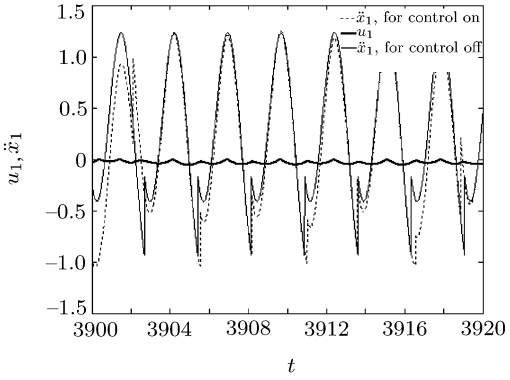


Fig.7. Comparison between control input u_1 and acceleration $\ddot{x}_1$ in different cases.

4. Conclusion

According to the damping control law, a feedback control technique, i.e. a controlled TDOF vibro-impact system, is presented and its behaviour is studied in this paper. The control signals are expressed by two piecewise-linear absolute value functions. The numerical simulation shows that this method is applicable to suppressing chaos and robust against even a strong noise for the system considered here. Finally, we find that this scheme cannot only suppress chaos, but also generate chaos in the system.

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