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Evolution of spiral wave and pattern formation in a vortical polarized electric field*

Ma Jun(马 军)[a)†], Yi Ming(易 鸣)[b)],
Li Bing-Wei(李炳伟)[c)], and Li Yan-Long(李延龙)[a)]

[a)] *Department of Physics, Lanzhou University of Technology, Lanzhou 730050, China*
[b)] *Department of Physics, Central China Normal University, Wuhan 430079, China*
[c)] *Zhejiang Institute of the Modern Physics, Zhejiang University, Hangzhou 310027, China*

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In this paper, the evolution of the pattern transition induced by the vortical electric field (VEF) is investigated. Firstly, a scheme is suggested to generate the VEF by changing the spatial magnetic field. Secondly, the VEF is imposed on the whole medium, and the evolutions of the spiral wave and the spatiotemporal chaos are investigated by using the numerical simulation. The result confirms that the drift and the breakup of the spiral wave and the new net-like pattern are observed when different polarized fields are imposed on the whole medium respectively. Finally, the pattern transition induced by the polarized field is discussed theoretically.

Keywords: spiral wave, pattern, vortical electric field (VEF)
PACC: 0545, 8730C, 0565, 8220M

1. Introduction

Spiral wave and spatiotemporal chaos[1,2] are observed in the chemical,[3−6] physical[4] and biological[7−9] systems. They have attracted more attention since some important evidences showed that the appearance of spiral wave was linked to a kind of cardiac arrhythmia[10] and the instability[11−13] of spiral wave could induce the ventricular fibrillation (VF).[14−16] Therefore, many schemes have been proposed to prevent the spiral wave from appearing and breaking up. For example, Wang *et al* [17] and Wang[18] proposed a weakly spatial perturbation for stabilizing and eliminating useless spatial pattern. Hendery *et al* [19] suggested that the local inhomogeneity and parameter perturbation could be introduced into the complex Ginzburg–Landau equation (CGLE) to induce target waves.[19] And it was proved that local periodical forcing was effective to eliminate the harmful spiral wave and spiral turbulence and the Winfree turbulence,[20−22] and the time-delay feedback scheme was also used to prevent the spiral wave from breaking up.[23−26] Furthermore, the relation between the complex Ginzburg–Landau equation, which is used to describe the oscillatory medium, and the reaction–diffusion system was discussed in Ref.[27], in which some schemes were proposed to remove the spiral wave and spiral turbulence in the excitable media,[28−35] including the scheme of local intermittent shock,[28] parameter perturbation,[29] local self-coupling feedback,[30] chaotic signal forcing[31,32] and stochastic phase perturbation,[33] etc. which are different from the pattern transition induced by the polarized field in Refs.[35, 36].

In this paper, we investigate the evolution and transition of pattern induced by the vortical electric field (VEF), which can be generated by using alternately changeable magnetic field in a long and straight solenoid. We will observe and discuss the transition between patterns when the VEF is introduced into the whole medium based by using the numerical simulation.

2. The distribution of vortical electric field

At first, a long and straight solenoid is used to generate a VEF, and its intensity, E is described by

$$\oint_L \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}\psi}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{S}, \qquad (1)$$

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†E-mail: hyperchaos@163.com

where Ψ is the flux in the confined area, and B is the intensity of the magnetic field. Equation (1) is equivalent to the following expression (2) when the area of magnetic field is big enough:

$$E = -\frac{r}{2}\frac{\mathrm{d}B}{\mathrm{d}t}, \tag{2}$$

where r is the distance between the centre of the solenoid and a field point. On the other hand, Eq.(1) can be transformed into the following expression (3) when the area of magnetic field is small.

$$E = -\frac{R^2}{2r}\frac{\mathrm{d}B}{\mathrm{d}t}, \tag{3}$$

where R is the diameter of the solenoid, and r is the distance from the field point to the centre of the solenoid. In this paper, introduced it a polarized field which is described in expression (2). Clearly, the VEF can be divided into two directions along the x- and y-axes in the Descartes, respectively. For example, a clockwise vortical electric field is given by

$$E_x = E\sin(\theta), \quad \text{and} \quad E_y = -E\cos(\theta), \tag{4}$$

where E is the external VEF and θ is the angle of the field point with respect to the positive x-axes. In fact, E can be evaluated from Eq.(1). For example, a constant $E = E_0 r$ and a changeable $E = E_0\cos(\omega t)r$ will be discussed in this paper. In the experiments, the intensity of the magnetic field is changeable to generate different VEFs.

3. Pattern transition in the polarized reaction–diffusion system

A reaction–diffusion equation system is given by

$$\begin{aligned} \partial_t u &= f(u,\ v) + D_u\nabla^2 u; \\ \partial_t v &= g(u,\ v) + D_v\nabla^2 v, \end{aligned} \tag{5}$$

where $f(u, v)$ and $g(u,\ v)$ are used to determine the local kinetics of the system, respectively. The two-dimensional Laplace operator $\nabla = \partial^2/\partial x^2 + \partial^2/\partial y^2$, and u and v are the activator and inhibitor variables, D_u and D_v are diffusion coefficients. The polarized medium is described by

$$\begin{aligned} \partial_t u &= f(u,\ v) + E_x\partial u/\partial x + E_y\partial u/\partial y + D_u\nabla^2 u; \\ \partial_t v &= g(u,\ v) + D_v\nabla^2 v. \end{aligned} \tag{6}$$

For simplicity, the famous modified Fitzhugh–Nagumo model (MFHNM)[37] is investigated under this scheme, and a clockwise rotating spiral wave is selected as the initial state and its evolution induced by a vortical electric field is investigated by using the numerical simulation. The MFHNM[37] subjected to a polarized electric field is given by

$$\begin{aligned} \partial_t u =& \varepsilon^{-1}u(1-u)(u-(v+b)/a) + E_x\partial u/\partial x \\ &+ E_y\partial u/\partial y + D\nabla^2 u; \\ \partial_t v =& f(u) - v, \end{aligned} \tag{7}$$

where u and v determine the activator and the inhibitor respectively, (for more details see Ref.[37]; the function $f(u)$=0 at $u < 1/3$, $f(u) = 1 - 6.75u(u-1)^2$ at $1/3 \le u \le 1$; $f(u)$=1 at $u > 1$; the parameter ε is the ratio of the time scale of the fast activator to that of the slow inhibitor variable; parameters a and b are selected as appropriate values such that a spiral wave could be generated under appropriate initial conditions. In our numerical simulation, parameters a=0.84, b=0.07, ε=0.03, and diffusion coefficient D=1 are required for a stable rotating spiral wave, while ε=0.073 is adopted for spatiotemporal chaos. And the time step size h=0.02, the space distance $\Delta x = \Delta y$=100/256, no-flux boundary condition and the Euler forward difference are used in the numerical algorithm.

Firstly, a spatial field $E = E_0 r$ (the magnetic field $B = k_0 t$) is investigated, and the images are shown in Fig.1 for $E = E_0 r = 0.03328r$ (or $\mathrm{d}B/\mathrm{d}t$=0.06656).

The numerical simulation results confirm that the rotating spiral wave is removed within 60 time units and the tip of the spiral wave is pulled out of the medium. It is confirmed that the spiral wave becomes sparse under a weaker E_0, while a stronger E_0 will speed up the death of the rotating spiral wave. Furthermore, an anticlockwise constant VEF fails to kill the spiral wave even when the intensity of the field is sufficiently big. As far as we know, if the direction of the vortical electric field is consistent with the direction of the rotating spiral wave, the tip of the spiral wave will be pulled out of the centre, and a collision with the boundary will make the spiral wave rapidly killed, otherwise, the tip of the spiral wave will be attracted to the centre of the medium, as a result, the spiral wave remains alive.

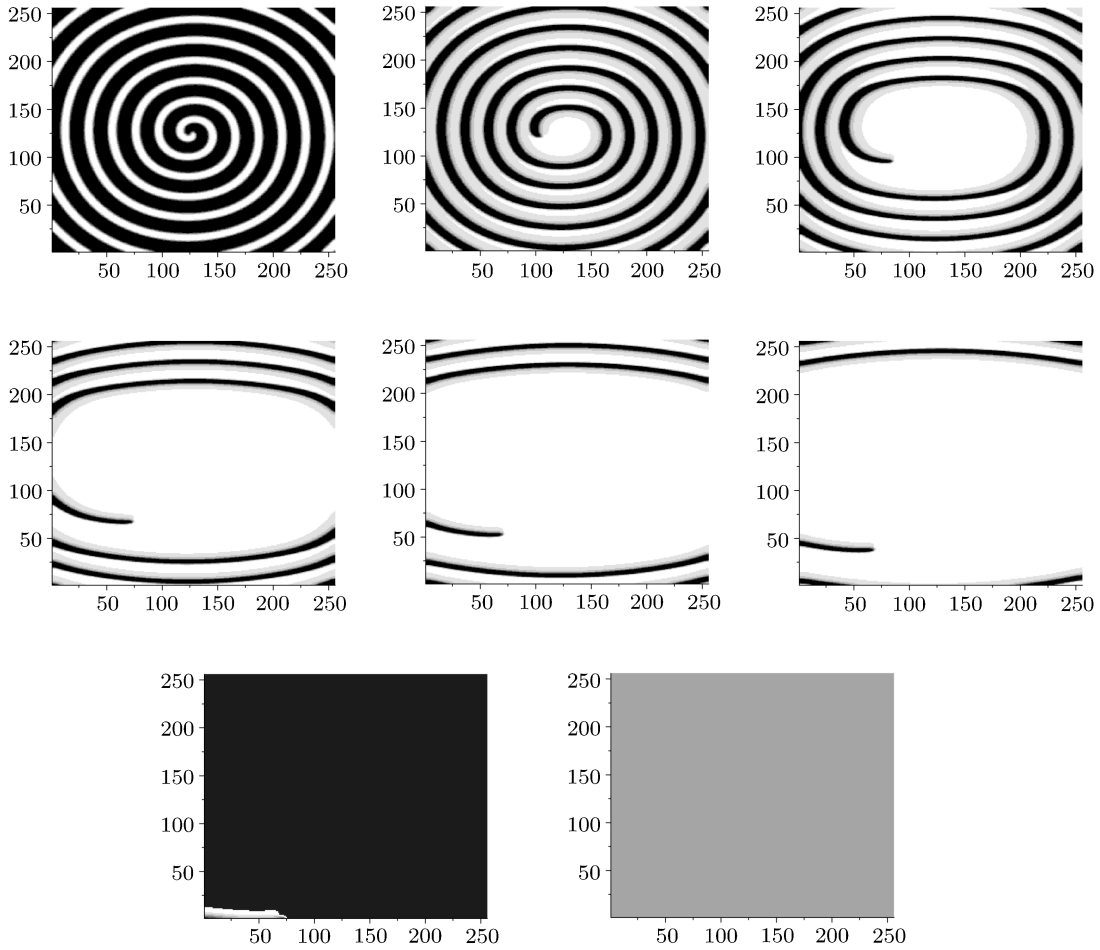


Fig.1. Evolution of the spiral wave at $E = E_0 r = 0.03328r$ (or $\mathrm{d}B/\mathrm{d}t$=0.06656) with time for t=0, 5, 15, 25, 30, 35, 55, 60 time units (the order is from left to right and from upper to lower, and the final state is shown in grey).

Secondly, we investigate the destiny of the spiral wave when the VEF varies periodically. In our numerical simulation, $E = E_0 \cos(\omega t)$. For simplicity, the frequency in the vicinity of eigenfrequency of the rotating spiral wave is investigated, and then the other frequencies will be discussed later. Here, E=0.03328$r$cos $(2\pi t/4.17)$ (or $\mathrm{d}B/\mathrm{d}t$=0.06656$r$cos $(2\pi t/4.17)$) is testified, and the numerical results are shown in Fig.2.

The numerical simulation results prove that the spiral wave is eliminated within 40 time units, and the whole medium becomes homogeneous when the VEF is imposed on the whole medium. As far as we know, the medium is polarized and the self-organization is enhanced when an appropriate intensity of the external VEF varies periodically. Furthermore, more numerical simulation results prove that a stronger VEF can shorten the transient period to reach a homogeneous state. For example, it takes about 39 time units for E=0.04096$r$cos$(2\pi t/4.17)$ and 8 time units for E=0.0512$r$cos$(2\pi t/4.17)$ to remove the stable spiral wave. The spiral wave just becomes sparse as compared with the previous one in the presence of a weaker VEF. Then other different frequencies of the external VEF are also investigated, and it is confirmed that the final states are determined by the intensity and the phase or the frequency of the external field. The whole medium becomes homogeneous at an appropriate intensity of external field (not too weak) even if the frequency is not equivalent to the eigenfrequency of the rotating spiral wave. For example, the transient period is about 16 time units at E=0.06144$r$cos$(\pi t/4.17)$ and 200 time units at E=0.04096$r$cos$(4\pi t/4.17)$, and the breakup of spiral wave is observed when the intensity of the external field is not sufficiently strong.

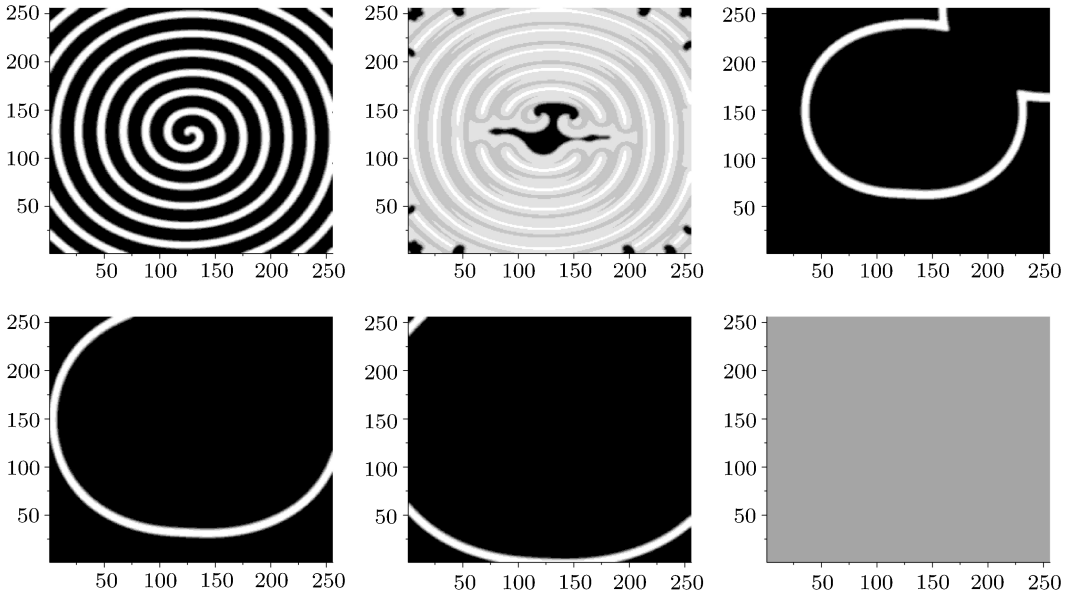


Fig.2. Evolution of the spiral wave at E=0.03328$r$cos (2πt/4.17) (or dB/dt=0.06656$r$cos (2πt/4.17)), with time for t=0, 10, 24, 28, 32, 40 time units (the order is from left to right, from upper to lower, and the final state is shown in grey).

Another interesting problem is about the evolution of the spatiotemporal chaos subjected to the external VEF. We select a parameter ε=0.073 such that the spatiotemporal chaos can be induced;[37] our interest is to investigate the evolution of the spatiotemporal chaos when the external VEF is imposed on the whole medium. And the numerical results are plotted in Fig.3.

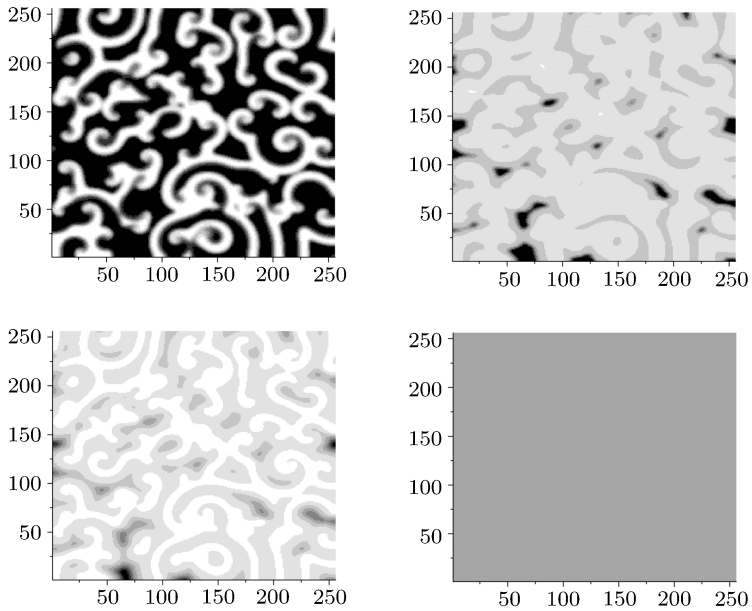


Fig.3. Evolution of the spatiotemporal chaos at E=0.03328r (or dB/dt=0.06656r) with time for t=0, 2, 8, 14 time units (the order is from left to right and from upper to lower, and the final state is shown in grey).

The numerical simulation results confirm that the whole medium can reach a homogeneous state within a few time units and the spatiotemporal chaos is suppressed when the VEF is constant. Then a stronger intensity of the VEF is investigated, and the whole medium still reaches a homogeneous state quickly.

Finally, we are to investigate the evolution of the spatiotemporal chaos when the external VEF varies periodically. Different intensities and periods of the external VEF are investigated, and the relevant numerical results are plotted in the Figs.4 to 8, and the images illustrate the evolution of the spatiotemporal chaos with time.

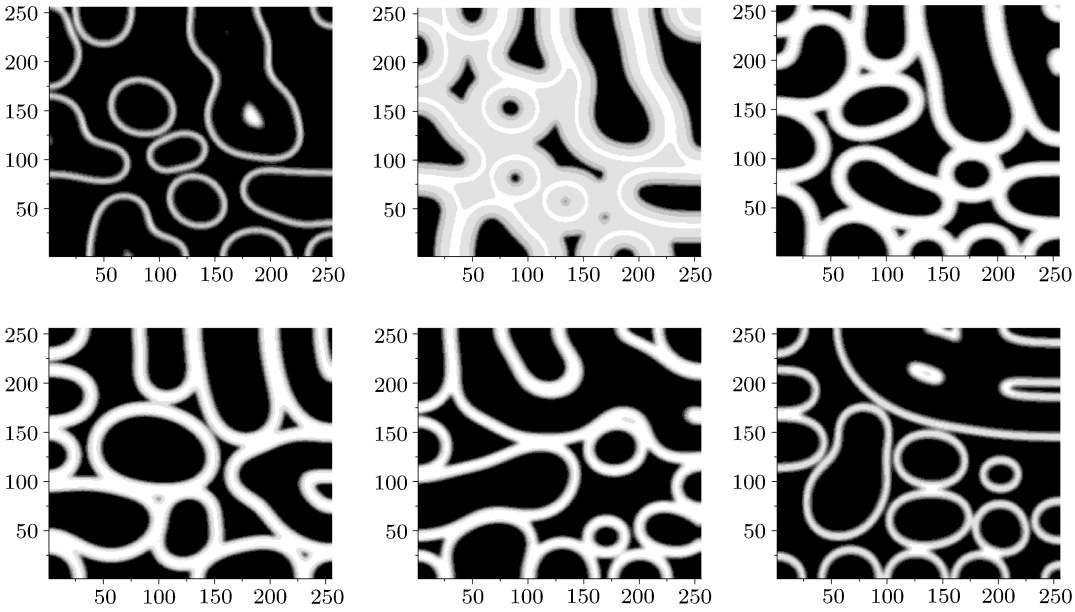


Fig.4. Evolution of the spatiotemporal chaos at $E = 0.03328r\cos(2\pi t/4.17)$ (or $\mathrm{d}B/\mathrm{d}t{=}0.06656r\cos(2\pi t/4.17)$) with time for t=40, 60, 100, 200, 500, 800 time units (the order is from left to right and from upper to lower, and the final state is shown in grey).

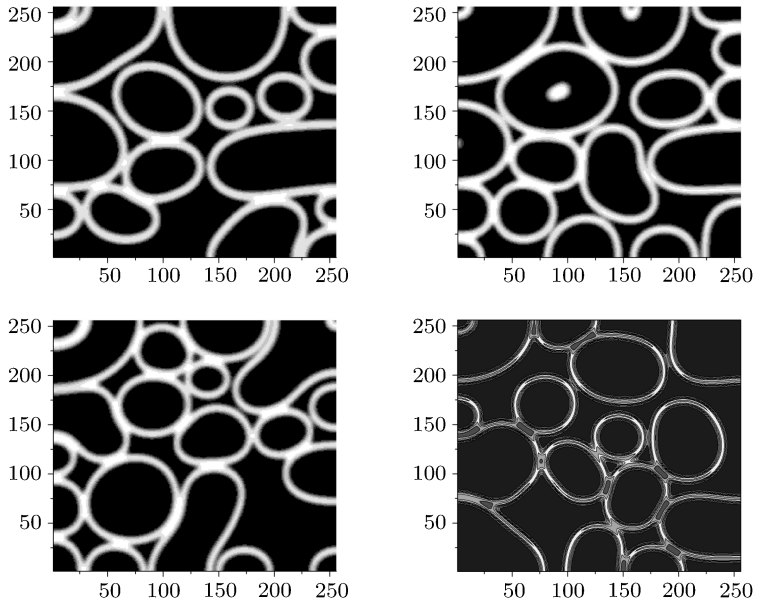


Fig.5. Evolution of the spatiotemporal chaos at $E{=}0.04096r\cos(2\pi t/4.17)$ (or $\mathrm{d}B/\mathrm{d}t{=}0.08192r\cos(2\pi t/4.17)$) with time for t=600, 700, 800, 900 time units (the order is from left to right, and upper from to lower, and the final state is shown in grey).

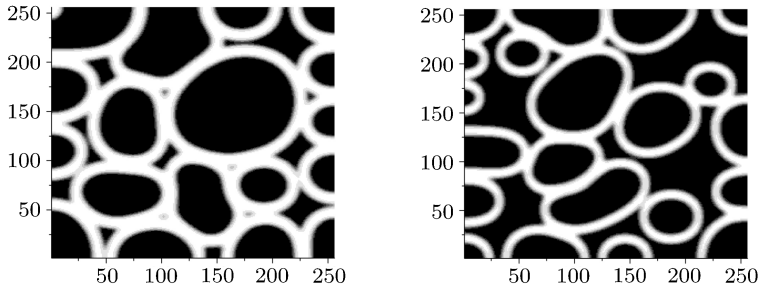


Fig.6. Evolution of the spatiotemporal chaos at E=0.04608$r$cos(2πt/4.17) (or dB/dt=0.09216$r$cos(2πt/4.17)) with time for t=250, 550 time units (the order is from left to right, and upper from to lower, and the final state is shown in grey).

It is confirmed that new pattern appears as the external VEF is adjusted. Then other frequencies are investigated and the spatiotemporal chaos develops into a new pattern that looks like a net.

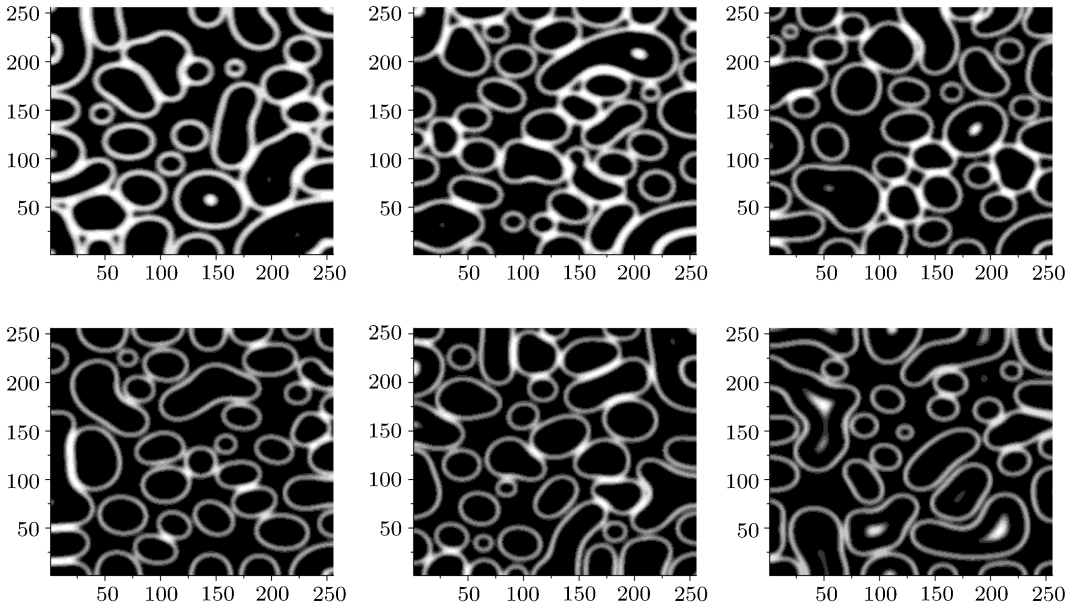
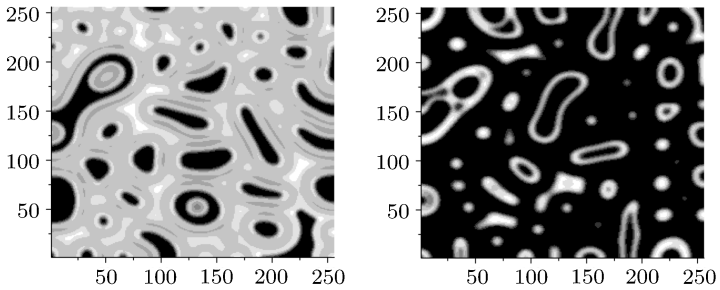


Fig.7. Evolution of the spatiotemporal chaos at E=0.04096$r$cos(3πt/4.17) (or dB/dt=0.08192$r$cos(3πt/4.17)) with time for t= 200, 400, 600, 700, 800, 1100 time units (the order is from left to right and from upper to lower, and the final state is shown in grey).



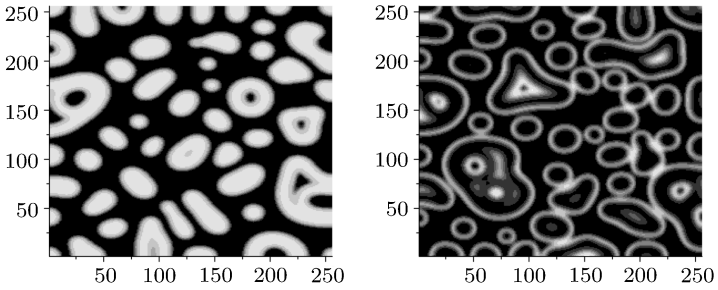


Fig.8. Evolution of the spatiotemporal chaos at E=0.04096$r$cos(3.2πt/4.17) (or dB/dt=0.08192$r$cos(3.2πt/4.17)) with time for t= 200, 300, 400, 500 time units (the order is left to right and from upper to lower, and the final state is shown in grey).

All the above numerical simulation results confirm that net-like patterns can be generated and developed from the spatiotemporal chaos state when the external VEF is changed periodically. In our view, it is the external VEF that enhances the mutual coupling and interaction among the cells in the medium, and the dynamics of the system is thus changed.

4. Conclusion

In this paper, the evolution of spiral wave and spiral turbulence induced by a real vortical electric field is investigated and discussed. It is suggested that the real polarized field can be generated from an alternately changeable magnetic field in a long and straight solenoid. It is confirmed in our numerical simulation that the tip of spiral wave can be removed and the whole medium becomes homogeneous soon after an appropriate intensity of VEF is applied, otherwise, the breakup of spiral wave will be observed. The spatiotemporal chaos or spiral turbulence can be removed in a few time units when the constant VEF is imposed on the whole medium, while new net-like pattern appears when the external VEF changes, as a sine or cosine condition, with time. As far as we know, it is due to the VEF by which the tip of the spiral wave can be driven out from the centre of the medium, thus the medium becomes homogeneous. Finally, the obtained numerical results need confirming in experiment.

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References

[1] Cross M C and Hohenberg P 1993 *Rev. Mod. Phys.* **65** 851
[2] Kuramoto Y 1984 *Chemical Oscillations, Waves, and Turbulence* (New York: Springer)
[3] Winfree A T 1972 *Science* **175** 634
[4] Vanag V K and Epstein I R 2001 *Science* **294** 835
[5] Ouyang Q and Flesselles J M 1996 *Nature* (London) **379** 143
[6] Steinbock O, Zykov V and Müller S C 1993 *Nature* (London) **366** 322
[7] Lee K J, Cox E C and Goldstein R E 1996 *Phys. Rev. Lett.* **76** 1174
[8] Lee K J, Cox E C and Goldstein R E 2001 *Phys. Rev. Lett.* **87** 068101
[9] Panfilov A V, Müller S C and Zykov V S 2000 *Phys. Rev.* E **61** 4644
[10] Winfree A T 1998 *Chaos* **8** 1
[11] Bär M, Brusch L and Or-Guil M 2004 *Phys. Rev. Lett.* **92** 119801
[12] Zhou L and Ouyang Q 2000 *Phys. Rev. Lett.* **85** 1650
[13] Zhao Y K, Wang G R and Chen S G 2007 *Chin. Phys.* **16** 1159
[14] Sinha S, Pande A and Pandit R 2001 *Phys. Rev. Lett.* **86** 3678
[15] Nash M P and Panfilov A V 2004 *Prog. Biophys. Molec. Biol.* **85** 501
[16] Winfree A T 1994 *Science* **266** 1003
[17] Wang P Y, Xie P and Yin H W 2003 *Chin. Phys.* **12** 647
[18] Wang P Y 2000 *Phys. Rev.* E **61** 5120
[19] Hendery M, Nam K, Guzdar P and Ott E *Phys. Rev.* E **62** 7627
[20] Hu BamBi and Zhang H 2003 *Int. J. Mod. Phys.* B **17** 3988
[21] Zhang H, Cao Z J, Wu N J, Ying H P and Hu G 2005 *Phys. Rev. Lett.* **94** 188301
[22] Alonso S, Sagues F and Mikhailov A S 2003 *Science* **299** 1722

[23] Xiao J H, Hu G and Zhang H 2005 *Europhys. Lett.* **69** 29
[24] Zykov V S and Engel H 2002 *Phys. Rev.* E **66** 16206
[25] Pollmann M, Bertram M and Rotermund H H 2001 *Chem. Phys. Lett.* **346** 123
[26] Zykov V S, Mikhailov A S and Müller S C 1997 *Phys. Rev. Lett.* **78** 3398
[27] Shao X, Ren Y and Ouyang Q 2006 *Chin. Phys.* **15** 513
[28] Ma J, Ying H P and Li Y L 2007 *Chin. Phys.* **16** 955
[29] Jun M, Li Y L, Jiang J L, Liu Y J and Wang C N 2006 *Phys.* A **369** 387
[30] Ma J, Gao J H, Wang C N and Su J Y *Chaos, Solitons and Fractals* doi:10.1016/j.chaos.2006.11.039
[31] Ma J, Wang C N, Li Y L and Li S R 2007 *Chaos, Solitons and Fractals* **33** 965
[32] Ma J, Ying H P and Pu Z S 2005 *Chin. Phys. Lett.* **22** 1065
[33] Ma J, Jin W Y, Li Y L and Chen Y 2007 *Acta Phys. Sin.* **56** 2456 (*in Chinese*)
[34] Ma J, Pu Z S, Feng W J and Li W X 2005 *Acta Phys. Sin.* **54** 4602 (in Chinese)
[35] Ma J, Ying H P and Pu Z S 2005 *Chin. Phys. Lett.* **22** 2176
[36] Chen J X, Zhang H and Li Y Q 2006 *J. Chem. Phys.* **124** 14505
[37] Hildebrand M, Bär M and Eiswirth M 1995 *Phys. Rev. Lett.* **75** 1503