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Control of beam halo-chaos by Gauss function in the triangle periodic-focusing channel*

Yu Hai-Jun(余海军)[a)†], Bai Long(白 龙)[b)], Weng Jia-Qiang(翁甲强)[c)], Luo Xiao-Shu(罗晓曙)[c)], and Fang Jin-Qing(方锦清)[d)]

[a)] *Department of Physics, Huainan Normal University, Huainan* 232001, *China*
[b)] *College of Science, China University of Mining and Technology, Xuzhou* 221008, *China*
[c)] *College of Physics and Information Technology, Guangxi Normal University, Guilin* 541004, *China*
[d)] *China Institute of Atomic Energy, Beijing* 102413, *China*

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This paper studies the Kapchinsky–Vladimirsky (K–V) beam through a triangle periodic-focusing magnetic field by using the particle-core model. The beam halo-chaos is found, and an idea of Gauss function controller is proposed based on the strategy of controlling the halo-chaos. It performs multiparticle simulation to control the halo by using the Gauss function control method. The numerical results show that the halo-chaos and its regeneration can be eliminated effectively, and that the radial particle density is uniform at the centre of the beam as long as the control method and appropriate parameter are chosen.

Keywords: beam halo-chaos, triangle periodic-focusing magnetic field, Gauss function
PACC: 0545, 4265

1. Introduction

Since Lorenz found the first classical chaotic attractor in 1963, as a very interesting nonlinear phenomenon, chaos has been intensively studied. Over the last several decades, a large variety of approaches have been proposed for chaos control and synchronization, such as the Ott–Grebogi–Yorke (OGY) method, adaptive method, time-delay feedback method, master–slave method, and impulsive control method.[1−3]

In recent years, the high intensity particle beam is utilized widely due to their attractive features in possible breakthrough applications, such as clean activity nuclear power systems, nuclear science, the production of radioactive isotopes for medical purpose.[4−22] However, beam halo-chaos has always taken place in high power accelerators. The halo of a particle beam not only reduces the accelerator efficiency, but also damages the body and environment. Therefore, the study of the beam halo-chaos becomes one of the key issues in current research projects.[7−22] Since the beam halo-chaos is a sensitive and unstable spatio-temporal chaotic motion in essential, many advanced control methods based on the chaos control strategy have been proposed to suppress the halo through many years of efforts, including nonlinear feedback method,[7−12] wavelet function method,[7−13] delay feedback control method,[14] logarithm-function control method,[18] and coupling pinning control based on complex networks.[20] Simulation results have shown that these methods are effective for controlling the halo-chaos complexity.

At present, study of controlling the beam halo-chaos through a rectangle periodic-focusing channel or a uniform-focusing channel has been carried out in Refs.[7–19], but the high intensity beam through a triangle periodic-focusing channel has not been studied. In this paper, we investigate the beam halo-chaos in the triangle periodic-focusing magnetic field, and propose the idea of Gauss function controller of strong nonlinearity and stability for controlling the halo. The character of particle beam is studied when the halo is not controlled or controlled by the Gauss function feedback controller. The simulative results indicate that we can use the method to control not only the halo-chaos, but also the beam distribution in some

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†E-mail: haijun20030@163.com

sense.

2. Numerical method and the controller

At present, the particle-core model is used to investigate the beam halo-chaos widely. In this model, assuming the Kapchinsky–Vladimirsky (K–V) beam to be round and continuous, we obtain the dimensionless nonlinear equations of the beam envelope and the transverse equations of motion for a single particle in a periodic-focusing system are as follows,[5,6]

$$\frac{\mathrm{d}^2 r_\mathrm{b}}{\mathrm{d}s^2} + k_z(s) r_\mathrm{b} - \frac{K}{r_\mathrm{b}} - \frac{1}{r_\mathrm{b}^3} = 0, \tag{1}$$

$$\frac{\mathrm{d}^2 x}{\mathrm{d}s^2} + k_z(s)\, x + \frac{q}{\gamma_\mathrm{b}^3 \beta_\mathrm{b}^2 mc^2} \frac{\partial \Phi^\mathrm{s}(x,y,s)}{\partial x} = 0, \tag{2}$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}s^2} + k_z(s)\, y + \frac{q}{\gamma_\mathrm{b}^3 \beta_\mathrm{b}^2 mc^2} \frac{\partial \Phi^\mathrm{s}(x,y,s)}{\partial y} = 0, \tag{3}$$

where r_b is the beam radius and $s = vt$ is the axial coordinate, in which v is the axial velocity of the beam particles, K is a measure of the beam self-field, c is the speed of light; $\beta_\mathrm{b} = v/c$, $\gamma_\mathrm{b} = 1/\sqrt{1-\beta_\mathrm{b}^2}$ is relativistic mass factor, q and m are the particle charge and rest mass, respectively; $\Phi^\mathrm{s}(x,y,s)$ is the self-electric potential. The periodic function $k_z(s)$ characterizes the strength of the periodic-focusing exterior magnetic field in Fig.1, $k_z(s) = k_z(s+S)$.

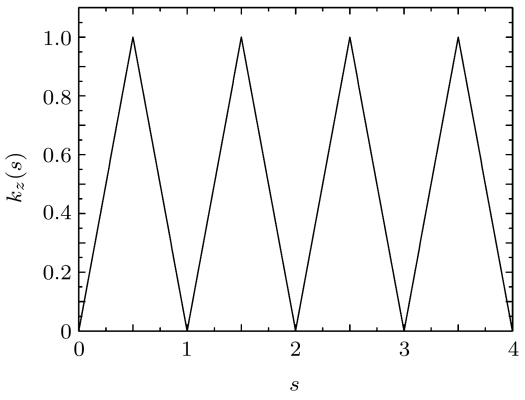


Fig.1. Periodic function $k_z(s)$.

Because space-charges cause the nonlinear effects and energy exchange take place between particles and the core, some particles can escape from the core and form a halo that is surrounding the core.[17,19] We find the radial distribution of beam particles being large and noncompact for the halo-chaos in the vicinity of the beam core as shown in Fig.3(a).

In the particle-core-interaction model, the self-field force acting on a particle is given by

$$F_r = -q\nabla \Phi^\mathrm{s}(x,y,s). \tag{4}$$

The nonlinear feedback control method is proposed on the basis of the general strategy of controlling chaos in Ref.[7]. The approach is to apply a nonlinear feedback controller G to the right-hand side of Eq.(4), that is

$$F_r = -q\nabla \Phi^\mathrm{s}(x,y,s) + G. \tag{5}$$

The force acting on a particle can be changed by controller G, at the same time the transverse velocity and energy of particles are reduced or transformed, so the tendency of particles escaping from the core is restrained. The loose particles in beam are compressed to the core field, and particles become concentrative after the halo-chaos is controlled, thus the goal of suppressing the halo-chaos is achieved.

It is critical to select an effective controller G to control the halo-chaos. In our work, we put forward the Gauss function, which has a strong nonlinearity, stability, and excellent localization property as shown in Fig.2, namely

$$f(x) = \mathrm{e}^{-x^2}. \tag{6}$$

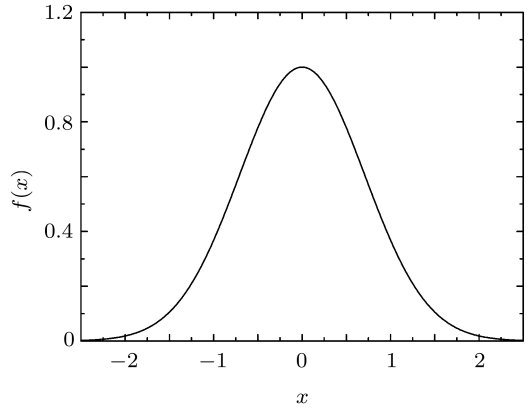


Fig.2. Gauss function.

And the Gauss function controller G is designed to be

$$G = 4\left[f(r_\mathrm{rms}) - f(a_\mathrm{m})\right], \tag{7}$$

where r_rms is the average root-mean-square radius, a_m is the matching radius. When $r_\mathrm{rms} - a_\mathrm{m} = 0$, $G = 0$. Therefore the purpose of controlling the halo is achieved.

3. Numerical results and discussion

The particle-in-cell (PIC) simulation is used for studying the beam under a triangle periodic-focusing channel. The main parameters used as follows: the total number of particles is 5×10^5, the vacuum phase advance $\sigma_0 = 115°$, tune-depression $\eta = 0.80$, $\eta = 1/a_{\rm m}^2\sigma_0$ characterizes the strength of space-charge effect. Mismatch factor $M = 1.5$, where the mismatch parameter gives the ratio of the initial beam radius to the matched radius. The calculated parameters are as follows: matched radius $a_{\rm m} = 0.7891642$ and perveance $K = 0.9032079$. The evolution periodic steps are 1800. The numerical simulation results are shown below.

3.1. Comparison of particle distributions in plane

Figure 3 shows the cross section of the beam before (a) and after controlling the halo-chaos (b). We find that the particle distribution in the beam is loose and nonuniform before the halo-chaos is suppressed, and the halo is visible as shown in Fig.3(a). After controlling the halo-chaos by the Gauss function controller, not only the halo is eliminated, but also the radial particle density becomes uniform. We also find that the ratio of the cross section of the beam after controlling the halo to that before controlling is approximately 1/7.

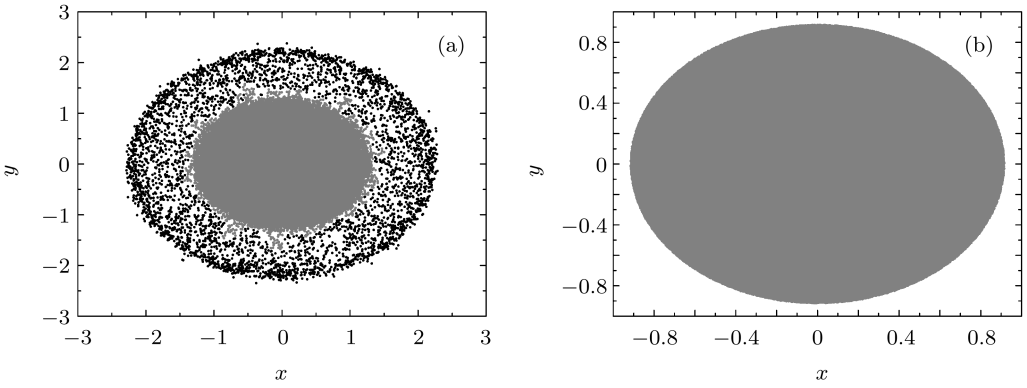


Fig.3. Particle distribution in the (x, y) space at periodic section ($S = 900$) before (a) and after (b) controlling the halo-chaos.

3.2. Comparison of statistical variables of beams

In the simulation, the halo-chaos strength factor H is defined, which is the ratio of particles outside the $1.75a_{\rm m}$ to the total particles. H is a measure of the halo control. From Fig.4(a), we find that H is not zero, which means that the halo exists at all times before controlling. After controlling the halo, H is zero, which shows that the halo is removed by the Gauss function controller.

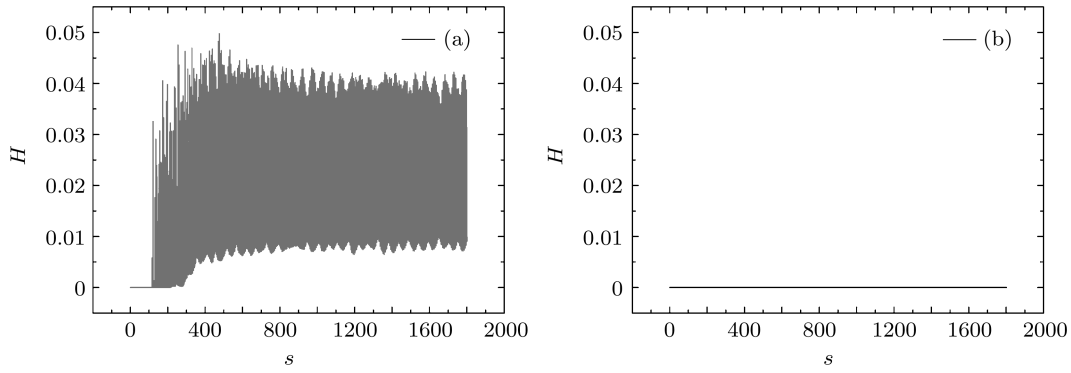


Fig.4. The halo-chaos strength factor H vs time s before (a) and after (b) controlling the halo-chaos.

The evolution of root-mean-square (rms) radius $r_{\rm rms}$ is smart and irregular before controlling the halo as shown in Fig.5(a). From Fig.5(b), we find that the evolution of $r_{\rm rms}$ is regular and its amplitude becomes small after the halo is controlled. These indicate that the tendency of particles escaping from the core is controlled effectively.

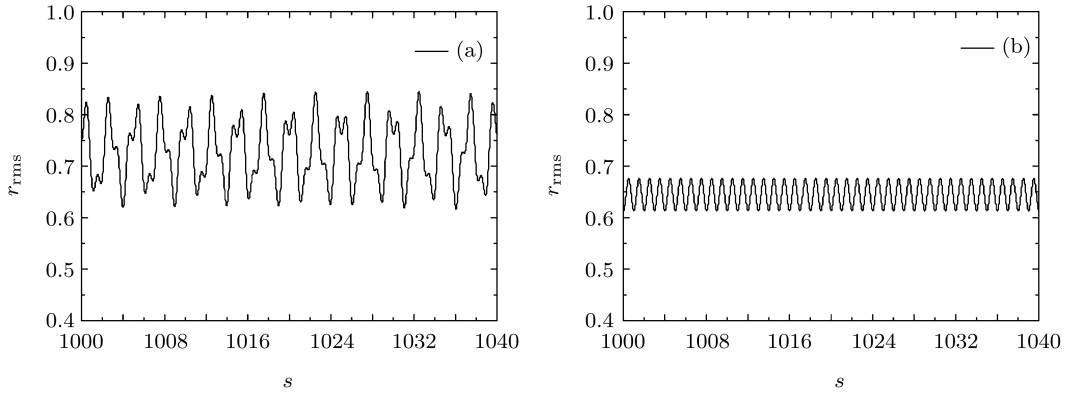


Fig.5. Evolution of average rms radius $r_{\rm rms}$ vs time s before (a) and after (b) controlling the halo-chaos.

3.3. Analysis of radial particle density in high-intensity particle beam

In order to understand how the halo can be controlled better, we further explore the radial particle density of K–V beam in the triangle periodic-focusing system.

The initial distribution of the K–V beam is uniform as shown in Fig.6. When the beam is not controlled, the simulation result indicates that the radial particle density varies rapidly and becomes nonuniform over the cross section of beam in comparison with the initial uniform distribution as shown in Fig.7(a). We find that lots of particles accumulate in the centre, few particles stay on the edge of the beam. From the particle-core-interaction model, the space-charge can cause the nonlinear effects and energy exchange between particles and the core, thus more and more particles can escape from the core, which causes the emittance growth and halo formation around the core. At the same time, particles of mismatched beam are affected by the external magnetic field. The complexity of the force acting on particles results in such a radial particle density curve of the beam.[21]

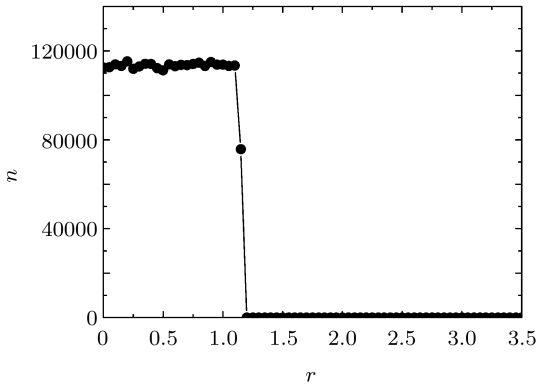


Fig.6. The initial density distribution of K–V beam.

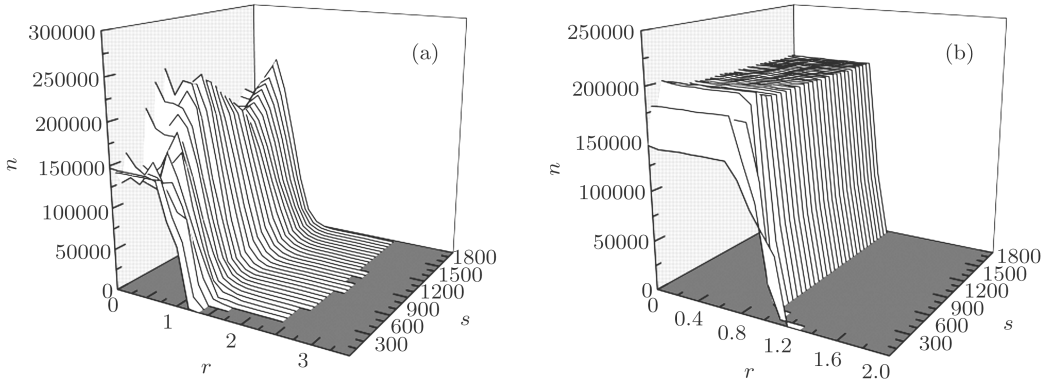


Fig.7. Evolution of radial particle density of beam before (a) and after (b) controlling the halo-chaos.

After the halo is suppressed effectively by the Gauss function controller, we can find that the uniform density appears at the centre of the beam, and that the area of particle distribution becomes smaller and more uniform than the initial density as shown in Fig.7(b). It is the Gauss function controller that can offer a transverse force to suppress the emission force of particles caused by the space-charge nonlinear effects. Thus the tendency of particles escaping from the core is restrained, and the loose particles beam are compressed to the core of the beam. So the nonuniform distribution of the beam before being controlled becomes the uniform distribution after the halo-chaos is controlled by the Gauss function controller. This indicates that the particle density may be made to produce some spatial distribution after controlling the halo-chaos.

4. Summary

From the simulative results, we can find that the beam halo-chaos also generates in the triangle periodic-focusing channel, and that not only can the halo be controlled by the Gauss function controller, but also the density uniformity of beam can be obtained as long as appropriate control method and appropriate system parameter are chosen, which may be a valuable try for the particle beam applications in the future.

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