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Adaptive control and synchronization of an uncertain new hyperchaotic Lorenz system*

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This paper is involved with the adaptive control and synchronization problems for an uncertain new hyperchaotic Lorenz system. Based on the Lyapunov stability theory, the adaptive control law is derived such that the trajectory of hyperchaotic Lorenz system with unknown parameters can be globally stabilized to an unstable equilibrium point of the uncontrolled system. Furthermore, an adaptive control approach is presented to the synchronizations between two identical hyperchaotic systems, particularly between two different uncertain hyperchaotic systems. Numerical simulations show the effectiveness of the presented method.

Keywords: hyperchaotic Lorenz system, control, synchronization, Lyapunov stability theory
PACC: 0545, 0547

1. Introduction

Since Pecora and Carroll[1] introduced a method to synchronize two identical chaotic systems with different initial conditions, synchronization in chaotic dynamic systems has received a great deal of interest of scientists in various research fields including secure communication, chemical reactions, biological systems, information science, plasma technologies, and so on. In the past two decades, a variety of approaches have been proposed for the control and synchronization of chaotic systems such as the Ott–Grebogi–Yorke (OGY) method,[2] active control,[3] backstepping design,[4,5] time-delay feedback control,[6] coupled synchronization method,[7] adaptive control,[8−11], etc. To our best knowledge, most of those studies involved mainly with low-dimensional attractors with only one positive Lyapunov exponent. However, a hyperchaotic system, possessing at least two positive Lyapunov exponents, has a more complex behaviour and more abundant dynamics than a chaotic system. Therefore, how to control and synchronize hyperchaotic systems is an interesting and challenging work. In recent years, some studies[12−18] have been made on the controlling hyperchaos and the achieving the synchronization of hyperchaos in hyperchaotic systems.

In this paper, we mainly study the control and synchronization of hyperchaotic systems with unknown parameters. Stabilizing hyperchaos to an unstable equilibrium is achieved. Synchronizations between two identical hyperchaotic systems and between two different uncertain hyperchaotic systems are realized according to the Lyapunov stability theory and the adaptive control theory.

Recently, a new hyperchaotic Lorenz system was constructed by Wang *et al.*[19] A four-dimensional autonomous hyperchaotic system is given by

$$\begin{aligned} \dot{x} &= a(y-x), \\ \dot{y} &= cx - y - xz + w, \\ \dot{z} &= xy - bz, \\ \dot{w} &= -kx, \end{aligned} \tag{1}$$

in which a,b, c and k are constant parameters.

With the parameters a=10, b=8/3, c=28, and k=5 adopted, system (1) has two positive Lyapunov exponents, i.e. λ_1=0.3997 and λ_2=0.3113. And it has a unique unstable equilibrium E_0 (0, 0, 0, 0). Thus, system (1) shows a hyperchaotic behaviour, as shown in Fig.1. For more detailed analysis of the complex dynamics of the system, one can see Ref.[19].

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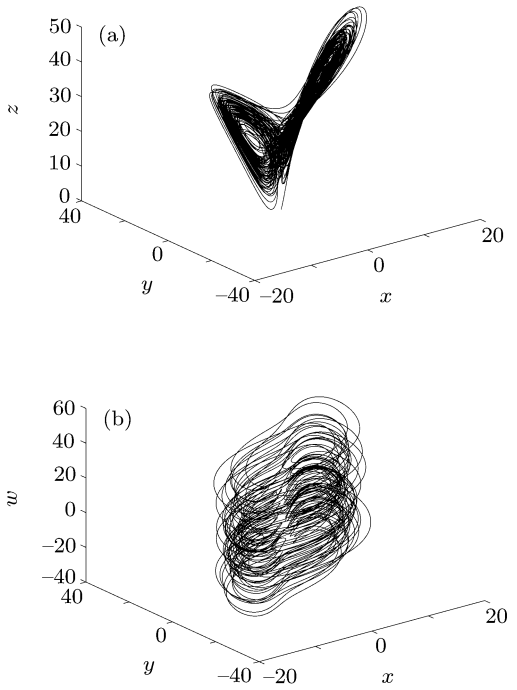


Fig.1. Hyperchaotic attractors of system (1) in $x-y-z$ space (a) and $x-y-w$ space (b).

2. Stabilization of the new hyperchaotic Lorenz system

In this section, the hyperchaos in dynamical system (1) is controlled to an unstable equilibrium E_0 in the presence of unknown system parameters. The adaptive control theory is applied to achieving this goal. For this purpose, we consider the following to -be-controlled system:

$$\begin{aligned}\dot{x} &= a(y-x),\\ \dot{y} &= cx - y - xz + w + u_1,\\ \dot{z} &= xy - bz + u_2,\\ \dot{w} &= -kx + u_3,\end{aligned} \tag{2}$$

in which a, b, c and k are unknown parameters, u_1, u_2 and u_3 are the controllers to be designed.

We choose the Lyapunov function for to-be-controlled system (2) as

$$V = \frac{1}{2}(x^2+y^2+z^2+w^2+\tilde{a}^2+\tilde{b}^2+\tilde{c}^2+\tilde{k}^2),$$

where $\tilde{a}=\hat{a}-a$, $\tilde{b}=\hat{b}-b$, $\tilde{c}=\hat{c}-c$ and $\tilde{k}=\hat{k}-k$, $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{k}$ are the estimate values of these unknown parameters.

The time derivative of V along the trajectories of system (2) is

$$\begin{aligned}\dot{V} =& ax(y-x)+y(cx-y-xz+w+u_1)\\ &+z(xy-bz+u_2)\\ &+w(-kx+u_3)+\tilde{a}\dot{\tilde{a}}+\tilde{b}\dot{\tilde{b}}+\tilde{c}\dot{\tilde{c}}+\tilde{k}\dot{\tilde{k}}\\ =&-x^2-y^2-z^2-w^2-(\hat{a}-1)x^2\\ &+(\hat{a}+\hat{c})xy+u_1y\\ &-(\hat{b}-1)z^2+u_2z-\hat{k}xw+w^2+yw+u_3w\\ &+\tilde{a}x^2-(\tilde{a}+\tilde{c})xy+\tilde{b}z^2\\ &+\tilde{k}xw+\tilde{a}\dot{\tilde{a}}+\tilde{b}\dot{\tilde{b}}+\tilde{c}\dot{\tilde{c}}+\tilde{k}\dot{\tilde{k}}\\ =&-x^2-y^2-z^2-w^2+y(u_1+(\hat{a}+\hat{c})x)\\ &+z\left(u_2-(\hat{b}-1)z-\frac{\hat{a}-1}{z}x^2\right)\\ &+w(u_3-\tilde{k}x+w+y)\\ &+\tilde{a}(\dot{\hat{a}}+x^2-xy)+\tilde{b}(\dot{\hat{b}}+z^2)\\ &+\tilde{c}(\dot{\hat{c}}-xy)+\tilde{k}(\dot{\hat{k}}+xw).\end{aligned} \tag{3}$$

Choose the controllers u_1, u_2, u_3 as

$$\begin{aligned}u_1 &= -(\hat{a}+\hat{c})x,\\ u_2 &= (\hat{b}-1)z+\frac{\hat{a}-1}{z}x^2,\\ u_3 &= \hat{k}x-y-w,\end{aligned} \tag{4}$$

and the following parameters estimation update law:

$$\begin{aligned}\dot{\hat{a}} &= xy-x^2-\tilde{a},\\ \dot{\hat{b}} &= -z^2-\tilde{b},\\ \dot{\hat{c}} &= xy-\tilde{c},\\ \dot{\hat{k}} &= -xw-\tilde{k},\end{aligned} \tag{5}$$

then the time derivative of the Lyapunov function will become

$$\dot{V} = -x^2-y^2-z^2-w^2-\tilde{a}^2-\tilde{b}^2-\tilde{c}^2-\tilde{k}^2 < 0.$$

It is clear that V is positive definite and $\dot{V}$ is negative definite in a neighbourhood of the zero solution for system (2). Therefore, based on the Lyapunov stability theory, the controlled system (2) can asymptotically converge to the unstable equilibrium E_0 (0, 0, 0, 0) with controllers (4) and parameters estimation update law (5).

3. Synchronization of a new hyperchaotic Lorenz system

In this section, synchronizations between two identical hyperchaotic systems, particularly between

two different uncertain hyperchaotic systems are achieved according to the Lyapunov stability theory and the adaptive control theory.

3.1. Synchronization between two identical hyperchaotic systems

In this subsection, based on the Lyapunov stability theory and the adaptive control theory, synchronization between two identical hyperchaotic systems with unknown parameters is achieved.

A drive system and a response system are respectively given as follows:

$$\begin{aligned}\dot{x}_1 &= a(y_1 - x_1),\\ \dot{y}_1 &= cx_1 - y_1 - x_1z_1 + w_1,\\ \dot{z}_1 &= x_1y_1 - bz_1,\\ \dot{w}_1 &= -kx_1,\end{aligned} \tag{6}$$

and

$$\begin{aligned}\dot{x}_2 &= a_1(y_2 - x_2) + u_1,\\ \dot{y}_2 &= c_1x_2 - y_2 - x_2z_2 + w_2 + u_2,\\ \dot{z}_2 &= x_2y_2 - b_1z_2 + u_3,\\ \dot{w}_2 &= -k_1x_2 + u_4,\end{aligned} \tag{7}$$

where a, b, c, k are unknown parameters of the drive system, and a_1, b_1, c_1, and k_1 are uncertain parameters, which need estimating in the response system, u_1, u_2, u_3, and u_4 are the controllers which are to be designed such that the two hyperchaotic systems can be synchronized with each other.

Subtracting drive system (6) from response system (7) yields the following error dynamical system:

$$\begin{aligned}\dot{e}_1 &= \tilde{a}(y_2 - x_2) + a(e_2 - e_1) + u_1,\\ \dot{e}_2 &= \tilde{c}x_2 + ce_1 - e_2 + e_4 - e_3x_2 - e_1z_1 + u_2,\\ \dot{e}_3 &= -\tilde{b}z_2 - be_3 + e_2x_2 + e_1y_1 + u_3,\\ \dot{e}_4 &= -\tilde{k}x_2 - ke_1 + u_4,\end{aligned} \tag{8}$$

where $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$, $e_3 = z_2 - z_1$, $e_4 = w_2 - w_1$, $\tilde{a} = a_1 - a$, $\tilde{b} = b_1 - b$, $\tilde{c} = c_1 - c$ and $\tilde{k} = k_1 - k$.

To guarantee that error dynamical system (8) converges to the origin asymptotically, we choose the controllers as

$$\begin{aligned}u_1 &= (a-1)e_1 - ae_2,\\ u_2 &= e_1z_1 - ce_1 - e_4,\\ u_3 &= (b-1)e_3 - e_1y_1,\\ u_4 &= ke_1 - e_4,\end{aligned} \tag{9}$$

and the parameters estimation update law as

$$\begin{aligned}\dot{a}_1 &= (x_2 - y_2)e_1 - \tilde{a},\\ \dot{b}_1 &= z_2e_3 - \tilde{b},\\ \dot{c}_1 &= -x_2e_2 - \tilde{c},\\ \dot{k}_1 &= x_2e_4 - \tilde{k}.\end{aligned} \tag{10}$$

Then, we have the following main result.

Theorem 1 For any initial conditions, drive system (6) and response system (7) are asymptotically synchronized with each other by adaptive control law (9) and update law (10) of parameters globally.

Proof We can select a Lyapunov function for Eq.(8) as

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{k}^2).$$

The time derivative of the Lyapunov function along the trajectories of Eq.(8) is

$$\begin{aligned}\dot{V} =& e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{k}\dot{\tilde{k}}\\ =& e_1(\tilde{a}(y_2 - x_2) + a(e_2 - e_1) + u_1)\\ &+ e_2(\tilde{c}x_2 + ce_1 + e_4 - e_3x_1 - e_1z_1 + u_2)\\ &+ e_3(-\tilde{b}z_2 - be_3 + e_2x_2 + e_1y_1 + u_3)\\ &+ e_4(-\tilde{k}x_2 - ke_1 + u_4)\\ &+ \tilde{a}\dot{a}_1 + \tilde{b}\dot{b}_1 + \tilde{c}\dot{c}_1 + \tilde{k}\dot{k}_1.\end{aligned} \tag{11}$$

Substituting Eqs.(9) and (10) into Eq.(11) yields

$$\dot{V} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 - \tilde{a}^2 - \tilde{b}^2 - \tilde{c}^2 - \tilde{k}^2 < 0.$$

Since the Lyapunov function V is positive definite and its derivative $\dot{V}$ is negative definite in a neighbourhood of the zero solution for system (8). In light of the Lyapunov stability theory, the error dynamical system can converge to the origin asymptotically. Consequently, drive system (6) is synchronous asymptotically with response system (7) under adaptive controller law (9) and parameters estimation update law (10).

3.2. Synchronization between two different hyperchaotic systems

In this subsection, the synchronization of new hyperchaotic system (1) with the hyperchaotic Liu system[20] is realized. An effective adaptive controller and a parameter estimation update law are designed according to the adaptive control theory.

The hyperchaotic Liu system is described as

$$\begin{aligned}\dot{x} &= l(y-x),\\ \dot{y} &= mx - nxz + w,\\ \dot{z} &= -pz + hx^2,\\ \dot{w} &= -qx,\end{aligned} \tag{12}$$

where l, m, n, p, q and h are constant parameters.

With l=10, m=40, n=1, p=2.5, q=10.6 and h=4 adopted, then this dynamical system will show a hyperchaotic behaviour.

We choose the hyperchaotic Liu system as a drive system

$$\begin{aligned}\dot{x}_1 &= l(y_1-x_1),\\ \dot{y}_1 &= mx_1 - nx_1z_1 + w_1,\\ \dot{z}_1 &= -pz_1 + hx_1^2,\\ \dot{w}_1 &= -qx_1.\end{aligned} \tag{13}$$

Let the new hyperchaotic system (1) be a response one, that is

$$\begin{aligned}\dot{x}_2 &= a(y_2-x_2) + u_1,\\ \dot{y}_2 &= cx_2 - y_2 - x_2z_2 + w_2 + u_2,\\ \dot{z}_2 &= x_2y_2 - bz_2 + u_3,\\ \dot{w}_2 &= -kx_2 + u_4,\end{aligned} \tag{14}$$

where u_1, u_2, u_3, u_4 are the controllers to be designed.

Supposing that all the parameters a, b, c, k, l, m, n, p, q and h are unknown, we will design an adaptive controller and a parameter estimation update law to achieve the synchronization between the two different uncertain hyperchaotic systems.

Then, an error dynamical system from Eqs.(13) and (14) can be expressed as

$$\begin{aligned}\dot{e}_1 &= a(y_2-x_2) - l(y_1-x_1) + u_1,\\ \dot{e}_2 &= cx_2 - y_2 - x_2z_2 - mx_1 + nx_1z_1 + e_4 + u_2,\\ \dot{e}_3 &= -bz_2 + pz_1 - hx_1^2 + x_2y_2 + u_3,\\ \dot{e}_4 &= -kx_2 + qx_1 + u_4.\end{aligned} \tag{15}$$

The goal of control is to find controllers u_i (i=1, 2, 3, 4) and a parameter estimation update law for Eq.(15) such that the states of response system (14) and the states of drive system (13) are asymptotically synchronized with each other globally.

Then we obtain the following theorem.

Theorem 2 Take the adaptive control law as

$$\begin{aligned}u_1 &= \hat{l}(y_1-x_1) - \hat{a}(y_2-x_2) - e_1,\\ u_2 &= \hat{m}x_1 - \hat{n}x_1z_1 - \hat{c}x_2 + y_1 + x_2z_2 - e_4,\\ u_3 &= \hat{b}z_2 - \hat{p}z_1 + \hat{h}x_1^2 - x_2y_2 - e_3,\\ u_4 &= \hat{k}x_2 - \hat{q}x_1 - e_4,\end{aligned} \tag{16}$$

where $\tilde{a} = \hat{a} - a$, $\tilde{b} = \hat{b} - b$, $\tilde{c} = \hat{c} - c$, $\tilde{k} = \hat{k} - k$, $\tilde{l} = \hat{l} - l$, $\tilde{m} = \hat{m} - m$, $\tilde{n} = \hat{n} - n$, $\tilde{p} = \hat{p} - p$, $\tilde{q} = \hat{q} - q$ and $\tilde{h} = \hat{h} - h$, and the parameter estimation update law as

$$\begin{aligned}\dot{\hat{a}} &= (y_2-x_2)e_1 - \tilde{a},\\ \dot{\hat{b}} &= -z_2e_3 - \tilde{b},\\ \dot{\hat{c}} &= x_2e_2 - \tilde{c},\\ \dot{\hat{k}} &= -x_2e_4 - \tilde{k},\\ \dot{\hat{l}} &= (x_1-y_1)e_1 - \tilde{l},\\ \dot{\hat{m}} &= -x_1e_2 - \tilde{m},\\ \dot{\hat{n}} &= x_1z_1e_2 - \tilde{n},\\ \dot{\hat{p}} &= z_1e_3 - \tilde{p},\\ \dot{\hat{q}} &= x_1e_4 - \tilde{q},\\ \dot{\hat{h}} &= -x_1^2e_3 - \tilde{h},\end{aligned} \tag{17}$$

then response system (14) will be synchronous with drive system (13) globally.

Proof Choose the following Lyapunov function:

$$\begin{aligned}V =& \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2 + \tilde{k}^2\\ &+ \tilde{l}^2 + \tilde{m}^2 + \tilde{n}^2 + \tilde{p}^2 + \tilde{q}^2 + \tilde{h}^2).\end{aligned}$$

The time derivative of the Lyapunov function along the solutions of Eq.(15) is

$$\begin{aligned}\dot{V} =& e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} + \tilde{k}\dot{\tilde{k}}\\ &+ \tilde{l}\dot{\tilde{l}} + \tilde{m}\dot{\tilde{m}} + \tilde{n}\dot{\tilde{n}} + \tilde{p}\dot{\tilde{p}} + \tilde{q}\dot{\tilde{q}} + \tilde{h}\dot{\tilde{h}}\\ =& e_1(a(y_2-x_2) - l(y_1-x_1) + u_1)\\ &+ e_2(cx_2 - y_2 - x_2z_2 - mx_1 + nx_1z_1 + e_4 + u_2)\\ &+ e_3(-bz_2 + pz_1 - hx_1^2 + x_2y_2 + u_3)\\ &+ e_4(-kx_2 + qx_1 + u_4)\\ &+ \tilde{a}\dot{\hat{a}} + \tilde{b}\dot{\hat{b}} + \tilde{c}\dot{\hat{c}} + \tilde{k}\dot{\hat{k}} + \tilde{l}\dot{\hat{l}}\\ &+ \tilde{m}\dot{\hat{m}} + \tilde{n}\dot{\hat{n}} + \tilde{p}\dot{\hat{p}} + \tilde{q}\dot{\hat{q}} + \tilde{h}\dot{\hat{h}}.\end{aligned} \tag{18}$$

Substituting Eqs.(16) and (17) into Eq.(18) yields

$$\dot{V} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 - \tilde{a}^2 - \tilde{b}^2 - \tilde{c}^2 - \tilde{k}^2 - \tilde{l}^2 - \tilde{m}^2 - \tilde{n}^2 - \tilde{p}^2 - \tilde{q}^2 - \tilde{h}^2 < 0.$$

Therefore, based on the Lyapunov stability theory, error dynamical system (15) is asymptotically stable at the origin with controllers (16) and parameters estimation update law (17). Thus, the states of drive system (13) and the states of response system (14) are ultimately synchronized with each other asymptotically.

4. Numerical simulations

In this section, to verify and demonstrate the effectiveness of the proposed methods, we consider three numerical examples. In the numerical simulations, the fourth-order Runge–Kutta method is used to solve the systems with a time step size of 0.001. The parameters of hyperchaotic systems are chosen as a=10, b=8/3, c=28, k=5, l=10, m=40, n=1, p=2.5, q=10.6 and h=4 to ensure the existence of hyperchaos in the absence of control.

Example 1 Controlling the hyperchaotic system (1) to an equilibrium E_0 (0, 0, 0, 0).

For this numerical simulation, the initial states of controlled system (2) are $(x(0), y(0), z(0), w(0)) = (2, 2, 2, -2)$ and the initial parameter estimation errors are $(\tilde{a}(0), \tilde{b}(0), \tilde{c}(0), \tilde{k}(0)) = (3, 3, 3, 3)$. The states of controlled system (2) are shown in Fig.2. It is clearly known from the figure that the system is driven to the origin immediately. Figure 3 shows the controller action u_1, u_2, and u_3. As can be seen from the figure, the controllers are bounded as $t \to \infty$.

Example 2 Adaptive synchronization of a new hyperchaotic Lorenz system with unknown parameters.

For the numerical simulation in this example, we consider the systems given as Eqs.(6) and (7). The initial values of the drive system and response system are taken as $(x_1(0), y_1(0), z_1(0), w_1(0)) = (-1, -1, -1, -1)$, and $(x_2(0), y_2(0), z_2(0), w_2(0)) = (1, 1, 1, 1)$, respectively. Hence, the initial errors are $e_1(0)$=2, $e_2(0)$=2, $e_3(0)$=2 and $e_4(0)$=2. The four initial uncertain parameters are chosen as ($a_1(0)$, $b_1(0)$, $c_1(0)$, $k_1(0)$)=(5, 3, 8, 10),

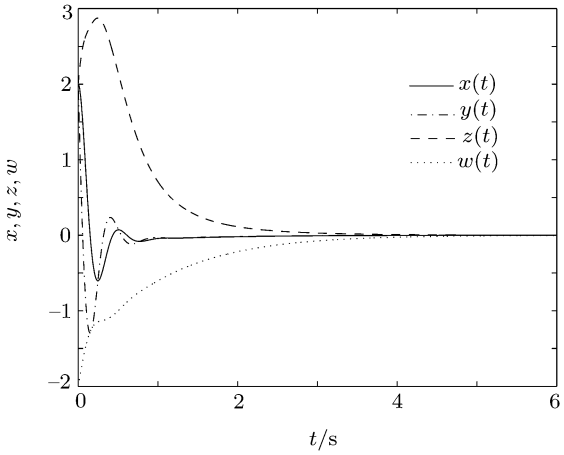


Fig.2. The time response of states(x, y, z, w): stabilizing the equilibrium at E_0.

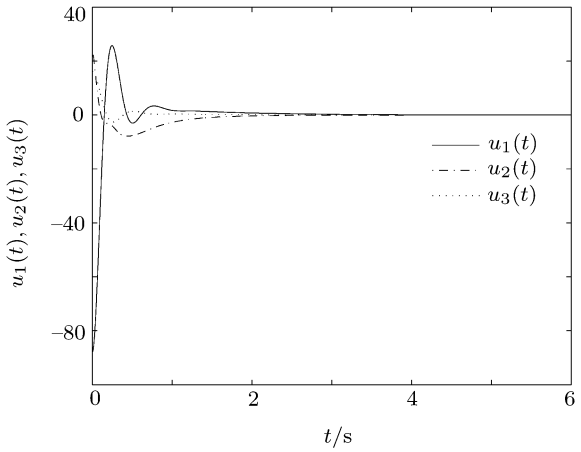


Fig.3. Control actions $u_1(t)$, $u_2(t)$ and $u_3(t)$.

The simulation results are illustrated in Figs.4–6. Figure 4 displays the time evolutions of the drive system and the response system. The dynamics of synchronization error is defined as $e(t) = \sqrt{e_1^2(t) + e_2^2(t) + e_3^2(t) + e_4^2(t)}$, whose time evolution is shown in Fig.5. Figure 6 shows the estimates of parameters $a_1(t)$, $b_1(t)$, $c_1(t)$, $k_1(t)$ as $t \to \infty$. From Figs.3 and 4, we can see that the two hyperchaotic systems starting from different conditions are synchronized with each other immediately. Furthermore, the estimated values of uncertain parameters converge to a=10, b=8/3, c=28 and k=5 as $t \to \infty$.

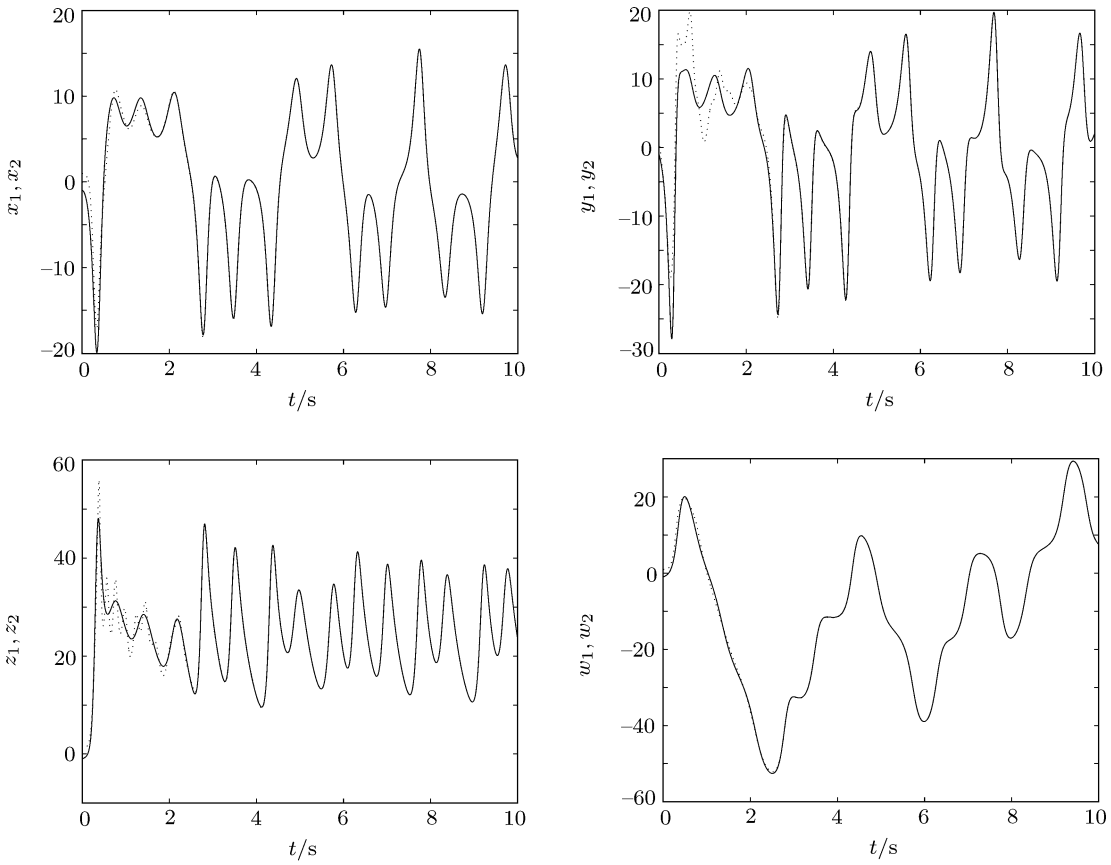


Fig.4. State trajectories of drive system states (—) and response system states ($\cdots$).

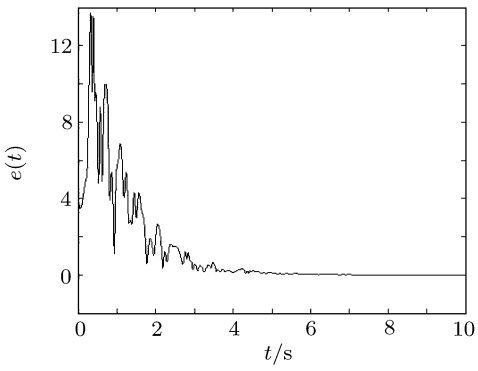


Fig.5. Error $e(t)$ with time t.

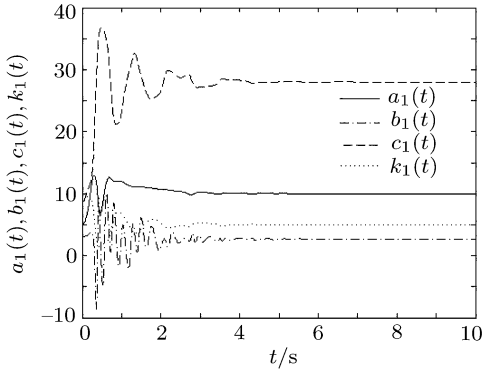


Fig.6. The response system parameters versus time t.

Example 3 Adaptive synchronization between two different structure hyperchaotic systems with unknown parameters.

For this numerical simulation, the initial values of drive system (13) and response system (14) are taken as $(x_1(0), y_1(0), z_1(0), w_1(0)) = (-1, -2, -3, -4)$, and $(x_2(0), y_2(0), z_2(0), w_2(0))$=(1, 2, 3, 4), respectively. Thus, the initial errors are $e_1(0)$= 2, $e_2(0)$=4, $e_3(0)$=6 and $e_4(0)$=8.

The simulation results are illustrated in Figs.7–9. Figure 7 demonstrates the state trajectories of drive system (13) and response system (14). Figure 8 gives the dynamics of synchronization error $e(t)$. From the figures, it can be seen that the synchronization errors

converge to zero and two different systems have indeed achieved chaos synchronization. The dynamical systems of the ten parameters estimation errors are shown in Fig.9.

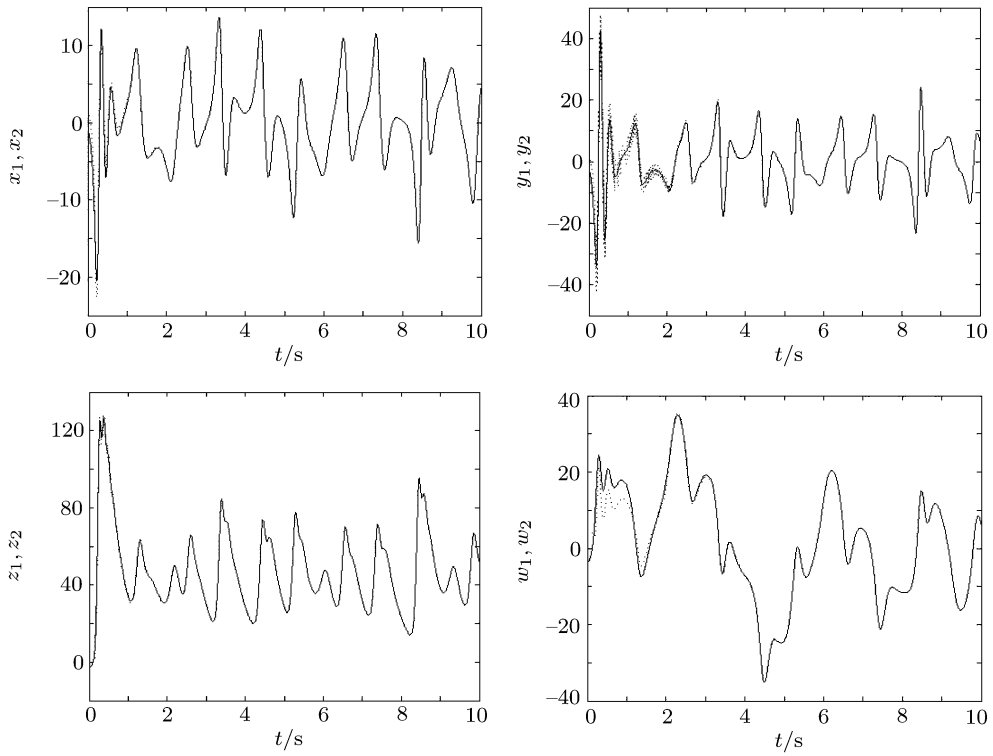


Fig.7. State trajectories of drive system states (—) and response system states ($\cdots$).

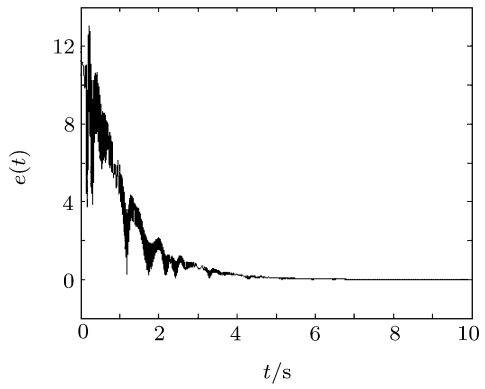


Fig.8. Error $e(t)$ versus time t.

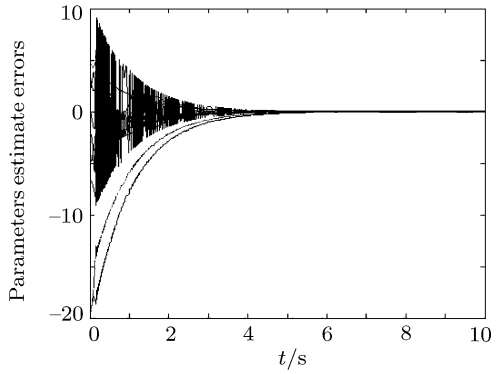


Fig.9. Evolutions of the ten parameter estimation errors with time t.

5. Conclusion

This paper is concerned with the adaptive control and synchronization of a new hyperchaotic Lorenz system with unknown parameters. An adaptive controller has been designed for controlling the hyperchaos to an unstable equilibrium point of the uncontrolled system. Furthermore, based on the Lyapunov stability theory and the adaptive control theory, synchronizations are achieved not only between two identical hyperchaotic systems, but also between two different hyperchaotic systems with uncertain parameters. Numerical simulations are given for the purpose of illustration and verification.

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