Vol 17 No 7, July 2008
1674-1056/2008/17(07)/2377-11

Chinese Physics B

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Impulsive control of nonlinear systems with time-varying delays

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(Received 8 October 2007; revised manuscript received 10 December 2007)

A whole impulsive control scheme of nonlinear systems with time-varying delays, which is an extension for impulsive control of nonlinear systems without time delay, is presented in this paper. Utilizing the Lyapunov functions and the impulsive-type comparison principles, we establish a series of different conditions under which impulsively controlled nonlinear systems with time-varying delays are asymptotically stable. Then we estimate upper bounds of impulse interval and time-varying delays for asymptotically stable control. Finally a numerical example is given to illustrate the effectiveness of the method.

Keywords: impulsive control, time-varying delays, impulsive delay systems, Lyapunov functions
PACC: 0545

1. Introduction

Impulsive control is a control paradigm based on impulsive differential equations. A impulsive dynamical system is first formulated by Mil'man and Myshkis,[1] which can be regarded as a subclass of hybrid systems and consists of three elements, that is, a continuous-time differential equation to govern the motion of the dynamical system between impulsive events or resetting events, a difference equation to govern the way the system states are changed instantaneously when an impulsive or a resetting event occurs, and a criterion for determining when the system states are to be reset. The fundamental theory of impulsive differential equations have been developed.[2−6] Recently, impulsive systems and impulsive control have attracted an increasing interest in engineering, economics, biology, and so on. Their examples include nanoelectronic circuits,[7] electronic networks,[8] orbital transfer of satellite,[9] financial modelling,[10] ecosystems management,[11] and so on.

Time delays including constant delays and variable delays are frequently encountered in linear or nonlinear systems such as electronic networks, communication and biological systems, where the state change rate depends on not only the current state of the system but also its state at some time in the past. Delay effects can introduce instability, oscillators, chaos and other complex dynamical behaviours. So time-delay dynamical systems have been a subject of extensive research during the past three-decades (see Refs.[12–38]). The proposed stability criteria are related to constant delays, time-varying delays, multiple time-delays, delay-dependence and delay-independence, separately. In many practical systems, since time-delays are commonly unknown and variable, it is natural to consider effects of time-varying delays in such systems.

The time delays and impulsive control can be described mathematically by impulsive delay differential equations. The impulsive delay differential equations describe a scheme of hybrid control and process with after-effect and state changing by jumps. In recent years, the impulsive delay differential equations have been studied by many researchers (see Refs.[21–30] and the references cited therein). By using the Lyapunov functions and the Razumikhin techniques, Liu and Ballinger[20] and Stamova and Stamov[21] have obtained some sufficient criteria on stability for a class of impulsive delay control systems. By establishing an impulsive delay differential inequality, Yang and Xu[26] have analysed the global exponential stability

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of the impulsive delay systems. By utilizing fuzzy control, Ho and Sun[27] studied the impulsive delay systems and obtained the criteria on uniform stability and uniform asymptotic stability. By using the method of Lyapunov functionals and feedback control, Liu[28] studied stabilities of the linear systems with time delay and impulsive effects. However, these proposed research methods do not consider comparison principle or consider it less in impulsive control of nonlinear systems with time-varying delays.

Significant progress has been made in the theory of impulsive control in recent years (see Refs.[29–36]). For example, Yang[7,8,29] and Li *et al*[30,31] have presented the concept and the principles of impulsive control and obtained some sufficient conditions on stability and stabilization for a class of nonlinear systems by using the results in Ref.[2]; Nersesov and Haddad[32] used control vector Lyapunov functions to study large-scale impulsive dynamical systems; Zheng and Chen[33] studied fuzzy impulsive control of chaotic systems based on the Takagi–Sugeno fuzzy model. In the process of the research development in the theory of the impulsive delay differential equations, the effects of delays and impulses on dynamical behaviours of linear or nonlinear systems bring about a number of difficulties. Therefore, the corresponding theory for impulsive delay control has not yet been fully developed. In this paper, we further develop the theory of impulsive delay differential equations to consider impulsive control of nonlinear systems with time-varying delays. Utilizing the Lyapunov functions and impulsive-type comparison principles, we present conditions under which the impulsively controlled delay systems are asymptotically stable, and estimate the upper bounds of impulse interval and time-varying delays. Compared with Refs.[19–28], this paper focuses on impulsive control instead of stability analysis of dynamical systems.

The rest of this paper is organized as follows. In the following section, we establish some inequalities and introduce some preliminary definitions and basic theory of impulsive differential equation. In Section 3, the impulsively controlled nonlinear systems with time-varying delays are studied, and new asymptotical stability theorems of the impulsively controlled nonlinear time-varying delay systems are given; then the estimate of upper bounds of impulse interval and time-varying delays is made; furthermore the impulsive controller design procedure is obtained. Following the illustrative numerical example in Section 4, finally, some concluding remarks are presented in Section 5.

2. Preliminaries

To start with, we prescribe the significance of some quantities that will be used later.

Let R denote the set of real numbers, R^+, R_+, and N represent the sets of all positive real numbers, nonnegative real numbers, and natural numbers respectively, R^n denotes the n-dimensional Euclidian space and $R^{n\times n}$ represents the real matrix space of dimension $n \times n$. For $A \in R^{n\times n}$, $A > 0 (< 0)$ means that A is a symmetrical positive (negative) definite matrix, $A > B (< B)$ implies $A - B > 0 (< 0)$, A^{-1} and A^{T} denote the inverse and the transpose of matrix A respectively. Let $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ represent respectively the minimal and maximal eigenvalues of matrix A. Let I be the identity matrix. For $x \in R^n$, let $|x|$ and $\|x\|$ be the absolute value and the Euclidian norm of x respectively.

Now, consider a general nonlinear system

$$\dot{x}(t) = f(t, x(t)), \tag{1}$$

where $f : R_+ \times R^n \mapsto R^n$ is continuous, and $x(t) \in R^n$ is the state variable satisfying

$$\dot{x}(t) = \frac{\mathrm{d}x}{\mathrm{d}t}. \tag{2}$$

Consider a discrete time set $\{t_k\}$ of control instants, that is, an impulsive time sequence, where

$$0 < t_1 < t_2 < \cdots < t_{k-1} < t_k < t_{k+1} < \cdots,$$
$$t_k \to \infty, \quad \text{as} \quad k \to \infty.$$

Let

$$U(k, x) = \Delta x|_{t=t_k} \triangleq x(t_k^+) - x(t_k^-),$$
$$x(t_k^+) = \lim_{t\to t_k^+} x(t),$$
$$x(t_k^-) = \lim_{t\to t_k^-} x(t), \quad k \in N$$

be the control law, which is the "jump" in the state variable at the time instant t_k, where $x(t_k^-) = x(t_k)$ is assumed, that is, $x(t)$ is left continuous at $t = t_k$. Then the impulsive dynamical system, which is an impulsive differential equation according to Ref.[2], is presented by

$$\begin{aligned} &\dot{x}(t) = f(t, x(t)), \quad t \neq t_k, \\ &\Delta x(t) = U(k, x), \quad t = t_k, \quad k \in N, \\ &x(t_0^+) = x_0, \quad t_0 \geq 0. \end{aligned} \tag{3}$$

Remark 1 Impulsive dynamical system (3) can be regarded as a subclass of hybrid systems and consists of three elements, that is, a continuous-time differential equation ($\dot{x}(t) = f(t, x(t))$), which governs the motion of the dynamical system between impulsive events; a difference equation ($\Delta x(t) = U(k, x)$), which governs the way the system states are changed instantaneously when an impulsive event occurs; and a criterion for determining ($x(t_0^+) = x_0$) when the system states are to be reset.

Remark 2 On the one hand, when $t \neq t_k$, the dynamics of the impulsive control system (3) is governed by the system (1). We can see that the controlled system is a free system when $t \neq t_k$. On the other hand, when $t = t_k, k = 1, 2, \cdots$, the state variable is changed from $x(t_k^-)$ to $x(t_k^+) = x(t_k^-) + \Delta x|_{t=t_k}$ instantaneously.

In order to establish sufficient conditions under which the impulsively controlled nonlinear system is stable, we need the following basic definitions and theory of impulsive differential equations.

Lemma 1 Let $n \times n$ matrix $P > 0$ and $P = Q^{\mathrm{T}}Q$. For any $x, \overline{x} \in R^n, A \in R^{n\times n}$, then

1) $\|x\| \triangleq \sqrt{x^{\mathrm{T}}x}, \quad \|A\| \triangleq \sqrt{\lambda_{\max}(A^{\mathrm{T}}A)}$, $\mu(A) \triangleq \dfrac{\lambda_{\max}(A^{\mathrm{T}} + A)}{2}$;
2) $x^{\mathrm{T}}(A^{\mathrm{T}}P + PA)x \leq 2\mu(QAQ^{-1})x^{\mathrm{T}}Px$;
3) $x^{\mathrm{T}}A^{\mathrm{T}}PAx \leq \|QAQ^{-1}\|^2 x^{\mathrm{T}}Px$;
4) $|x^{\mathrm{T}}P\overline{x}| \leq \sqrt{x^{\mathrm{T}}Px}\sqrt{\overline{x}^{\mathrm{T}}P\overline{x}}$.

Lemma 1 is important for impulsively controlling nonlinear systems. And the proof of Lemma 1 is given in the Appendix.

Definition 1[3] Let $V : R_+ \times R^n \to R_+$, then V is said to belong to Class v_0 if

1) V is continuous in $(t_{k-1}, t_k] \times R^n$ and for each $x \in R^n, k = 1, 2, \cdots$.

$$\lim_{(t,y)\to(t_k^+,x)} V(t, y) = V(t_k^+, x)$$

exists;

2) V is locally Lipschitzian in x.

Definition 2[3] Let V: $R_+ \times R^n \to R_+$. For $(t, x) \in (t_{k-1}, t_k] \times R^n$, we define the right upper derivative of $V(t, x)$ as

$$D^+V(t, x) = \limsup_{h\to 0^+} \frac{1}{h}[V(t + h, x + hf(t, x(t)) - V(t, x)].$$

We also need to define a comparison system, which plays an important role in impulsively controlling nonlinear systems.

Definition 3 [2]–Comparison System Let $V \in v_0$ and assume that

$$\begin{aligned} &D^+V(t, x) \leq g(t, V(t, x)),\ t \neq t_k,\\ &V(t, x + U(k, x)) \leq \psi_k(V(t, x)),\\ &t = t_k, \end{aligned} \tag{4}$$

where $g : R_+ \times R_+ \mapsto R$ is continuous and ψ_k: $R_+ \mapsto R_+$ is nondecreasing. Then the following system:

$$\begin{aligned} &\dot{w}(t) = g(t, w),\ t \neq t_k,\\ &w(t_k^+) = \psi_k(w(t_k)),\\ &w(t_0^+) = w_0 \geq 0 \end{aligned} \tag{5}$$

is the comparison system of impulsive dynamical system (3). Let $S(\rho) = \{x \in R^n \mid \|x\| < \rho\}$. Then we can obtain the following theorem to give sufficient conditions in a unified way for various stability criteria.

Lemma 2 [2] Assume that the following three conditions are satisfied:

1) V: $R_+ \times S(\rho) \mapsto R_+$, $\rho > 0$, $V \in v_0$, $D^+V(t, x) \leq g(t, V(t, x))$, $t \neq t_k$.

2) there exists a ρ_0 such that $x \in S(\rho_0)$ implies that $x + U(k, x) \in S(\rho_0)$ for all k and $V(t, x + U(k, x)) \leq \psi_k(V(t, x)),\ t = t_k,\ x \in S(\rho_0)$.

3) there exist $\alpha(\cdot)$ and $\beta(\cdot) \in \aleph$(class of continuous strictly increasing functions $\alpha : R_+ \to R_+$ such that $\alpha(0) = 0$), such that $\beta(\|x\|) \leq V(t, x) \leq \alpha(\|x\|)$ on $R_+ \times S(\rho)$.

Then, the stability properties of the trivial solution $w = 0$ of the comparison system (5) imply the corresponding stability properties of the trivial solution $x = 0$ of the impulsive dynamical system (3).

Let $g(t, w) = \dot{\lambda}(t)w(t)$, $\lambda \in C^1[R_+, R_+]$, $\psi_k(w) = d_k w(t_k), d_k \geq 0$ for all k. Then we have the following stability criteria.

Lemma 3 [2] The origin of the impulsive dynamical system (3) is asymptotically stable if conditions

$$\int_{t_k}^{t_{k+1}} \dot{\lambda}(t)\mathrm{d}t + \ln(\gamma d_k) \leq 0, \quad k \in N,\ \ \gamma > 1 \tag{6}$$

and

$$\dot{\lambda}(t) \geq 0 \tag{7}$$

are satisfied.

Lemma 4 [31] The origin of the impulsive dynamical system (3) is asymptotically stable if the following conditions hold:

1) $d_0 = \inf_k\{d_k\}$ exists and $d_0 > 0$;

2) there exists an $r(0 < r < 1)$ such that

$$\int_{t_{2k+1}}^{t_{2k+3}} \dot{\lambda}(t)\mathrm{d}t + \ln\left(\frac{d_{2k+2}d_{2k+1}}{r^2}\right) \le 0, \quad k \in N; \tag{8}$$

3) $\lambda(t)$ satisfies

$$\dot{\lambda}(t) \ge 0; \tag{9}$$

4) there exist $\alpha(\cdot)$ and $\beta(\cdot) \in \aleph$ such that

$$\beta(\|x\|) \le V(t,x) \le \alpha(\|x\|). \tag{10}$$

Remark 3 The comparison system, Eq.(5), is a scalar impulsive differential equation. If we can find the impulsive-type comparison system of a high-order impulsive control system without or with time delay, then the stability analysis may be simplified into a scalar impulsive differential equation.

Remark 4 Lemmas 3 and 4 ensure the asymptotic stability of an impulsive differential system without time delay (see Refs.[2, 29–31] and the references cited therein). The two lemmas are also applied to designing an impulsive control for a class of nonlinear systems with time-varying delays.

Remark 5 In equality (8) can be generalized to the following condition: there exist a finite integer $m_0 > 0$ and an $r(0 < r < 1)$ such that

$$\int_{t_{m_0k+1}}^{t_{m_0(k+1)+1}} \dot{\lambda}(t)\mathrm{d}t + \ln\left(\frac{d_{m_0(k+1)}\cdots d_{m_0k+1}}{r^{m_0}}\right) \le 0,$$
$$k \in N. \tag{11}$$

Remark 6 Lemma 3 presents some conditions for asymptotic stability of impulsive dynamical system and impulsive control within one impulsive interval. In contrast with Lemma 3, Lemma 4 represents less conservative conditions for asymptotic ability of the impulsive dynamical system and impulsive control within two successive impulsive intervals.

3. Main results

In this section, we shall analyse the impulsive control of nonlinear systems with time-varying delays. First, we use different Lyapunov functions, the inequalities of Lemma 1 and impulsive-type comparison principles to establish some sufficient conditions under which the impulsively controlled nonlinear system with time-varying delays is asymptotically stable. Then we estimate the upper bounds of impulse interval and time-varying delays. Finally we obtain the impulsive controller design procedure.

Consider a nonlinear system with time-varying delays

$$\begin{aligned}&\dot{x}(t) = Ax(t) + f(t, x(t), x(t - r_1(t)), \cdots,\\ &x(t - r_m(t))),\\ &y(t) = Cx(t), \quad t \ge 0,\end{aligned} \tag{12}$$

where $x(t) \in R^n$ is the state variable, A is an $n \times n$ constant matrix, $y(t) \in R^m$ is the output variable, C is an $m \times n$ constant matrix, $r_i(t)$ is the time-varying delay satisfying

$$\begin{aligned}&0 \le r_i(t) \le \tau_i, \quad i = 1, 2, \cdots, m,\\ &\|x(t - r_i(t))\| \le M_i\|x(t)\|, \quad M_i > 0,\end{aligned} \tag{13}$$

and the nonlinearity $f : R^+ \times \underbrace{R^n \times \cdots \times R^n}_{m+1} \to R^n$ is a continuous function satisfying

$$\begin{aligned}&\|f(t, x(t), x(t - r_1(t)), \cdots,\\ &x(t - r_m(t)))\| \le K\|x(t)\| + \sum_{i=1}^{m} L_i\|x(t - r_i(t))\|,\\ &K, L_i > 0, \quad \text{forany} \quad x(t),\\ &x(t - r_i(t)) \in R^n, t \ge 0.\end{aligned} \tag{14}$$

From in equalities (13) and (14), we obtain

$$\begin{aligned}&\|f(t, x(t), x(t - r_1(t)), \cdots, x(t - r_m(t)))\|\\ &\le K\|x(t)\| + \sum_{i=1}^{m} L_iM_i\|x(t)\|\\ &\le \left(K + \sum_{i=1}^{m} L_iM_i\right)\|x(t)\|, \ K, L_i, M_i > 0.\end{aligned} \tag{15}$$

In order to study the impulsive control of the delay system (12), we design an impulsive control law $U(k, x) = By(t) = BCx(t), \quad k \in N$, where B is an $n \times m$ constant matrix. Then, we can obtain an impulsive control nonlinear system with time-varying delays as follows:

$$\begin{aligned}&\dot{x}(t) = Ax(t) + f(t, x(t), x(t - r_1(t)), \cdots,\\ &x(t - r_m(t))), \quad t \ne t_k,\\ &\Delta x(t) = U(k, x) = BCx(t), \quad t = t_k, \quad k \in N,\\ &x(t) = \phi(t), \quad -\tau \le t \le 0,\end{aligned} \tag{16}$$

where $\tau = \max_{1\le i\le m}\{\tau_i\}$ satisfying $\tau \le (t_{k+1} - t_k), k \in N$, and the initial function ϕ is continuous for $t \in [-\tau, 0]$.

3.1. Asymptotical stability of impulsive control of nonlinear systems with time-varying delays

By applying the theory presented in the previous section, we can obtain the following results on the design of impulsive control of nonlinear systems with time-varying delays.

Theorem 1 Let $n \times n$ matrix $P > 0$ and BC be symmetric. Let $\rho(I + BC)$ be the spectral radius of $I + BC$ and satisfy $\rho(I + BC) \leq 1$. Let

$$\lambda_1 \triangleq \lambda_{\max}(A^{\mathrm{T}} + PAP^{-1} + P),$$
$$\lambda_2 \triangleq \frac{(K + \sum_{i=1}^{m} L_i M_i)^2}{\lambda_{\min}(P)},$$

and

$$\lambda_3 \triangleq \lambda_{\max}[P^{-1}(I + (BC)^{\mathrm{T}})P(I + BC)]. \quad (17)$$

The origin of the impulsive delay system (16) is asymptotically stable if

$$(\lambda_1 + \lambda_2)r_i(t) \leq (\lambda_1 + \lambda_2)(t_{k+1} - t_k) \leq -\ln(\gamma\lambda_3),$$
$$k \in N, \gamma > 1, i = 1, 2, \cdots, m, \quad (18)$$

and

$$\lambda_1 + \lambda_2 \geq 0. \quad (19)$$

Proof Construct a Lyapunov function $V(t, x) = x^{\mathrm{T}}(t)Px(t)$. It yields that

$$\lambda_{\min}(P)\|x(t)\|^2 \leq V(t, x) \leq \lambda_{\max}(P)\|x(t)\|^2,$$

from which one can see that condition 3 of Lemma 2 also satisfies $\beta(x) = \lambda_{\min}(P)x$ and $\alpha(x) = \lambda_{\max}(P)x$.

When $t \neq t_k, k \in N$ we have

$$\begin{aligned}
D^+V(t, x) &= \dot{x}^{\mathrm{T}}(t)Px(t) + x^{\mathrm{T}}(t)P\dot{x}(t) \\
&= x^{\mathrm{T}}(t)(A^{\mathrm{T}}P + PA)x(t) + 2x^{\mathrm{T}}(t)Pf(t, x(t), x(t - r_1(t)), \cdots, x(t - r_m(t))) \\
&\leq x^{\mathrm{T}}(t)(A^{\mathrm{T}}P + PA)x(t) + x^{\mathrm{T}}(t)P^2x(t) + \|f(t, x(t), x(t - r_1(t)), \cdots, x(t - r_m(t)))\|^2 \\
&= x^{\mathrm{T}}(t)(A^{\mathrm{T}}P + PA + P^2)x(t) + \|f(t, x(t), x(t - r_1(t)), \cdots, x(t - r_m(t)))\|^2 \\
&\leq \lambda_1 V(t, x) + \|f(t, x(t), x(t - r_1(t)), \cdots, x(t - r_m(t)))\|^2 \\
&\leq \lambda_1 V(t, x) + \left(K + \sum_{i=1}^{m} L_i M_i\right)^2 \|x(t)\|^2 \\
&\leq (\lambda_1 + \lambda_2)V(t, x), \quad t \neq t_k.
\end{aligned}$$

Therefore, condition 1 of Lemma 2 satisfies $g(t, w) = \dot{\lambda}(t)w = (\lambda_1 + \lambda_2)w$.

Since BC is symmetric, we know that $I + BC$ is also symmetric. By utilizing the Euclidean norm we have

$$\rho(I + BC) = \|I + BC\|.$$

Given any $\rho_0 > 0$ and $x(t) \in S(\rho_0) = \{x(t) \in R^n \mid \|x(t)\| < \rho_0\}$, we have

$$\begin{aligned}
\|x(t) + U(k, x)\| &= \|x(t) + BCx(t)\| \\
&\leq \|I + BC\| \, \|x(t)\| \\
&= \rho(I + BC)\|x(t)\| \\
&\leq \|x(t)\|.
\end{aligned}$$

Hence, $\|x(t) + U(k, x)\| \leq \|x(t)\| < \rho_0$, and we know that $x(t) + U(k, x) \in S(\rho_0)$.

When $t = t_k$, we have

$$\begin{aligned}
&V(x(t) + U(k, x) \\
&= V(x(t) + BCx(t))|_{t=t_k} \\
&= (x(t) + BCx(t))^{\mathrm{T}}P(x(t) + BCx(t)) \\
&= x^{\mathrm{T}}(t)(I + BC)^{\mathrm{T}}P(I + BC)x(t) \\
&\leq \lambda_3 V(t, x).
\end{aligned}$$

Therefore, condition 2 of Lemma 2 satisfies $\psi_k(w) = d_k w = \lambda_3 w$.

It follows from Lemma 2 that the asymptotic stability of the impulsive delay control system in (16) is implied by that of the following comparison system:

$$\dot{w}(t) = g(t, w) = \dot{\lambda}(t)w(t) = (\lambda_1 + \lambda_2)w(t), \ t \neq t_k,$$
$$w(t_k^+) = \psi_k(w(t_k)) = d_k w(t) = \lambda_3 w(t_k),$$
$$w(t_0^+) = w_0 \geq 0. \quad (20)$$

It follows from Lemma 3 that if

$$\int_{t_k}^{t_{k+1}} (\lambda_1+\lambda_2)dt + ln(\gamma\lambda_3) \le 0, \quad \gamma > 1,$$

i.e.

$$(\lambda_1+\lambda_2)(t_{k+1}-t_k) \le -ln(\gamma\lambda_3), \quad \gamma > 1$$

and

$$\lambda_1+\lambda_2 \ge 0$$

hold true, then the origin of the impulsive delay control system (16) is asymptotically stable. Furthermore, we know

$$0 \le r_i(t) \le \tau_i \le \tau \le t_{k+1}-t_k,$$
$$i = 1, 2, \cdots, m, \quad k \in N.$$

So, we can obtain

$$\begin{aligned}(\lambda_1+\lambda_2)r_i(t) &\le (\lambda_1+\lambda_2)\tau_i \\ &\le (\lambda_1+\lambda_2)\tau \\ &\le (\lambda_1+\lambda_2)(t_{k+1}-t_k) \\ &\le -\ln(\gamma\lambda_3),\end{aligned}$$

$$\gamma > 1, \quad k \in N, \quad i = 1, 2, \cdots, m.$$

The proof is complete.

When $\lambda_1+\lambda_2 > 0$, we can have

$$r_i(t) \le \tau_i \le \tau \le t_{k+1}-t_k \le -\frac{ln\gamma\lambda_3}{\lambda_1+\lambda_2},$$
$$\gamma > 1, k \in N, i = 1, 2, \cdots, m.$$

So we can deduce an estimate of the upper bounds of impulse interval and time-varying delays for an asymptotically stable control as follows:

$$\begin{aligned}&\Delta_{\max} \triangleq t_{k+1}-t_k \le -\frac{ln\gamma\lambda_3}{\lambda_1+\lambda_2}, \quad \gamma \to 1^+, \quad k \in N; \\ &0 \le r_i(t) \le \tau_i = -\frac{ln\gamma\lambda_3}{\lambda_1+\lambda_2}, \\ &\gamma \to 1^+, \quad i = 1, 2, \cdots, m. \end{aligned} \tag{21}$$

From the above we can see that $0 < \lambda_3 < 1$ and $\lambda_1+\lambda_2 > 0$.

From Lemma 4 and the proof of Theorem 1, it is easy to obtain the following theorem.

Theorem 2 The origin of the impulsive delay system (16) is asymptotically stable if the following conditions hold:

1) $d_0 = \lambda_3 > 0$ exists and $\lambda_1+\lambda_2 \ge 0$

2) there exists an $r(0 < r < 1)$ such that

$$(\lambda_1+\lambda_2)(t_{2k+3}-t_{2k+1}) \le -2\ln\left(\frac{\lambda_3}{r}\right), \quad k \in N. \tag{22}$$

The proof is similar to that of Theorem 1, only with the following noted

$$\int_{t_{2k+1}}^{t_{2k+3}} (\lambda_1+\lambda_2)\mathrm{d}t + \ln\left(\frac{\lambda_3\lambda_3}{r^2}\right) \le 0, \quad 0 < r < 1,$$

i.e.

$$(\lambda_1+\lambda_2)(t_{2k+3}-t_{2k+1}) \le -2\ln\left(\frac{\lambda_3}{r}\right), \quad 0 < r < 1.$$

So it is omitted here.

Remark 7 We can obtain condition (22) as follows:

$$\begin{aligned}(\lambda_1+\lambda_2)(t_{2k+3}-t_{2k+1}) &= (\lambda_1+\lambda_2)[(t_{2k+3}-t_{2k+2})+(t_{2k+2}-t_{2k+1})],\ k \in N \\ &= (\lambda_1+\lambda_2)(t_{2k+3}-t_{2k+2}) + (\lambda_1+\lambda_2)(t_{2k+2}-t_{2k+1}) \\ &\le -2\ln(\gamma\lambda_3), \quad \gamma > 1 \\ &= -2\ln\frac{\lambda_3}{r}, \quad 0 < r = \frac{1}{\gamma} < 1.\end{aligned}$$

However, condition (18) cannot be derived from condition (22), for condition (22) is only required along a subsequence of $\{t_k\}$. Therefore, Theorem 2 is less conservative than Theorem 1, and this method can be applied to a wider class of nonlinear systems.

Remark 8 Condition (22) can be generalized to the following condition: there exist a finite integer $m_0 > 0$ and an $r(0 < r < 1)$ such that

$$(\lambda_1+\lambda_2)(t_{m_0(k+1)+1}-t_{m_0k+1})$$

$$\leq -\ln\frac{\lambda_3^{m_0}}{r^{m_0}} = -m_0\ln\frac{\lambda_3}{r}, \quad k \in N. \tag{23}$$

Note that the choice of m_0 is related to the actual system considered.

On the base of Theorems 1 and 2, by the use of Lemma 1, we can establish different conditions under which the impulsively controlled nonlinear system with time-varying delays is asymptotically stable.

Corollary 1 Let $n \times n$ matrix Q be nonsingular. Let $BC \in R^{n\times n}$ be symmetric and $\rho(I+BC)$ be the spectral radius of $I+BC$ and satisfy $\rho(I+BC) \leq 1$. Suppose that

$$2\mu(QAQ^{-1}) + 2\left(K + \sum_{i=1}^{m} L_i M_i\right)\sqrt{\frac{\lambda_{\max}(Q^{\mathrm{T}}Q)}{\lambda_{\min}(Q^{\mathrm{T}}Q)}} \triangleq \lambda_4 \geq 0,$$

and

$$\|I + QBCQ^{-1}\|^2 \leq \lambda_5, \quad 0 < \lambda_5 < 1. \tag{24}$$

The origin of the impulsive delay system, Eq.(16), is asymptotically stable if the following condition holds:

$$\lambda_4 r_i(t) \leq \lambda_4(t_{k+1} - t_k) \leq -\ln(\gamma\lambda_5), \quad \gamma > 1, \quad k \in N, \quad i = 1, 2, \cdots, m. \tag{25}$$

Proof Take $P = Q^{\mathrm{T}}Q$, then P will be symmetric and positive definite. Construct a Lyapunov function $V(t,x) = x^{\mathrm{T}}(t)Px(t)$. It yields that

$$\lambda_{\min}(P)\|x(t)\|^2 \leq V(t,x) \leq \lambda_{\max}(P)\|x(t)\|^2,$$

from which one can see that condition 3 of Lemma 2 also satisfies $\beta(x) = \lambda_{\min}(P)x$ and $\alpha(x) = \lambda_{\max}(P)x$.

When $t \neq t_k, k \in N$, by Lemma 1, we have

$$\begin{aligned}
D^+V(t,x) &= \dot{x}^{\mathrm{T}}(t)Px(t) + x^{\mathrm{T}}(t)P\dot{x}(t)\\
&= x^{\mathrm{T}}(t)(A^{\mathrm{T}}P + PA)x(t) + 2x^{\mathrm{T}}(t)Pf(t,x(t),x(t-r_1(t)),\cdots,x(t-r_m(t)))\\
&\leq 2\mu(QAQ^{-1})x^{\mathrm{T}}(t)Px(t) + 2x^{\mathrm{T}}(t)Pf(t,x(t),x(t-r_1(t)),\cdots,x(t-r_m(t)))\\
&\leq 2\mu(QAQ^{-1})x^{\mathrm{T}}(t)Px(t) + 2|x^{\mathrm{T}}(t)Pf(t,x(t),x(t-r_1(t)),\cdots,x(t-r_m(t)))|\\
&\leq 2\mu(QAQ^{-1})V(t,x) + 2\sqrt{x^{\mathrm{T}}(t)Px(t)}\sqrt{f^{\mathrm{T}}(\cdot)Pf(\cdot)}\\
&\leq 2\mu(QAQ^{-1})V(t,x) + 2\sqrt{V(t,x)}\sqrt{\lambda_{\max}(P)}\|f(\cdot)\|\\
&\leq 2\mu(QAQ^{-1})V(t,x) + 2\sqrt{V(t,x)}\sqrt{\lambda_{\max}(P)}\left(K + \sum_{i=1}^{m} L_i M_i\right)\|x(t)\|\\
&\leq \left[2\mu(QAQ^{-1}) + 2\left(K + \sum_{i=1}^{m} L_i M_i\right)\sqrt{\frac{\lambda_{\max}(Q^{\mathrm{T}}Q)}{\lambda_{\min}(Q^{\mathrm{T}}Q)}}\right]V(t,x)\\
&= \lambda_4 V(t,x), \quad t \neq t_k.
\end{aligned}$$

Therefore, condition 1 of Lemma 2 satisfies $g(t,w) = \dot{\lambda}(t)w = \lambda_4 w$.

Since BC is symmetric, we know that $I+BC$ is also symmetric. Given any $\rho_0 > 0$ and $x(t) \in S(\rho_0) = \{x(t) \in R^n \mid \|x(t)\| < \rho_0\}$, we have

$$\begin{aligned}
\|x(t) + U(k,x)\| &= \|x(t) + BCx(t)\|\\
&\leq \|I + BC\|\,\|x(t)\|\\
&= \rho(I+BC)\|x(t)\|\\
&\leq \|x(t)\|.
\end{aligned}$$

Hence, $\|x(t) + U(k,x)\| \leq \|x(t)\| < \rho_0$, we know that $x(t) + U(k,x) \in S(\rho_0)$.

When $t = t_k$, by Lemma 1, we have

$$\begin{aligned}
&V(x(t) + U(k,x))\\
&= V(x(t) + BCx(t))|_{t=t_k}\\
&= (x(t) + BCx(t))^{\mathrm{T}}P(x(t) + BCx(t))\\
&= x^{\mathrm{T}}(t)(I+BC)^{\mathrm{T}}P(I+BC)x(t)\\
&\leq \|Q(I+BC)Q^{-1}\|^2 x^{\mathrm{T}}(t)Px(t)\\
&= \|I + QBCQ^{-1}\|^2 V(t,x)\\
&\leq \lambda_5 V(t,x).
\end{aligned}$$

Therefore, condition 2 of Lemma 2 satisfies $\psi_k(w) = d_k w = \lambda_5 w$.

It follows from Lemma 2 that the asymptotic sta-

bility of the impulsive delay control system (16) is implied by that of the following comparison system:

$$\begin{aligned} &\dot{w}(t) = \lambda_4 w(t),\ t \neq t_k,\\ &w(t_k^+) = \lambda_5 w(t_k),\\ &w(t_0^+) = w_0 \geq 0. \end{aligned} \tag{26}$$

It follows from Lemma 3 that if

$$\int_{t_k}^{t_{k+1}} \lambda_4 \mathrm{d}t + \ln(\gamma\lambda_5) \leq 0, \quad \gamma > 1,$$

i.e.

$$\lambda_4(t_{k+1} - t_k) \leq -\ln(\gamma\lambda_5), \quad \gamma > 1,$$

and

$$\lambda_4 \geq 0$$

hold true, then the origin of impulsive delay control system (16) is asymptotically stable.

Note that

$$\begin{aligned} &\lambda_4 r_i(t) \leq \lambda_4 \tau_i \leq \lambda_4 \tau \leq \lambda_4(t_{k+1} - t_k) \leq -\ln(\gamma\lambda_5),\\ &\gamma > 1, \quad k \in N, \quad i = 1, 2, \cdots, m. \end{aligned}$$

So the above theorem also gives an estimate of the upper bounds of impulse interval and time-varying delays for asymptotically stable control, as follows:

$$\begin{aligned} &\Delta_{\max} \triangleq t_{k+1} - t_k \leq -\frac{\ln(\gamma\lambda_5)}{\lambda_4}, \quad \gamma \to 1^+, \quad k \in N,\\ &0 \leq r_i(t) \leq \tau_i = -\frac{\ln\gamma\lambda_5}{\lambda_4},\\ &\gamma \to 1^+, \quad i = 1, 2, \cdots, m. \end{aligned} \tag{27}$$

From the above we can see that $\lambda_4 > 0$.

From Lemma 4 and the proof of Corollary 1, it is easy to obtain the following theorem.

Corollary 2 The origin of the impulsive delay system (16) is asymptotically stable if the following conditions hold:

1) $d_0 = \lambda_5 > 0$ exists and $\lambda_4 \geq 0$

2) there exists an $r(0 < r < 1)$ such that

$$\lambda_4(t_{2k+3} - t_{2k+1}) \leq -2\ln\left(\frac{\lambda_5}{r}\right), \quad k \in N. \tag{28}$$

The proof is similar to those of Corollary 1 and Theorem 2, so it is omitted here.

Remark 9 Condition (28) can be generalized to condition (29) given below: there exist a finite integer $m_0 > 0$ and an $r(0 < r < 1)$ such that

$$\begin{aligned} &\lambda_4(t_{m_0(k+1)+1} - t_{m_0 k+1})\\ &\leq -\ln\frac{\lambda_5^{m_0}}{r^{m_0}} = -m_0\ln\frac{\lambda_5}{r}, \quad k \in N. \end{aligned} \tag{29}$$

In practical applications, we want to use a simple Lyapunov function to establish some sufficient conditions under which the impulsively controlled nonlinear delay system is asymptotically stable, and estimate the upper bounds of impulse interval and time-varying delays.

When we choose $P = I$, we can construct a simple and practical Lyapunov function $V(t, x) = x^{\mathrm{T}}(t)x(t)$ and deal with a simple case. According to the proofs of Theorem 1 and Theorem 2, it is easy to obtain the following corollaries. Here we omit the proofs to avoid the repetition.

Corollary 3 Let BC be symmetric, and $\rho(I + BC)$ be the spectral radius of $I + BC$ and satisfy $\rho(I + BC) \leq 1$. Let

$$\begin{aligned} &\lambda_{\max}(A^{\mathrm{T}} + A + I) + \left(K + \sum_{i=1}^{m} L_i M_i\right)^2\\ &\triangleq \lambda_6 \geq 0, \end{aligned}$$

and

$$(I + (BC)^{\mathrm{T}})(I + BC) \leq \lambda_7 I. \tag{30}$$

The origin of the impulsive delay system (16) is asymptotically stable if the following condition holds:

$$\begin{aligned} &\lambda_6 r_i(t) \leq \lambda_6(t_{k+1} - t_k) \leq -\ln(\gamma\lambda_7),\\ &k \in N, \quad \gamma > 1, \quad i = 1, 2, \cdots, m. \end{aligned} \tag{31}$$

At the same time, the above theorem gives an estimate of the upper bounds of impulse interval and time-varying delays for asymptotically stable control as follows:

$$\begin{aligned} &\Delta_{\max} \triangleq t_{k+1} - t_k \leq -\frac{\ln\gamma\lambda_7}{\lambda_6}, \quad \gamma \to 1^+, \quad k \in N,\\ &0 \leq r_i(t) \leq \tau_i = -\frac{\ln\gamma\lambda_7}{\lambda_6},\\ &\gamma \to 1^+, \quad i = 1, 2, \cdots, m. \end{aligned} \tag{32}$$

From the above we can see that $\lambda_6 > 0$.

Corollary 4 The origin of the impulsive delay system (16) is asymptotically stable if the following conditions hold:

1) $d_0 = \lambda_7 > 0$ exists and $\lambda_6 \geq 0$,

2) there exists an $r(0 < r < 1)$ such that

$$\lambda_6(t_{2k+3} - t_{2k+1}) \leq -2\ln\left(\frac{\lambda_7}{r}\right), \quad k \in N. \tag{33}$$

Moreover, condition (33) can be generalized as follows: there exist a finite integer $m_0 > 0$ and an $r(0 < r < 1)$ such that

$$\lambda_6(t_{m_0(k+1)+1} - t_{m_0k+1}) \leq -\ln\frac{\lambda_7^{m_0}}{r^{m_0}} = -m_0\ln\frac{\lambda_7}{r}, \quad k \in N. \tag{34}$$

Remark 10 By applying Lemma 3 to designing an impulsive control for a class of nonlinear systems with time-varying delays, we obtain Theorem 1, Corollary 1 and Corollary 3, and establish different conditions under which the impulsively controlled nonlinear system with time-varying delays is asymptotically stable; at the same time, by applying Lemma 4 to designing the impulsive controller for a class of nonlinear systems with time-varying delays, we also obtain Theorem 2, Corollary 2 and Corollary 4, and derive some less conservative conditions under which the impulsively controlled nonlinear system with time-varying delays is asymptotically stable.

Remark 11 Time-varying delays $r_i(t)$, $i = 1, 2, \cdots, m$ are unknown but bounded by some conditions given in Theorem 1, Corollary 1 and Corollary 3. On the other hand, Theorem 1, Corollary 1 and Corollary 3 also present the relation between impulsive time sequence and time-varying delays.

3.2. Impulsive controller design procedure

On the basis of the Theorem 1, we present the following impulsive controller design procedure to asymptotically stabilize the time-varying delay system (12):

Step 1). obtain the matrices A and C and the positive constants K, L_i and M_i given nonlinear system with time-varying delays;

Step 2). construct the matrix P satisfying $P > 0$;

Step 3). calculate the parameters λ_1, λ_2 according to the condition (17);

Step 4). determine a set of impulsive control instants $\{t_k, k \in N\}$ and calculate the impulse interval $t_{k+1} - t_k$;

Step 5). choose the parameter γ to determine the parameter λ_3 according to the condition (18);

Step 6). obtain the impulsive controller B according to the condition (17).

4. Numerical example

In this section, we shall give an example to illustrate the effectiveness of impulsive control method on the basis of the above analysis.

Consider the following nonlinear system with time-varying delays made up of Mackey–Glass equation (see Ref.[39])

$$\begin{aligned} \dot{x}(t) &= \frac{bx(t-\tau)}{1+x^c(t-\tau)} - ax(t), \\ y(t) &= x(t), \quad t \geq 0. \end{aligned} \tag{35}$$

When $a = 0.1$, $b = 0.2$, $c = 10$, $\tau = 14$ and $\tau = 17$, the Mackey–Glass equations

$$\dot{x}(t) = \frac{0.2x(t-14)}{1+x^{10}(t-14)} - 0.1x(t)$$

and

$$\dot{x}(t) = \frac{0.2x(t-17)}{1+x^{10}(t-17)} - 0.1x(t)$$

exhibit chaotic behaviours (see Figs.1(a) and 1(b)).

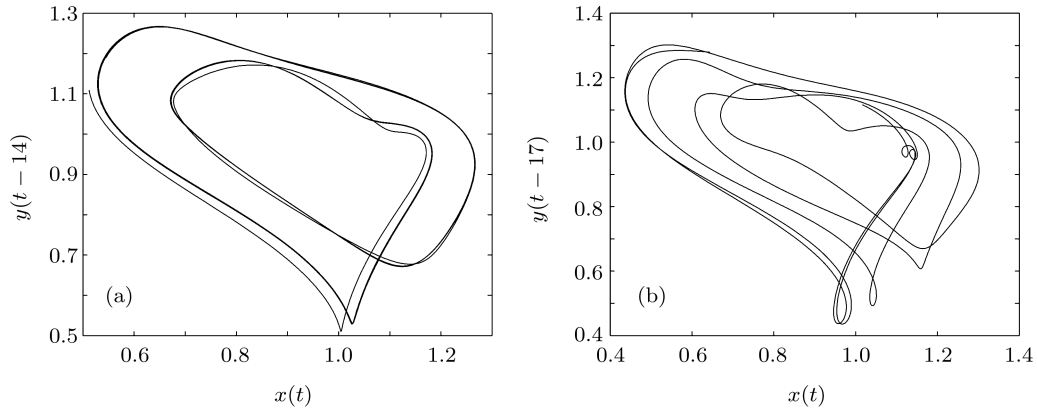


Fig.1. Chaotic behaviours of Mackey–Glass equation.

According to the impulsive controller design procedure, we design the impulsive control law as

$$B_1 = \begin{pmatrix} -0.1033 & 0 & 0 \\ 0 & -0.1033 & 0 \\ 0 & 0 & -0.1033 \end{pmatrix}, \quad \text{and} \quad B_2 = \begin{pmatrix} -1.8967 & 0 & 0 \\ 0 & -1.8967 & 0 \\ 0 & 0 & -1.8967 \end{pmatrix}.$$

Let the initial condition be $x_0 = 1.8$. As for the impulsive controllers B_1 and B_2, the simulation results are shown in Figs.2(a) and 2(b) respectively. The figures illustrate that the impulsively controlled Mackey–Glass system is asymptotically stable.

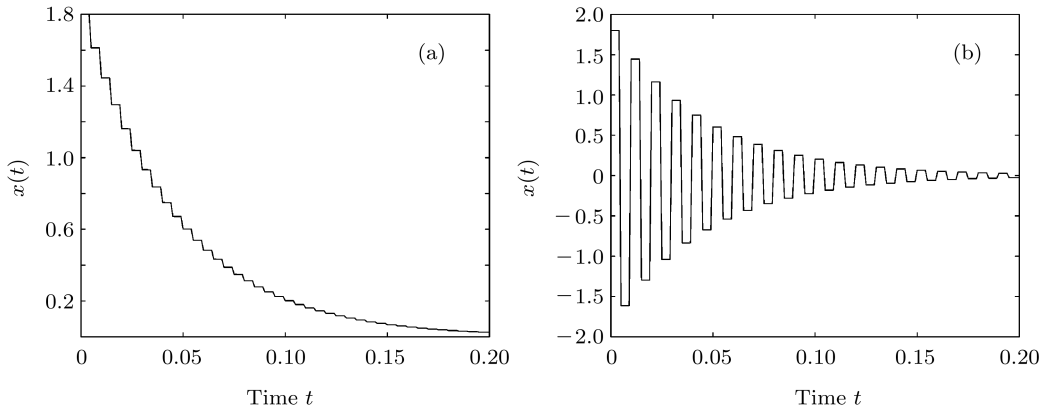


Fig.2. Impulsive control of Mackey-Glass system.

5. Conclusions

Impulsive control of nonlinear systems with time-varying delays, which can be regarded as an extension for the nonlinear systems without time delay in Refs.[29, 30], has been studied.

The findings of this paper lie in the following aspects. Firstly, we extend the theory of impulsive differential equations into impulsive delay differential equations. Secondly, we establish the relation between impulsive time sequence and time-varying delays to simplify the design of impulsive control law, and we estimate the upper bounds of impulse interval and time-varying delays for asymptotically stable control. Thirdly, we use some inequalities and construct different Lyapunov functions to study the impulsive control of nonlinear systems with time-varying delays. Finally, we list impulsive-type comparison principles and impulsive control criteria as Lemmas 3 and 4 to obtain a series of different conditions under which impulsively controlled nonlinear systems with time-varying delays are asymptotically stable.

Appendix

Proof of Lemma 1 Q is a nonsingular matrix because $P > 0$. If $x = 0$, the result is obvious. Let $x \neq 0$ and $z = Qx$. So, $z^{\mathrm{T}}z = x^{\mathrm{T}}Q^{\mathrm{T}}Qx = x^{\mathrm{T}}Px = (Qx)^{\mathrm{T}}Qx = \|Qx\|^2$, and $\|Qx\| = \sqrt{x^{\mathrm{T}}Px}$.

1) $\|x\|$ is the Euclid vector norm. $\|A\|$ and $\mu(A)$ denote the matrix norm and the matrix measure respectively.

2)

$$\begin{aligned} & x^{\mathrm{T}}(A^{\mathrm{T}}P + PA)x \\ &= x^{\mathrm{T}}(A^{\mathrm{T}}Q^{\mathrm{T}}Q + Q^{\mathrm{T}}QA)x \\ &= x^{\mathrm{T}}Q^{\mathrm{T}}(Q^{-1})^{\mathrm{T}}(A^{\mathrm{T}}Q^{\mathrm{T}}Q + Q^{\mathrm{T}}QA)Q^{-1}Qx \\ &= x^{\mathrm{T}}Q^{\mathrm{T}}[(QAQ^{-1})^{\mathrm{T}} + QAQ^{-1}]Qx \\ &\leq 2\mu(QAQ^{-1})z^{\mathrm{T}}z \end{aligned}$$

$$= 2\mu(QAQ^{-1})x^{\mathrm{T}}Px,$$

3)

$$\begin{aligned} & x^{\mathrm{T}}A^{\mathrm{T}}PAx \\ &= x^{\mathrm{T}}A^{\mathrm{T}}Q^{\mathrm{T}}QAx \\ &= x^{\mathrm{T}}Q^{\mathrm{T}}(Q^{-1})^{\mathrm{T}}(QA)^{\mathrm{T}}QAQ^{-1}Qx \\ &= z^{\mathrm{T}}(QAQ^{-1})^{\mathrm{T}}(QAQ^{-1})z \\ &\leq \|QAQ^{-1}\|^2 z^{\mathrm{T}}z \end{aligned}$$

$$= \|QAQ^{-1}\|^2 x^{\mathrm{T}}Px,$$

4)

$$\begin{aligned} & |x^{\mathrm{T}}P\overline{x}| \\ &= |x^{\mathrm{T}}Q^{\mathrm{T}}Q\overline{x}| \\ &= |(Qx)^{\mathrm{T}}(Q\overline{x})| \\ &\leq \|Qx\|\|Q\overline{x}\| \\ &= \sqrt{x^{\mathrm{T}}Px}\sqrt{\overline{x}^{\mathrm{T}}P\overline{x}}. \end{aligned}$$

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