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New insights into traffic dynamics: a weighted probabilistic cellular automaton model*

Li Xing-Li(李兴莉)[a], Kuang Hua(邝 华)[a)b)], Song Tao(宋 涛)[a)], Dai Shi-Qiang(戴世强)[a)†], and Li Zhi-Peng(李志鹏)[c)]

[a)] *Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai* 200072, *China*
[b)] *College of Physics and Electronic Engineering, Guangxi Normal University, Guilin* 541004, *China*
[c)] *School of Electronics and Information Engineering, Tongji University, Shanghai* 200092, *China*

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From the macroscopic viewpoint for describing the acceleration behaviour of drivers, a weighted probabilistic cellular automaton model (the WP model, for short) is proposed by introducing a kind of random acceleration probabilistic distribution function. The fundamental diagrams, the spatiotemporal patterns, are analysed in detail. It is shown that the presented model leads to the results consistent with the empirical data rather well, nonlinear flow–density relationship existing in lower density regions, and a new kind of traffic phenomenon called neo-synchronized flow. Furthermore, we give the criterion for distinguishing the high-speed and low-speed neo-synchronized flows and clarify the mechanism of this kind of traffic phenomenon. In addition, the result that the time evolution of distribution of headways is displayed as a normal distribution further validates the reasonability of the neo-synchronized flow. These findings suggest that the diversity and the randomicity of drivers and vehicles have indeed a remarkable effect on traffic dynamics.

Keywords: WP model, unitary random acceleration, neo-synchronized flow, nonlinear flow–density relationship

PACC: 0520, 0550

1. Introduction

In the last decades, traffic dynamics has attracted much attention from physicists and mathematicians, because it is a typical subject in non-equilibrium statistical mechanics of self-driven many-particle systems. There are a lot of models for traffic flows which have been proposed thus far, from various viewpoints, such as macroscopic and microscopic, differential equations and cellular automata, deterministic and probabilistic, etc.[1−13] Among these traffic models, the cellular automata (CA) model achieves a remarkable success although it is conceptually simple compared with other dynamical approaches.[11] Presently, there are two basic CA models describing single-lane traffic flows: the Nagel–Schreckenberg (NaSch) model[12] and the Fukui–Ishibashi (FI) model.[13] Both of them are defined on a one-dimensional lattice consisting of L sites with periodic boundary conditions. Either site is not occupied by a vehicle, but it is empty. The speed of each vehicle is an integer between zero and $V_{\max}$. Let x_i^t denote the position of the ith car at time t, and x_{i+1}^t the position of its preceding car at time t, then the system will evolve according to the synchronous rules, resulting in

$$x_i^{t+1} = x_i^t + v_i^{t+1}, \tag{1}$$

where

$$v_i^{t+1} = \min\{v_{\max}, v_i^t + 1, \Delta x_i\} \tag{2}$$

in the NaSch model and

$$v_i^{t+1} = \min\{v_{\max}, \Delta x_i\} \tag{3}$$

in the FI model, in which Δx_i is the headway of the ith car with its preceding car. If we do not consider the randomization caused by other complicated influences, these two models differ only in acceleration rules, that is, the NaSch model restricts the cars to gradual acceleration, while the FI model allows for abrupt increase if there is an enough empty spacing ahead. In both the update rules, the speed of the ith car depends on headway. Based on the two models, many modified models have been proposed, among

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†Corresponding author. E-mail: sqdai@shu.edu.cn

which are the VDR models,[14] the T^2 model,[15] the BJH model,[16] and the VE model.[17] In addition, some relevant analytical and improved work about the FI model has been done by Wang *et al*[18−20] and others.[21,22]

However, because the study of the CA traffic model has relatively short history, there has not yet been the 'best' CA traffic model which should be both realistic and simple. In this paper, we consider globally the different acceleration behaviours of drivers, which were not elucidated well in all previous work. In real traffic, there are various kinds of drivers and vehicles on a road, and this diversity leads to different responses in the same traffic situation, e.g. aggressive drivers and pessimistic drivers (or fast vehicles and slow vehicles) have entirely different driving behaviours. In addition, such other stochastic factors as noise and external influences could affect the reactions to a certain extent. These reasons lead to the fact that not all drivers accelerate the way those described in the FI model and NaSch model do, where all drivers accelerate from zero to the possible maximum speed or accelerate at the same probability under the same conditions. Therefore, it is more realistic to take into account the diversity and randomicity of drivers and vehicle properties.

In view of the above analysis, we propose another new CA model from a totally different macroscopic viewpoint based on the probabilistic theory instead of the traditional CA theory. Our basic idea is to introduce a kind of weighted random acceleration probability distribution function to synthetically describe the differences of drivers' own intentions, vehicle responses, noise and external influences. Here the introduction of this function indicates that the dependence of the driving strategy on the traffic state becomes essential. At the same time, it also implies the expectation of driving comfortably, i.e. avoiding both crashes and unnecessary large or small acceleration compared with the FI model and NaSch model.

2. Outline of the weighted probabilistic cellular automaton (WP) model

First of all, we explain the general framework of our model for single-lane traffic. N vehicles are moving on a single-lane road divided into a one-dimensional array of L sites. In this paper, parallel updating is adopted. Each site contains one vehicle at most, and colliding and overtaking are thus prohibited (the so-called hard-core exclusion rule). Let M_i^t be a stochastic variable which denotes the number of sites through which the ith vehicle moves at time t. We introduce a probability distribution function $w_i^t(m)$ which gives the probability of the ith vehicle hopping m ($m = 0, 1, 2, \cdots$) sites ahead at time t, i.e. $M_i^t = m$ ($m = 0, 1, 2, \cdots$). The stochastic variable incorporates such uncertain effects as different driver and vehicle characteristics, incorrect driving operations, noises, and external influences. In this paper, we assume $m_{\max} = 5$. It should be noted that $m_{\max} = 5$ dose not imply that the maximum speeds of each vehicle are equal to 5 and we only use the probability density function to describe the mean effect of drivers, vehicles and external influences. The probability distribution function $w_i^t(m)$ is defined as

$$w_i^t(m) = \begin{cases} 1, & \Delta x_i = 0, \\ \dfrac{1}{\Delta x_i}\left(1 - \dfrac{\alpha}{\gamma^{m+1}}\right), & m = 0, 1, 2, \cdots, \Delta x_i - 1; \\ & \Delta x_i \neq 0, \\ \dfrac{1}{\Delta x_i}\left(1 - \dfrac{\beta}{\gamma^m}\right), & m = \Delta x_i; \Delta x_i \neq 0, \end{cases} \tag{4}$$

where α, β and γ are undetermined parameters which should meet the requirements that $\alpha, \beta, \gamma \in Z^+$ and $\alpha + \beta = \gamma$; $\Delta x_i = x_{i+1}^t - x_i^t - 1$ denotes the headway as before. Here it is assumed that if $\Delta x_i > 5$, the vehicle moves at the probability $w_i^t(m)$ of $\Delta x_i = 5$ with considering the speed limit on roads. It is necessary to point out that $w_i^t(m)$ has the following properties:

(1) $w_i^t(m)$ ($m = 0, 1, 2, \cdots, \Delta x_i$) increases monotonically with the increase of m at a given Δx_i, which means $w_i^t(m)/w_i^t(m-1) > 1$, for we know that drivers generally intend to move as fast as possible at a constant Δx_i, but the ratio is close to 1 as m increases, which reflects that the acceleration effect is almost the same at larger headways;

(2) $\sum_{m=0}^{\Delta x_i} w_i^t(m) = 1$, and $w_i^t(m) = 1$ for $\Delta x_i = 0$, which shows that the probability of random acceleration, $w_i^t(m)$, obeys the unitary condition.

Based on the above analysis and realistic mathematical and physical considerations, the reasonable parameters are suggested as $\alpha = 2$, $\beta = 1$ and $\gamma = 3$.

The updating procedure consists of the following three steps.

• Determination of the randomization parameter $w_i^t(m)$ based on Eq.(4) at a given Δx_i.

• Determination of the number of sites through which a vehicle passes (i.e. the probability speed) according to the intention $w_i^t(m)$.

According to Eq.(4), the set of probability density is composed of the following $\Delta x_i + 1$ subsets:

1) $[0, w_i^t(1))$;

2) $[w_i^t(1), w_i^t(1) + w_i^t(2))$;

$\cdots$;

Δx_i) $[w_i^t(1) + \cdots + w_i^t(m-2), w_i^t(1) + \cdots + w_i^t(m-1))$;

$\Delta x_i + 1$) $[w_i^t(1) + \cdots + w_i^t(m-1), 1]$.

During the computation, as to a certain Δx_i, the random number rand$(\cdot)$ is produced by computer, and rand$(\cdot) \in [0, 1]$. If rand$(\cdot)$ belongs to the jth ($j = 1, 2, \cdots, \Delta x_i + 1$) one of the above subsets, the number of sites which a vehicle hops is thus $m = j - 1$.

$$v_i^t = m. \tag{5}$$

• Updating the new position of each vehicle

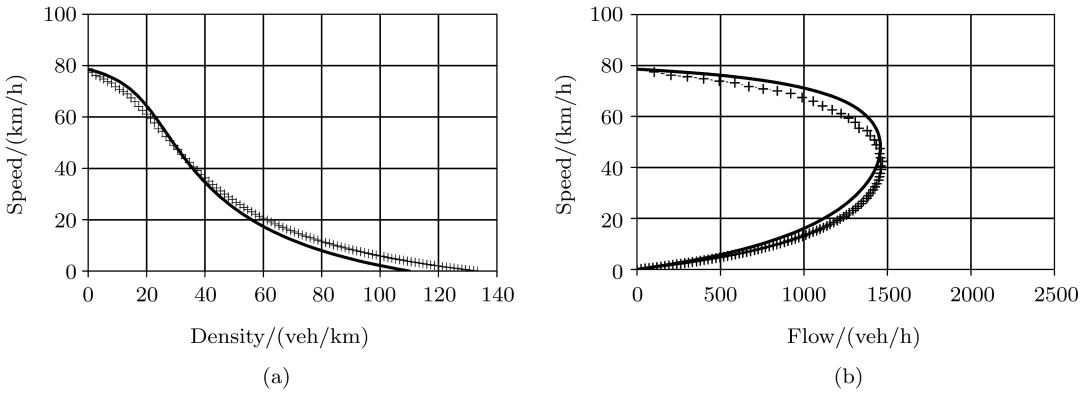
$$x_i^{t+1} = x_i^t + v_i^t. \qquad (\forall i) \tag{6}$$

We call the probability distribution w_i^t the intention because it is an intrinsic variable of driver himself. It brings an uncertainty of operation into the traffic model and reflects a certain statistical mean effect.

3. Simulation results and discussion

The numerical simulations are performed according to the above updating rules under the periodic boundary condition. A one-dimensional lattice of L sites and vehicles moving unidirectionally are considered. Each site is set to be 7.5 m long, L to be 1000, one time step to be 1 s, on the order of the reaction time for humans, and the maximum speed to be $v_{\max} = 5$, corresponding to 135 km/h. In order to characterize the behaviour of the model, we determine globally the macroscopic quantities, including the global density ρ, the mean speed V, the mean flow J, defined as $\rho = N/L$, $V = \frac{1}{T}\sum_{t=t_0}^{t=T+t_0-1} \frac{1}{N}\sum_{n=1}^{N} v_n(t)$ and $J = \rho V$. In numerical simulation, the first 5×10^4 time-steps of each run are put away in order to remove the transient effects, and then the data are recorded in successive 1×10^4 time-steps. The mean speed is obtained by averaging over 30 runs of simulations.

Figure 1 presents a comparison between the results obtained from the WP model and the Van Aerde model, presented in Ref.[23]. Here it should be noted that the reason why we make a comparison between our simulation results and those in Ref.[23] is that the WP model describes the road condition with speed limit and the Van Aerde model was also used to investigate the speed limit freeway. In Ref.[23], the Van Aerde model described the empirical observation well,



(a) (b)

Fig.1. A comparison between the results obtained from the WP model (++ line) and Van Aerde model (solid line) in Ref.[23], showing the speed–density curves (a) and the speed–flow curves (b).

so through the indirect comparison, we can see that the fundamental diagrams are in rather good agreement with the empirical data (see Fig.1). The results also validate the conclusion of general CA models well,[24] i.e. on a single-lane road, the average speed of vehicles is mainly determined by those of the slow vehicles. Meanwhile, the variation tendency of mean speed is in good agreement with the empirical

observations[25] instead of the occurrence of a plateau as appearing in other CA models,[12,13] and a nonlinear flow–speed–density relationship exists.[26]

In order to analyse the traffic behaviour revealed by the WP model, we further study the fundamental diagram (see Fig.2) and the spatiotemporal patterns (see Fig.3). From the figures, we can find a new traffic state called neo-synchronized flow, whose difference from the conventional synchronized flow will be expounded below. As the density is very low ($\rho < \rho_1$), the free flow exists. As the density exceeds ρ_1, the high-speed neo-synchronized flow begins to emerge (see Fig.3(a)). As the density is in the range $\rho_1 < \rho < \rho_2$ and increases further, the neo-synchronized flow region gradually invades the free flow region and, simultaneously, the region of the high-speed neo-synchronized flow is gradually amplified. As the density is equal to ρ_2, the flux reaches its maximum. In the range $\rho_1 < \rho < \rho_2$, the speed of vehicles decreases from 2.6 to 1.5, whereas the flux increases from 0.17 to 0.41, which is the most basic phenomenon in the synchronized flow.[27] As the density is in the range $\rho_2 < \rho < \rho_3$, the high-speed neo-synchronized flow gradually transforms into the low-speed neo-synchronized flow (see Figs.3(b) and 3(c)), in which the speed is lower than that in the range $\rho_1 < \rho < \rho_2$ and the flux is still high, but a large area of jam cannot appear. As the density exceeds ρ_3, both the speed and flow are very low and the wide moving jam will occur.

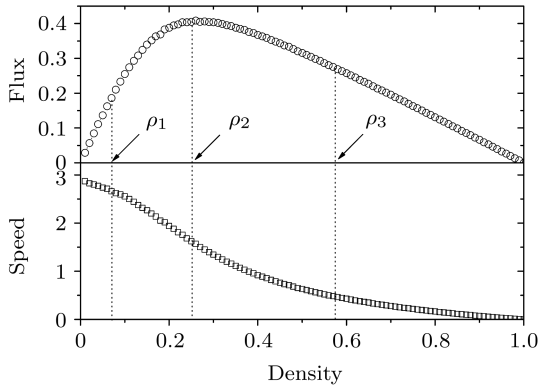


Fig.2. The fundamental diagram for the WP model with the flux–density curve (upper) and the speed–density curve (lower).

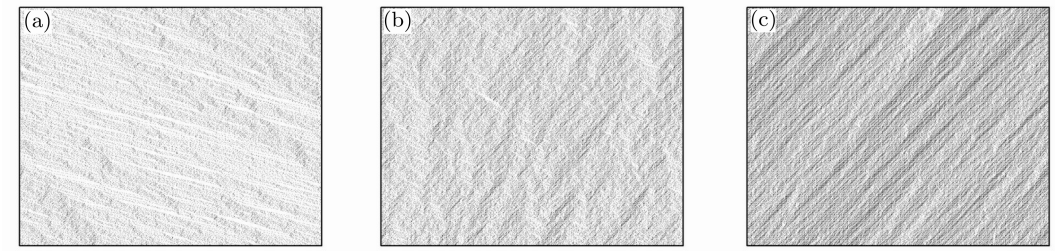


Fig.3. The spatiotemporal patterns of the WP model from a stochastic initial distribution with values of ρ being 0.15 (a), 0.3 (b), and 0.5 (c). The horizontal direction is space between 200 and 700 cells and the vertical direction (downwards) is time between periods of 49000 and 49600.

The above analysis mainly focuses on the traffic phenomenon, and now we turn our energies to elucidating the mechanism of it. We first give a definition of the neo-synchronized flow consisting of high-speed and low-speed neo-synchronized flows. We consider the speed–density curve given in Fig.2, and introduce the variables ξ and η defined as

$$\xi = \frac{\Delta v}{\Delta \rho}$$

and

$$\eta = \frac{\Delta^2 v}{\Delta \rho^2}, \tag{7}$$

where ξ and η are the first- and second-order difference quotients of speed with respect to density, respectively. ξ denotes for the magnitude of change of speed with respect to density and meets the condition: $\xi < 0$ and $\xi \to 0$ as $\rho \to 1$; and η refers to the magnitude of change of ξ with respect to density, which, in nature, discloses the degree of drivers' response to the changing of density and can be described as the 'speed fluctuation'.

Figure 4 shows the difference quotients of speed corresponding to Fig.2 with the definition in Eq.(7). We can see clearly that: (1) as $\rho_1 < \rho < \rho_2$, ξ decreases first and then increases a little with density increasing up to ρ_2. Besides, the fluctuations of ξ and η become larger and the difference quotients turn unstable. These can be the criterion of high-speed neo-synchronized flow, and its speed variation range

is 56.7 ± 13.5 km/h; (2) as $\rho_2 < \rho < \rho_3$, ξ is increasing, and the fluctuations of ξ and η are almost zero. These can be the criterion of low-speed neo-synchronized flow, and its speed variation scope is 28 ± 15 km/h. These results reflect that in the high-speed neo-synchronized flow, the drivers can change vehicle speeds more easily with the variation of density because of the introduction of random probabilistic distribution function, where drivers have more choices at large headways than at small headways, which causes the speed to fluctuate intensively. This conclusion is similar to that in Refs.[28,29], where for synchronized flows, a certain range of speed fluctuations of vehicle should be acceptable. In addition, the random probabilistic distribution function could also explain why the spatiotemporal patterns qualitatively look the same and have some common characteristic features in Fig.3, because there are always some drivers who continuously have the tendency to almost collide with each other. As a consequence, the system is in a littered with mini-jams in both the low and the high density regimes. This also reflects that all spatiotemporal patterns in the system are of a chaotic nature (i.e. nonlinear with stochastic noise).[30] These obtained results, to a certain extent, validate the fact that the appearance of synchronized flow can be reproduced under the assumption of the admixture of different vehicle types[31] as well as give a new insight into the explanation of traffic phenomena from the viewpoint of probability.

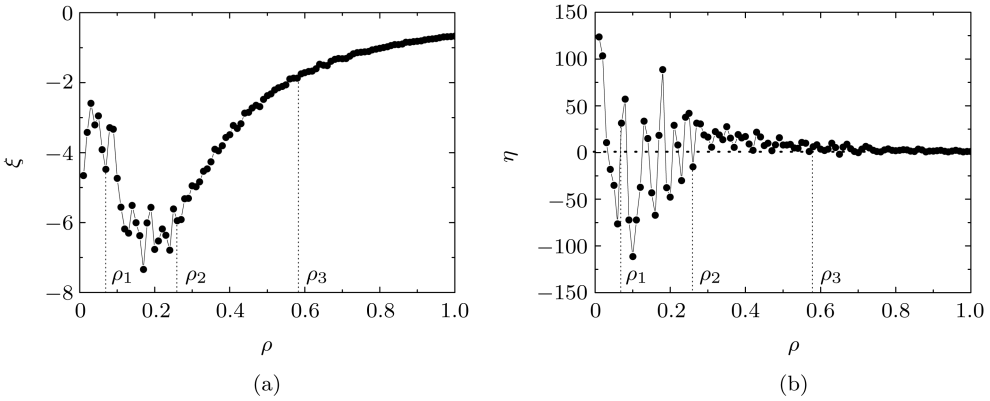


Fig.4. The difference quotients of speed corresponding to Fig.2. $\xi = \dfrac{\Delta v}{\Delta \rho}$ (a) and $\eta = \dfrac{\Delta^2 v}{\Delta \rho^2}$ (b).

Note that the term about "light and heavy synchronized flows" has been found in Ref.[32], but with the above specific analysis, we find some obvious differences in synchronized flow between in this paper and in Ref.[32] as follows: (1) the scope of light and heavy synchronized flows is 24–60 km/h in Ref.[32], while in our model the scope of neo-synchronized flow extends from 13 to 70 km/h and the acceptable speed fluctuations of high-speed and low-speed neo-synchronized flows are also given; (2) the spatiotemporal patterns display no obvious large jam in the appearance of a neo-synchronized traffic flow, in which vehicles are all moving although they have a relatively low speed; (3) spatiotemporal pattern in Fig.3 is obtained from a stochastic initial distribution, and the same spatiotemporal patterns can also be obtained from an initial homogeneous distribution and from an initial mega-jam distribution, which indicates that the initial distribution has no influence on the patterns. Moreover, in this paper, the WP model mainly describes the road condition with speed limit. Knowing from the synchronized flow in Ref.[32], we temporarily call this new phenomenon the neo-synchronized flow.

Last, we study the time evolution of distribution of headway. Figure 5 shows the distributions of headways at two time stages under different values of density ρ: 0.15, 0.30 and 0.50, respectively. We find that the distribution of headways of the WP model changes significantly. In particular, the ratio of vehicles with null headway decreases evidently and the vehicles centralize in an adjacent region $1/\rho - 1$ after a sufficient time evolution, displayed as a normal distribution, which is consistent with the spatiotemporal patterns in Fig.3. In other words, on the one hand, this does not represent jams easily appearing in this model, on the other hand, it shows that the normal distribution of vehicles leads the availability of road and flux to increase. All these results further validate the reasonability of neo-synchronized flow.

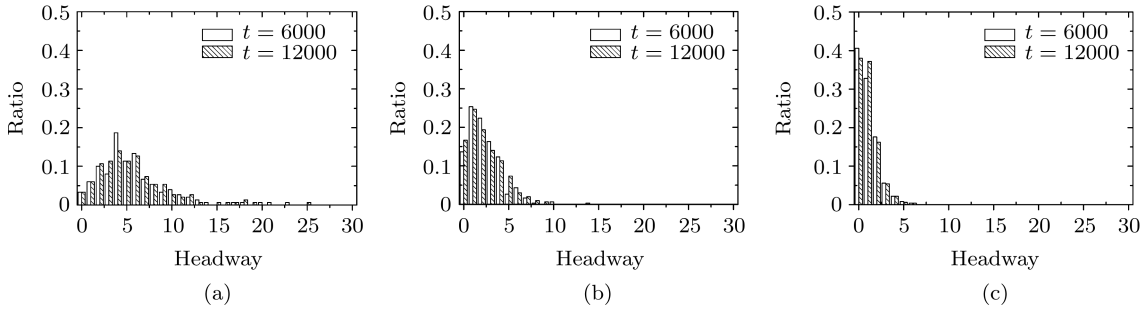


Fig.5. The distributions of the headways with which the vehicles move at time steps 6000 and 12000 of the WP model and at values of ρ: 0.15 (a), 0.3 (b) and 0.5 (c).

4. Summary and outlook

Our aim in this paper is to deliver a better understanding of the simulation of the traffic flow by using a kind of totally different CA theory. To do so, beginning with a general scheme, we have proposed a new weighted probabilistic CA model, in which we have introduced a probability distribution function. The new model can be more adapted to reflecting some uncertain effects such as different driver and vehicle characteristics, driver error false operation and external influences. Through numerical simulations and comparison with the empirical data, it can be concluded that (i) the obtained fundamental diagrams are in good agreement with the empirical observations; (ii) the nonlinear flow–density relationship is further validated; (iii) a new kind of traffic behaviour called neo-synchronized flow can be found; (iv) the criterion of neo-synchronized flow is rational and practicable; (v) time evolution of distribution of headways is displayed as a normal distribution; (vi) the basic idea of the new model is different from that of the conventional CA model and it has a simpler form and reasonable physical meaning.

Finally, the method, especially the modelling idea developed here for all kinds of discrete dynamical systems with statistical behaviour, is actually more general. The similarities in some statistical properties of the investigated vehicle systems, pedestrian flow systems and granular systems strongly suggest that a wide class of different phenomena can be described by the same type of modelling approach. The corresponding universality could complement the developed interdisciplinary research in the recent years. Further studies, e.g. researching into the mechanism of this neo-synchronized flow deeply, extending the WP model to the more general case (i.e. without speed limit) and comparing this model with other CA models, will be given in subsequent work.

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