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Thermal entanglement and teleportation of a thermally mixed entangled state of a Heisenberg chain through a Werner state*

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The thermal entanglement and teleportation of a thermally mixed entangled state of a two-qubit Heisenberg XXX chain under the Dzyaloshinski–Moriya (DM) anisotropic antisymmetric interaction through a noisy quantum channel given by a Werner state is investigated. The dependences of the thermal entanglement of the teleported state on the DM coupling constant, the temperature and the entanglement of the noisy quantum channel are studied in detail for both the ferromagnetic and the antiferromagnetic cases. The result shows that a minimum entanglement of the noisy quantum channel must be provided in order to realize the entanglement teleportation. The values of fidelity of the teleported state are also studied for these two cases. It is found that under certain conditions, we can transfer an initial state with a better fidelity than that for any classical communication protocol.

Keywords: thermal entanglement, concurrence, entanglement teleportation
PACC: 0365, 0367, 7510

1. Introduction

Quantum entanglement has been studied intensively in recent years due to its potential applications in quantum communication and information processing,[1−8] such as quantum key distribution,[9] superdense coding,[10] and telecloning.[11] A lot of efforts have been made to study and characterize the quantum entanglement.

The quantum entanglement in solid state systems such as spin chains is an important emerging field.[12−19] Spin chains are natural candidates for the realization of quantum entanglement compared with other physics systems.[20] As a simplest spin chain, the Heisenberg chain has been extensively studied in the context of quantum information science. Recently, an interesting and natural type of quantum entanglement, i.e., thermal entanglement, was introduced and analysed in a one-dimensional anisotropic Heisenberg model.[21] In addition, the nonlocal property of quantum mechanics has become a striking phenomenon called quantum teleportation.[22−27] It was first proposed by Bennett *et al*,[28] and is one of the most important applications of quantum entanglement. In a recent paper, the entanglement teleportation of a two-body pure entangled state via the noisy quantum channel represented by a Werner state[29] was investigated.[30] And the information transfer of a two-qubit pure entangled state through two independent one-dimensional Heisenberg XXX chains in a uniform magnetic field as a quantum channel was studied.[31] Also Yeo *et al*[32] discussed the influences of anisotropy and magnetic field on the entanglement teleportation of a two-qubit pure entangled state via a Heisenberg XY chain. What is more, the entanglement teleportation of a Werner state through a thermally mixed entangled state of a two-qubit Heisenberg XX chain was analysed.[33] While only the entanglement teleportation of a two-qubit pure entangled state and the entanglement teleportation of a Werner state were considered, the entanglement teleportation of a thermally mixed entangled state of a two-qubit Heisenberg XXX chain is rarely taken into account. However, as is well known, the thermal entanglement differs from the other kinds of entanglement. It is stable against the reduction due to various factors, such as decoherence and thermal fluctuation, for the entanglement at a finite temperature takes thermal

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decoherence into account implicitly. And the thermal entanglement requires neither measurement nor controlled switching of interactions in the preparing process, so it becomes an important quantity of solid state systems for the purpose of quantum computation. Compared with other quantum communication protocols, it is much easy to make use of thermal entanglement to realize various quantum operations such as quantum computation in solid state systems, e.g., spin chains. So it is significant to study the thermal entanglement and teleportation of a thermally mixed entangled state of a spin chain. In view of the above results, in this paper we are going to study the thermal entanglement and teleportation of a thermally mixed entangled state of a two-qubit Heisenberg XXX chain under the DM anisotropic antisymmetric interaction through a noisy quantum channel given by a Werner state.

2. Measure of entanglement and quantum channel

Since the decoherence from the environment always exerts an influence on the degree of quantum entanglement, the resource of maximally entangled states is hard to prepare in a real world. Certainly, a mixed entangled state as the resource approximates to a real circumstance, which is called noisy quantum channel. Generally, for the noisy quantum channel, it is important and nontrivial to know how much the entanglement is transferred to the teleported state from the initial state and how close to the initial state the teleported state is , and both the two problems are dependent on the entanglement of the initial state and the entanglement of the channel state.

Therefore, before considering the quantum channel and the state to be transferred, we review a measure of entanglement. Concurrence[34] is one of the most prevalently used entanglement monotones for two qubits. Let ρ_{12} be the joint density matrix of the system consisting of qubits 1 and 2, which may be pure or mixed. The concurrence corresponding to ρ_{12} is defined as

$$C_{12} = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}, \tag{1}$$

where λ_1, λ_2, λ_3 and λ_4 are the square roots of the four eigenvalues of ϱ_{12}=$\rho_{12}(\sigma^y\otimes\sigma^y)\rho^*_{12}$ $(\sigma^y\otimes\sigma^y)$ in descending order, with the asterisk denoting the complex conjugation and σ^y being the Pauli operator. The value of C_{12} ranges from 0 for completely disentangled states to 1 for maximally entangled states.

The Werner state as the quantum channel is represented by the density matrix $\rho_{\rm w}$ whose purity is $\frac{C_w+1}{2}$,[29] expressed as

$$\rho_w = \frac{1}{4}\left(\sigma_1^0\otimes\sigma_2^0 - \frac{2C_w+1}{3}\sum_{i=x,y,z}\sigma_1^i\otimes\sigma_2^i\right), \quad (-1 \le C_{\rm w} \le 1), \tag{2}$$

where $\sigma_\alpha^i (i = 0, x, y, z)$ signify the identity matrix I and three components of Pauli matrix $\boldsymbol{\sigma}$ of the αth qubit correspondingly.

According to the definition of expression (1), the concurrence associated with $\rho_{\rm w}$ is just $C(\rho_{\rm w}) = \max\{C_{\rm w}, 0\}$. Thus we substitute $C_{\rm w}$ for $C(\rho_{\rm w})$. In particular, when $C_{\rm w} = 1$, $\rho_{\rm w} = |\Psi^-\rangle\langle\Psi^-|$ is a pure maximally entangled state. When $0 < C_{\rm w} < 1$, $\rho_{\rm w}$ is a mixed entangled state. And when $-1 \le C_{\rm w} \le 0$, $\rho_{\rm w}$ is a mixed separable state. Since we are interested in the entanglement teleportation, we restrict our attention to $0 < C_{\rm w} \le 1$.

3. The model

We consider as an input state a thermally mixed entangled state of a two-qubit Heisenberg XXX chain in the presence of the Dzyaloshinski–Moriya (DM) anisotropic antisymmetric interaction. The Hamiltonian of the system is given by[20]

$$H_{\rm DM} = \frac{J}{2}[(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y + \sigma_1^z\sigma_2^z) + \boldsymbol{D}\cdot(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)], \tag{3}$$

where J is the real coupling constant and $\boldsymbol{D}$ is the DM vector coupling. The DM anisotropic antisymmetric interaction arises from spin–orbit coupling.[35,36] The positive J corresponds to the antiferromagnetic case, and the negative J refers to the ferromagnetic case. For simplicity, we choose $\boldsymbol{D} = D\boldsymbol{z}$, so the Hamiltonian is rewritten as

$$H_{\rm DM} = \frac{J}{2}[(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y + \sigma_1^z\sigma_2^z) + D(\sigma_1^x\sigma_2^y - \sigma_1^y\sigma_2^x)]. \tag{4}$$

Recently, the presence of quantum thermal entanglement in solid state systems at finite temperatures has been investigated by a number of authors. The state of a spin chain with the above Hamiltonian H

at a thermal equilibrium can be described by a density matrix

$$\rho(T) = \exp(-\beta H)/Z, \tag{5}$$

where $\beta = 1/(kT)$, k is the Boltzmann constant, which is henceforth taken as 1, and T is the temperature, H is the system Hamiltonian and $Z = \mathrm{tr}[\exp(-\beta H)]$ is the partition function. As $\rho(T)$ represents a thermal state, the entanglement in the thermal state is called thermal entanglement.[37]

4. Thermal entanglement and teleportation

The eigenvalues E_k and the eigenstates $|\psi_k\rangle$ $(k = 0, 1, 2, 3)$ of the Hamiltonian H_{DM} can be calculated,[20] then the state at a thermal equilibrium is expressed by the following density matrix[38] in the standard basis$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$:

$$\begin{aligned}\rho_{\mathrm{in}} =& \frac{1}{Z}\exp(-\beta H)\\ =& \frac{1}{Z}\sum_{k=0}^{3}\exp(-\beta E_k)|\psi_k\rangle\langle\psi_k|\\ =& \frac{1}{Z}\begin{pmatrix} \mathrm{e}^{-\frac{\beta J}{2}} & 0 & 0 & 0\\ 0 & \frac{1}{2}\mathrm{e}^{\frac{1}{2}\beta(J-\delta)}(1+\mathrm{e}^{\beta\delta}) & \frac{1}{2}\mathrm{e}^{\mathrm{i}\theta}\mathrm{e}^{\frac{1}{2}\beta(J-\delta)}(1-\mathrm{e}^{\beta\delta}) & 0\\ 0 & \frac{1}{2}\mathrm{e}^{-\mathrm{i}\theta}\mathrm{e}^{\frac{1}{2}\beta(J-\delta)}(1-\mathrm{e}^{\beta\delta}) & \frac{1}{2}\mathrm{e}^{\frac{1}{2}\beta(J-\delta)}(1+\mathrm{e}^{\beta\delta}) & 0\\ 0 & 0 & 0 & \mathrm{e}^{-\frac{\beta J}{2}} \end{pmatrix},\end{aligned} \tag{6}$$

where $Z = 2\mathrm{e}^{-\frac{\beta J}{2}}\left(1 + \mathrm{e}^{\beta J}\cosh\dfrac{\beta\delta}{2}\right)$, $\theta = \arctan D$ and $\delta = 2J\sqrt{1+D^2}$. Based on the expressions (1) and (6), the thermal entanglement of the initial thermally mixed entangled state can be measured by the concurrence C which has been defined as

$$C_{\mathrm{in}} = \frac{2}{Z}\max\left\{\frac{1}{2}\mathrm{e}^{\frac{1}{2}\beta(J-\delta)}|1-\mathrm{e}^{\beta\delta}| - \mathrm{e}^{-\frac{\beta J}{2}}, 0\right\}, \tag{7}$$

where $C(\rho_{\mathrm{in}})$ is expressed by C_{in}.

From the above equation, we can find that when $D = 0$, i.e., there exists no spin–orbit coupling interaction, there is no thermal entanglement for $J < 0$ at any temperature, while the critical temperature T_{c}, above which the concurrence is zero, is $T_{\mathrm{c}} = \dfrac{2J}{\ln 3}$ for $J > 0$. These conclusions accord with the known results.[20]

While D increases, the C_{in} is created and then enhanced for $J < 0$ and strengthened for $J > 0$.

After the entanglement teleportation, the initial state becomes the teleported state which is given by[39]

$$\rho_{\mathrm{out}} = \sum_{ij} p_{ij}(\sigma^i \otimes \sigma^j)\rho_{\mathrm{in}}(\sigma^i \otimes \sigma^j), \tag{8}$$

where $p_{ij} = p_i p_j = \mathrm{tr}(E^i\rho_{\mathrm{w}}) \cdot \mathrm{tr}(E^j\rho_{\mathrm{w}})$, p_{00} denotes the maximal possibility of successful teleportation with $p_i = \mathrm{tr}(E^i\rho_{\mathrm{w}})$ being the ratio of maximally entangled state to the mixed entangled state ρ_{w}. $\sum_{ij} p_{ij} = 1$, and $E^0 = |\Psi^-\rangle\langle\Psi^-|$, $E^x = |\Phi^-\rangle\langle\Phi^-|$, $E^y = |\Phi^+\rangle\langle\Phi^+|$, and $E^z = |\Psi^+\rangle\langle\Psi^+|$, in which $|\Psi^\pm\rangle = \dfrac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$, and $|\Phi^\pm\rangle = \dfrac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$. After some straightforward algebra, we can obtain the amount of thermal entanglement of ρ_{out}

$$\begin{aligned}C_{\mathrm{out}} =& \frac{2}{Z}\max\left\{\frac{4C_{\mathrm{w}}^2 + 4C_{\mathrm{w}} + 1}{18}\mathrm{e}^{\frac{1}{2}\beta(J-\delta)}|1-\mathrm{e}^{\beta\delta}|\right.\\ &\left. - \frac{2C_{\mathrm{w}}^2 + 2C_{\mathrm{w}} + 5}{9}\mathrm{e}^{-\frac{\beta J}{2}} - \frac{4 - 2C_{\mathrm{w}} - 2C_{\mathrm{w}}^2}{18}\mathrm{e}^{\frac{1}{2}\beta(J-\delta)}(1+\mathrm{e}^{\beta\delta}), 0\right\},\end{aligned} \tag{9}$$

where the $C(\rho_{\mathrm{out}})$ is expressed by C_{out}.

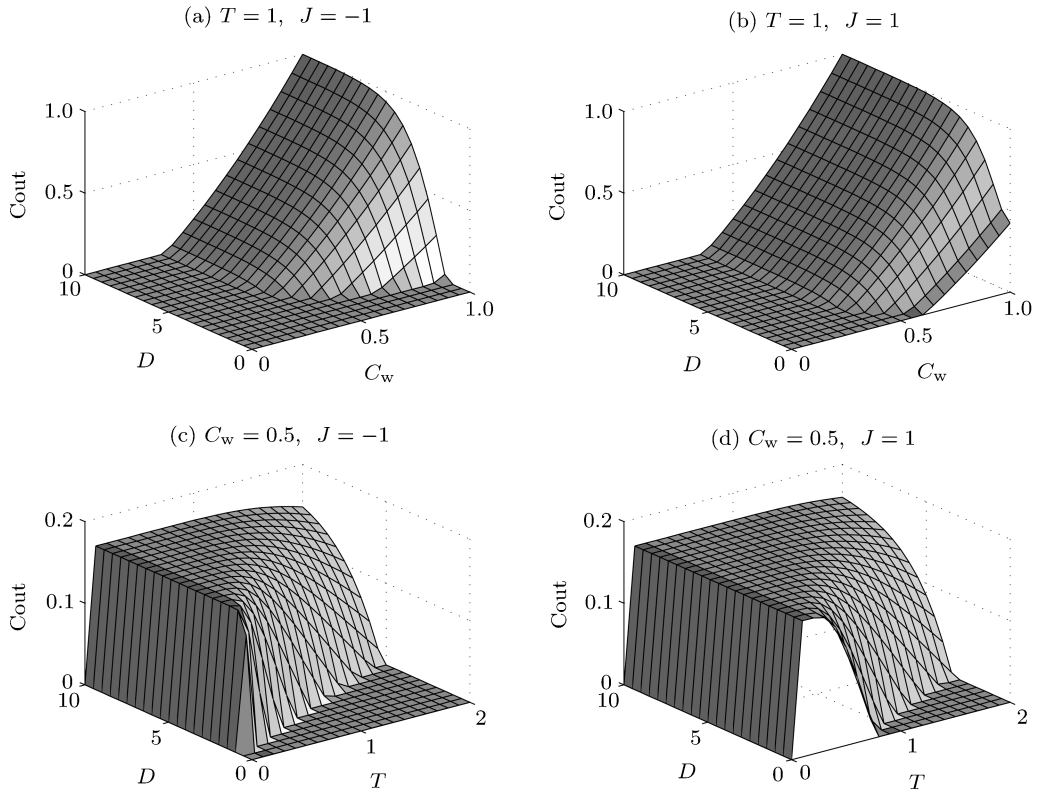


Fig.1. The thermal entanglements of the teleported state measured by the concurrence $C_{\rm out}$ as a function of the DM coupling constant D when the concurrence $C_{\rm w}$ of the noisy quantum channel represented by a Werner state and the temperature T are changed for both the ferromagnetic and the antiferromagnetic cases. (a) $T = 1$, $J = -1$; (b) $T = 1$, $J = 1$; (c) $C_{\rm w} = 0.5$, $J = -1$; (d) $C_{\rm w} = 0.5$, $J = 1$.

The dependences of the entanglement of the teleported state measured by the concurrence $C_{\rm out}$ on the DM coupling constant D are shown in Fig.1 when the concurrence $C_{\rm w}$ of the noisy quantum channel represented by a Werner state and the temperature T are changed for both the ferromagnetic and the antiferromagnetic cases. Figure 1(a) and 1(b) are the plots of $C_{\rm out}$ versus $C_{\rm w}$ and D for a given T. It can be seen that, generally, the $C_{\rm out}$ increases as the values of D and $C_{\rm w}$ increase. Particularly, when $C_{\rm w}$ is less than a critical value $C_{\rm w}^{\rm c} \approx 0.35$, there is no thermal entanglement in the teleported state, no matter what the value of the D is. Therefore, in order to realize entanglement teleportation, the quantum channel should possess a critical value of minimum entanglement. The value of $C_{\rm out}$ is equal to $C_{\rm in}$ which can be obtained easily from expression (9) as long as $C_{\rm w} = 1$, which means that the perfect quantum channel can transfer the thermal entanglement of the initial state completely. And in general circumstances, the initial entanglement is lost partially during entanglement teleportation even when the quantum channel has nonzero entanglement but not maximal entanglement, namely, $C_{\rm out} \leq C_{\rm in}$. Figure 1(c) and 1(d) show the plots of the $C_{\rm out}$ for a fixed value of $C_{\rm w}$ versus D and T. It is found that the value of $C_{\rm out}$ increases with D increasing and decreases with T increasing. This stems from the fact that $C_{\rm in}$ increases with D increasing and decreases with T increasing. And the critical value of T, above which the $C_{\rm out}$ is zero, increases with D increasing for a given $C_{\rm w}$.

However, from Fig.1, it is notable that for the ferromagnetic case, when $D = 0$, $C_{\rm out}$ is equal to zero regardless of what the values of $C_{\rm w}$ and T are, which is different from the antiferromagnetic case. It is due to the fact that, as we have mentioned, if $D = 0$, there is no initial thermal entanglement at any temperature for $J < 0$. Since the thermal entanglement cannot be increased under local operations, it is natural that there is no output thermal entanglement regardless of what the values of T and $C_{\rm w}$ are.

For $J > 0$, $C_{\rm out}$ becomes

$$C_{\rm out}=\frac{4C_{\rm w}^2+4C_{\rm w}+1}{9}C_{\rm in}+\frac{2C_{\rm w}^2+2C_{\rm w}-4}{9}. \quad (10)$$

It is evident from expression (10) that $C_{\rm out}$ depends on initial entanglement $C_{\rm in}$ and entanglement $C_{\rm w}$ of the quantum channel, and increases linearly with the value of $C_{\rm in}$ increasing. It is surprising to find that the conclusion is consistent with the results under the condition that the input state is a two-body pure entangled state which has been discussed by Lee and Kim.[30] It can be comprehended that when there is no DM coupling, i.e., there is no spin–orbit coupling, the Heisenberg XXX chain itself alone interacts with the quantum channel, so $C_{\rm in}$ can be the entanglement of the pure state or the mixed state. On the other hand, considering the DM coupling, not only spin–spin coupling but also spin–orbit coupling interact with the quantum channel, so the connection between $C_{\rm out}$ and $C_{\rm in}$ is not direct.

5. The fidelity of entanglement teleportation

To characterize the quality of the teleported state $\rho_{\rm out}$, an important parameter is the fidelity of the teleported state which is defined as[40]

$$\begin{aligned}F(\rho_{\rm in},\rho_{\rm out}) =&\{{\rm tr}[\sqrt{(\rho_{\rm in})^{\frac{1}{2}}\rho_{\rm out}(\rho_{\rm in})^{\frac{1}{2}}}]\}^2\\ =&\frac{1}{18{\rm e}^{-\beta J}(1+{\rm e}^{\beta J}\cosh\left(\dfrac{\beta\delta}{2}\right))^2}\cdot\Big[(3C_{\rm w}^2+3C_{\rm w}+3){\rm e}^{\beta J}\cosh(\beta\delta)\\ &+(4-2C_{\rm w}-2C_{\rm w}^2)\Big(\cosh(\beta J)+2\cosh\left(\frac{1}{2}\beta\delta\right)\Big)+(C_{\rm w}^2+C_{\rm w}+7){\rm e}^{-\beta J}\Big]. \quad (11)\end{aligned}$$

The values of fidelity F of the teleported state are plotted as a function of the DM coupling constant D in Fig.2 when the concurrence $C_{\rm w}$ of the noisy quantum channel represented by a Werner state and the temperature T are changed for the ferromagnetic and the antiferromagnetic cases. We can see that F increases with D and $C_{\rm w}$ increasing for a definite T from Figs.2(a) and 2(b), and increases with D increasing and decreases with T increasing for a fixed $C_{\rm w}$ from Fig.2(c) and 2(d), which means that F increases with $C_{\rm in}$ and $C_{\rm w}$ increasing. These relations can be seen directly in the following expression when $D=0$:

$$F=\frac{C_{\rm w}^2(4+28C_{\rm in}+22C_{\rm in}^2)+C_{\rm w}(4+28C_{\rm in}+22C_{\rm in}^2)+(46-2C_{\rm in}+10C_{\rm in}^2)}{162}. \quad (12)$$

So in order to obtain a better F, we should increase the values of $C_{\rm w}$ and $C_{\rm in}$.

From Fig.2, it is also found that the variation rules of the F with D, T and $C_{\rm w}$ are approximately the same for both the ferromagnetic and the antiferromagnetic cases. The only difference is that, for the ferromagnetic case, when $D=0$, the F decreases with $C_{\rm w}$ increasing for a definite T, while there exists an opposite result for the antiferromagnetic case. This is due to the fact that in the ferromagnetic case, the initial thermal entanglement is zero, namely, the initial state is a separable state. During the teleportation, the initial separable state interacts with the entangled quantum channel, so after the teleportation, the teleported state may be partially entangled, as a result, F decreases with $C_{\rm w}$ increasing.

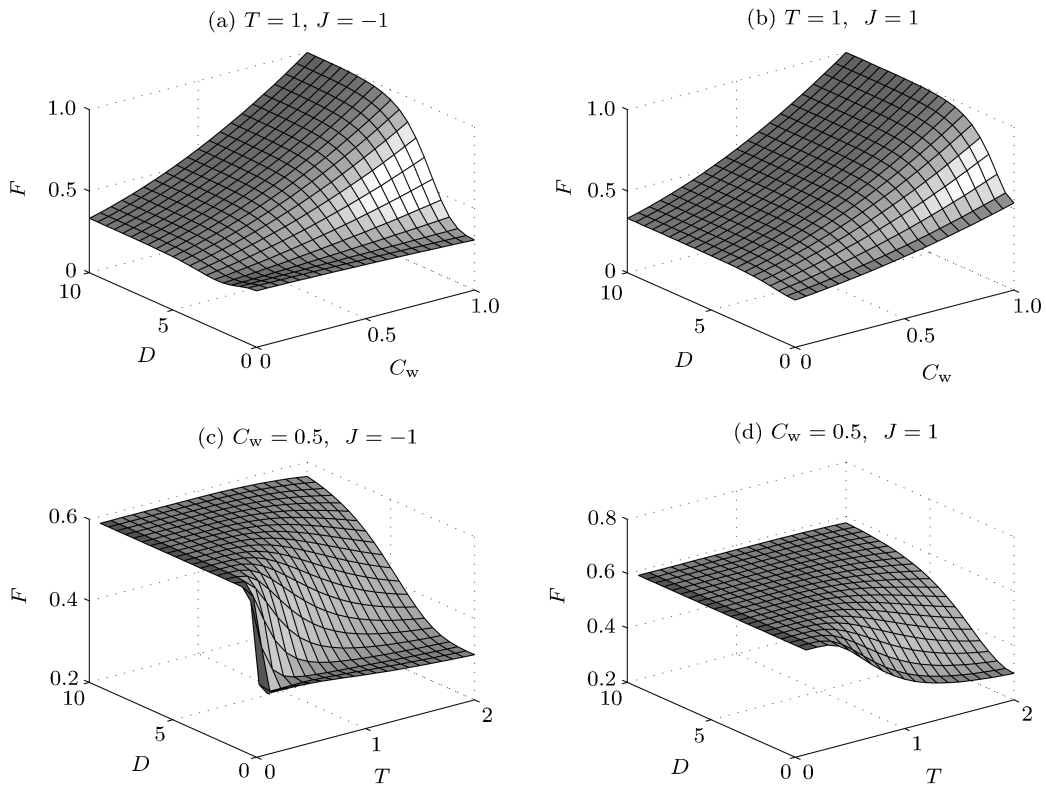


Fig.2. The values of fidelity F of the teleported state as a function of the DM coupling constant D when the concurrence $C_{\rm w}$ of the noisy quantum channel represented by a Werner state and the temperature T are changed for both the ferromagnetic and the antiferromagnetic cases. (a) $T = 1$ and $J = -1$; (b) $T = 1$ and $J = 1$; (c) $C_{\rm w} = 0.5$ and $J = -1$; (d) $C_{\rm w} = 0.5$ and $J = 1$.

In order to transfer an initial state with a better fidelity than that for any classical communication protocol, we require Eq.(11) to be greater than 2/3. When $J > 0$, $D = 0$, and $C_{\rm w} = 1$, the requirement becomes $T < \dfrac{2J}{\ln(6+3\sqrt{5})}$, that is to say, F is larger than 2/3 if $0 < T < \dfrac{2J}{\ln(6+3\sqrt{5})}$. While for $J < 0$, the F is always smaller than 2/3 at any temperature in this case. And we find that the critical value

$$T_{\rm c}' = \frac{2J}{\ln(6+3\sqrt{5})},$$

below which the fidelity is better than that for any classical communication protocol is smaller than

$$T_{\rm c} = \frac{2J}{\ln 3},$$

above which the entanglement of the initial thermally mixed entangled state is zero, which means that when

$$T_{\rm c}' = \frac{2J}{\ln(6+3\sqrt{5})} < T < T_{\rm c} = \frac{2J}{\ln 3},$$

although the degree of the initial entanglement is nonzero, the possibility that the fidelity is better than that for any classical communication protocol cannot be guaranteed.

6. Summary

We have investigated the thermal entanglement and teleportation of a thermally mixed entangled state of a two-qubit Heisenberg XXX chain under the DM anisotropic antisymmetric interaction through a noisy quantum channel given by a Werner state for both the ferromagnetic and the antiferromagnetic cases. The dependences of the thermal entanglement of the teleported state on the DM coupling constant, the temperature and the entanglement of the noisy quantum channel have been revealed for both the ferromagnetic and the antiferromagnetic cases. The result shows that a minimum entanglement of the noisy quantum channel must be provided in order to realize entanglement teleportation. The values of fidelity for the teleported state are also studied for these two cases. It is found that under certain conditions, we can transfer an initial state with a better fidelity than that for any

classical communication protocol. Our results are important for realizing the quantum information processing in the solid state systems due to the fact that the thermal entanglement, which differs from the other kinds of entanglement by its stability, requires neither measurement nor controlled switching of interactions in the preparing process.

References

[1] Liu X J and Fang M F 2003 *Chin. Phys.* **12** 971
[2] Wu Y, Payne M G, Hagley E W and Deng L 2004 *Phys. Rev.* A **69** 063803
[3] Wu Y and Deng L 2004 *Opt. Lett.* **29** 1144
[4] Zhou L, Yi X X, Song H S and Guo Y Q 2005 *Chin. Phys.* **14** 1168
[5] Zheng X J, Fang M F, Cai J W and Liao X P 2006 *Chin. Phys.* **15** 492
[6] Zheng X J, Fang M F, Cai J W and Liao X P 2006 *Chin. Phys.* **15** 2840
[7] Zhu Y, Zhu S Q and Hao X 2007 *Chin. Phys.* **16** 2229
[8] Bennett C H and Divincenzo D P 2000 *Nature* **404** 247
[9] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661
[10] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881
[11] Murao M, Jonathan D, Plenio M B and Vedral V 1999 *Phys. Rev.* A **59** 156
[12] Sun Y, Chen Y and Chen H 2003 *Phys. Rev.* A **68** 044301
[13] Khveshchenko D V 2003 *Phys. Rev.* B **68** 193307
[14] Zhang G F, Li S S and Liang J 2005 *Opt. Commun.* **245** 457
[15] Zhang G F and Li S S 2005 *Phys. Rev.* A **72** 034302
[16] Loss D and Divincenzo D P 1998 *Phys. Rev.* A **57** 120
Burkard G, Loss D and Divincenzo D P 1999 *Phys. Rev.* B **59** 2070
[17] Lidar D A, Bacon D and Whaley K B 1999 *Phys. Rev. Lett.* **82** 4556
[18] Divincenzo D P, Bacon D, Kempe J, Burkard G and Whaley K B 2000 *Nature* **408** 339
[19] Santos L F 2003 *Phys. Rev.* A **67** 062306
[20] Zhang G F 2007 *Phys. Rev.* A **75** 034304
[21] Arnesen M C, Bose S and Vedral V 2001 *Phys. Rev. Lett.* **87** 017901
[22] Zheng Y Z, Gu Y J, Wu G C and Guo G C 2003 *Chin. Phys.* **12** 1070
[23] Cai X H 2003 *Chin. Phys.* **12** 1066
Cao H J, Guo Y Q and Song H S 2006 *Chin. Phys.* **15** 915
[24] Ye S Y 2007 *Chin. Phys.* **16** 1678
[25] Zhan Y B 2007 *Chin. Phys.* **16** 2557
[26] Yang Z B, Wu H Z and Su W J 2007 *Chin. Phys.* **16** 2563
[27] Zhou Y L, Dai H Y and Yang L J 2007 *Chin. Phys.* **16** 3692
[28] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[29] Werner R F 1989 *Phys. Rev.* A **40** 4277
[30] Lee J and Kim M S 2000 *Phys. Rev. Lett.* **84** 4236
[31] Hao X and Zhu S Q 2005 *Phys. Lett.* A **338** 175
[32] Yeo Y, Liu T, Lu Y E and Yang Q Z 2005 *J. Phys.* A **38** 3235
[33] Yeo Y 2002 *Phys. Rev.* A **66** 062312
[34] Hill S and Wootters W K 1997 *Phys. Rev. Lett.* **78** 5022
Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245
Coffman V, Kundu J and Wootters W K 2000 *Phys. Rev.* A **61** 052306
[35] Dzyaloshinski I 1958 *J. Phys. Chem. Solids* **4** 241
[36] Moriya T 1960 *Phys. Rev. Lett.* **4** 228
Moriya T 1960 *Phys. Rev.* **117** 635
Moriya T 1960 *Phys. Rev.* **120** 91
[37] Nielsen M A 2000 quant-ph/0011036
[38] Wang X G, Fu H C and Solomon A I 2001 *J. Phys.* A **34** 11307
[39] Bowen G and Bose S 2001 *Phys. Rev. Lett.* **87** 267901
[40] Jozsa R 1994 *J. Mod. Opt.* **41** 2315