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Periodic-soliton solutions of the (2+1)-dimensional Kadomtsev–Petviashvili equation*

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$2N$ line-soliton solutions of the (2+1)-dimensional Kadomtsev–Petviashvili equation can be presented by resorting to the Hirota bilinear method. By extending the real parameters into complex parameters, this paper obtains N periodic-soliton solutions of the (2+1)-dimensional Kadomtsev–Petviashvili equation from the $2N$ line-soliton solutions.

Keywords: periodic-soliton, Kadomtsev–Petviashvili equation, Hirota bilinear method
PACC: 0340K, 0290

1. Introduction

The interaction of the solitons plays an important role in many applications, therefore its study for nonlinear integrable models is very important.[1−22] Recently, the y periodic-soliton solutions for the (2+1)-dimensional Sawada–Kotera equation and the (2+1)-dimensional Nizhnik–Novikov–Veselov equation are introduced in Refs.[7,8]. We are enlightened by the above references, the periodic-soliton solutions of the (2+1)-dimensional Kadomtsev–Petviashvili (KP) is discussed systematically and completely in this paper. The Hirota bilinear method will be applied to obtain $2N$ line-soliton solutions of the (2+1)-dimensional KP equation. Using the $2N$ line-soliton solutions, we present N periodic-soliton solutions of the (2+1)-dimensional KP equation through extending the real parameters into complex parameters. As an application, we obtain the solutions of multiple soliton interactions, including interactions between two periodic-solitons, one periodic-soliton and one line-soliton and two line-solitons.

2. The explicit solutions of the (2+1)-dimensional KP equation

The (2+1)-dimensional KP equation[1−4] is

$$(u_t + 6uu_x + u_{xxx})_x + 3\sigma^2 u_{yy} = 0 \tag{1}$$

with $\sigma^2 = \pm 1$. Equation (1) is known as the first type of KP equation (KP I equation), when $\sigma^2 = -1$, and the second type of KP equation (KP II equation), when $\sigma^2 = 1$. We shall consider the periodic soliton solutions of the (2+1)-dimensional KP equation Eq.(1).

Let

$$u = 2(\ln f)_{xx} \tag{2}$$

and substitute it into Eq.(1), we obtain a corresponding bilinear form

$$(\mathrm{D}_t\mathrm{D}_x + \mathrm{D}_x^4 + 3\sigma^2\mathrm{D}_y^2) f \cdot f = 0, \tag{3}$$

where D-operator is defined by[1]

$$\begin{aligned}&\mathrm{D}_x^n\mathrm{D}_y^m\mathrm{D}_t^l a \cdot b\\ &= \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^n \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^l\\ &\cdot a(x,y,t)b(x',y',t')|_{x'=x,y'=y,t'=t}.\end{aligned}$$

By using the Hirota bilinear method, we know that Eq.(1) has $2N$ line-soliton solution[1]

$$u = 2\,(\ln f)_{xx} = 2\left[\ln\left(\sum_{\mu=0,1}\exp\left(\sum_{j=1}^{2N}\mu_j\eta_j + \sum_{1\le j<l}^{2N}\mu_j\mu_l A_{jl}\right)\right)\right]_{xx}, \tag{4}$$

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where the first $\sum_{\mu=0,1}$ means a summation over all possible combinations of $\mu_1 = 0, 1$, $\mu_2 = 0, 1, \cdots$, $\mu_{2N} = 0, 1$ and $\sum_{1\le j<l}^{2N}$ means a summation over all possible pair (j, l) chosen from the set $\{1, 2, \cdots, 2N\}$, with the condition that $j < l$ and

$$\eta_j = p'_j x + q'_j y + \omega'_j t + \eta_j^{0'}, \tag{5}$$

$$\exp(A_{jl}) = -\frac{(\omega'_j - \omega'_l)(p'_j - p'_l) + (p'_j - p'_l)^4 + 3\sigma^2(q'_j - q'_l)^2}{(\omega'_j + \omega'_l)(p'_j + p'_l) + (p'_j + p'_l)^4 + 3\sigma^2(q'_j + q'_l)^2}, \tag{6}$$

$$\omega'_j p'_j = -3\sigma^2 {q'_j}^2 - {p'_j}^4 \ (1 \le j < l \le 2N). \tag{7}$$

By extending the real parameters into complex parameters in Eqs.(5)–(7), we have

$$\eta_{2m-1} = \eta^*_{2m} = p_m x + q_m y + \omega_m t + \eta^0_m, \tag{8}$$

where η^*_{2m} denotes the complex conjugation of η_{2m} and

$$\begin{aligned} &p'_{2m-1} = p_m,\ p'_{2m} = p^*_m,\ q'_{2m-1} = q_m,\ q'_{2m} = q^*_m,\\ &\omega'_{2m-1} = \omega_m,\ \omega'_{2m} = \omega^*_m,\ {\eta^0}'_{2m-1} = \eta^0_m,\\ &{\eta^0}'_{2m} = {\eta^0_m}^* \ \ (m = 1, 2, \cdots, N). \end{aligned} \tag{9}$$

Suppose

$$\begin{aligned} &p_m = (a_m + \mathrm{i}b_m),\quad q_m = (c_m + \mathrm{i}d_m),\\ &\omega_m = (e_m + \mathrm{i}f_m),\quad \eta^0_m = (\Omega^0_m + \mathrm{i}\theta^0_m), \end{aligned} \tag{10}$$

where a_m, b_m, c_m, d_m, e_m, f_m, Ω^0_m, θ^0_m are real parameters. Substituting Eqs.(9)–(10) into Eq.(7), we obtain

$$\begin{aligned} e_m = \frac{1}{a_m^2 + b_m^2}\Big[&- a_m^5 + 2a_m^3 b_m^2 + 3a_m b_m^4\\ &+\sigma^2(-3a_m c_m^2 - 6b_m c_m d_m + 3a_m d_m^2)\Big], \end{aligned} \tag{11}$$

$$\begin{aligned} f_m = \frac{1}{a_m^2 + b_m^2}\Big[&- 3a_m^4 b_m - 2a_m^2 b_m^3 + b_m^5\\ &+\sigma^2(3b_m c_m^2 - 6a_m c_m d_m - 3b_m a_m^2)\Big], \end{aligned} \tag{12}$$

$$(m = 1, 2, \cdots, N).$$

Substituting Eqs.(8)–(12) into Eq.(4), we can obtain an explicit solution of the (2+1)-dimensional KP equation. In general, the parameters are suitably chosen, Eq.(4) represent periodic-soliton and line-soliton solutions of the (2+1)-dimensional KP equation. (i) When $b_m^2 + d_m^2 \neq 0$, $a_m^2 + b_m^2 \neq 0$ and $\exp(A_{2m-1,2m}) > 1$ $(m = 1, 2, \cdots, N)$, Eq.(4) represent the interaction of N periodic-soliton solutions. Further selecting the parameters in the above, when $b_m = 0$, $d_m a_m \neq 0$ $(m = 1, 2, \cdots, N)$, we can obtain y periodic-soliton solutions from Eq.(4) and when parameters $d_m = 0$, $b_m \neq 0\,(m = 1, 2, \cdots, N)$, we can obtain x periodic-soliton solutions from Eq.(4). (ii) When the parameters $b_k^2 + d_k^2 \neq 0$, $b_s = d_s = 0$, $a_k^2 + b_k^2 \neq 0$, $a_s \neq 0$, $\exp(A_{2k-1,2k}) > 1$, $\{k\} \cup \{s\} = \{1, 2, \cdots, N\}$ and $\{k\} \cap \{s\} = \emptyset$, Eq.(4) denotes the interaction of periodic-soliton and line-soliton solutions. (iii) When the parameters $b_m = d_m = 0$, $a_m \neq 0, (m = 1, 2, \cdots, N)$, Eq.(4) represents the interaction of N line-soliton solutions.

3. The periodic-soliton and line-soliton solutions of the (2+1)-dimensional KP I equation

In the following, we give the periodic-soliton solutions of the KP I equation ($\sigma^2 = -1$). For the sake of simplicity, we shall discuss the two basic cases $N = 1$ and $N = 2$.

Case 1 ($N = 1$) Supposing

$$\eta_1 = \eta^*_2 = \Omega_1 + \mathrm{i}\theta_1, \tag{13}$$

where $\Omega_1 = a_1 x + c_1 y + e_1 t + \Omega^0_1$, $\theta_1 = b_1 x + d_1 y + f_1 t + \theta^0_1$, without loss of generality, we assume that $\Omega^0_1 = \theta^0_1 = 0$. Noticing Eqs.(6), (11) and (12), we have

$$\exp(A_{12}) = -\frac{-f_1 b_1 + 4b_1^4 + 3d_1^2}{e_1 a_1 + 4a_1^4 - 3c_1^2}, \tag{14}$$

$$\begin{aligned} e_1 = \frac{1}{a_1^2 + b_1^2}[&-a_1^5 + 2a_1^3 b_1^2 + 3a_1 b_1^4\\ &-(-3a_1 c_1^2 - 6b_1 c_1 d_1 + 3a_1 d_1^2)], \end{aligned} \tag{15}$$

$$f_1 = \frac{1}{a_1^2+b_1^2}[-3a_1^4b_1 - 2a_1^2b_1^3 + b_1^5 - (3b_1c_1^2 - 6a_1c_1d_1 - 3b_1a_1^2)]. \quad (16)$$

Substituting Eqs.(13)–(16) into f in Eq.(4), we find

$$\begin{aligned} f &= 1 + [\exp(\eta_1) + \exp(\eta_2)] \\ &\quad + \exp(A_{12})\exp(\eta_1+\eta_2) \\ &= 1 + 2\exp(\Omega_1)\cos\theta_1 \\ &\quad + \exp(A_{12})\exp(2\Omega_1). \end{aligned} \quad (17)$$

Therefore, we obtain an explicit solution of the (2+1)-dimensional KP I equation

$$\begin{aligned} u = {} & 2\frac{(a_1^2-b_1^2)\cos\theta_1 - 2a_1b_1\sin\theta_1 + 2a_1^2\exp(A_{12})\exp(\Omega_1)}{\sqrt{\exp(A_{12})}\cosh\left(\Omega_1 + \ln\sqrt{\exp(A_{12})}\right) + \cos\theta_1} \\ & -2\left(\frac{a_1\cos\theta_1 - b_1\sin\theta_1 + a_1\exp(A_{12})\exp(\Omega_1)}{\sqrt{\exp(A_{12})}\cosh\left(\Omega_1 + \ln\sqrt{\exp(A_{12})}\right) + \cos\theta_1}\right)^2. \end{aligned} \quad (18)$$

The parameters are suitably chosen, when $b_1^2 + d_1^2 \neq 0$, $a_1^2 + b_1^2 \neq 0$ and $\exp(A_{12}) > 1$, Eq.(18) represents one periodic-soliton solution, when $b_1 = d_1 = 0$ and $a_1 \neq 0$, Eq.(18) denotes one line-soliton solution. Plots of these solutions are given in Fig.1.

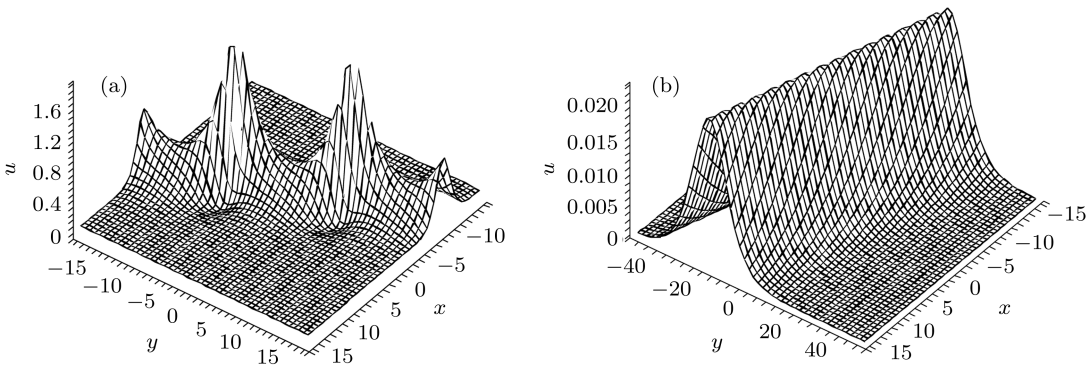


Fig.1. (a) One periodic-soliton solution Eq.(18) with $a_1 = 0.59$, $b_1 = 0.01$, $c_1 = 0.2$, $d_1 = 0.45$, $t = 0$; (b) One line-soliton solution Eq.(18) with $a_1 = 0.2$, $b_1 = 0$, $c_1 = 0.2$, $d_1 = 0$, $t = 0$.

Case 2 $(N = 2)$ Supposing

$$\eta_1 = \eta_2^* = \Omega_1 + \mathrm{i}\theta_1, \quad \eta_3 = \eta_4^* = \Omega_2 + \mathrm{i}\theta_2, \quad (19)$$

where $\Omega_m = a_m x + c_m y + e_m t + \Omega_m^0$, $\theta_m = b_m x + d_m y + f_m t + \theta_m^0$, without loss of generality, we assume that $\Omega_m^0 = \theta_m^0 = 0$ $(m = 1, 2)$. Noticing Eqs.(6), (11) and (12), we have

$$\begin{aligned} \exp(A_{12}) &= -\frac{-f_1b_1 + 4b_1^4 + 3d_1^2}{e_1a_1 + 4a_1^4 - 3c_1^2}, \\ \exp(A_{34}) &= -\frac{-f_2b_2 + 4b_2^4 + 3d_2^2}{e_2a_2 + 4a_2^4 - 3c_2^2}, \end{aligned} \quad (20)$$

$$\begin{aligned} \exp(A_{13}) &= (\exp(A_{24}))^* \\ &= -\{[(e_1 - e_2) + \mathrm{i}(f_1 - f_2)][(a_1 - a_2) \\ &\quad + \mathrm{i}(b_1 - b_2)] + [(a_1 - a_2) + \mathrm{i}(b_1 - b_2)]^4 \\ &\quad - 3[(c_1 - c_2) + \mathrm{i}(d_1 - d_2)]^2\}/\{[(e_1 + e_2) \\ &\quad + \mathrm{i}(f_1 + f_2)][(a_1 + a_2) + \mathrm{i}(b_1 + b_2)] \\ &\quad + [(a_1 + a_2) + \mathrm{i}(b_1 + b_2)]^4 - 3[(c_1 + c_2) \\ &\quad + \mathrm{i}(d_1 + d_2)]^2\}, \end{aligned} \quad (21)$$

$$\begin{aligned} \exp(A_{14}) &= (\exp(A_{23}))^* \\ &= -\{[(e_1 - e_2) + \mathrm{i}(f_1 + f_2)][(a_1 - a_2) \\ &\quad + \mathrm{i}(b_1 + b_2)] + [(a_1 - a_2) + \mathrm{i}(b_1 + b_2)]^4 \\ &\quad - 3[(c_1 - c_2) + \mathrm{i}(d_1 + d_2)]^2\}/\{[(e_1 + e_2) \\ &\quad + \mathrm{i}(f_1 - f_2)][(a_1 + a_2) + \mathrm{i}(b_1 - b_2)] \\ &\quad + [(a_1 + a_2) + \mathrm{i}(b_1 - b_2)]^4 - 3[(c_1 + c_2) \\ &\quad + \mathrm{i}(d_1 - d_2)]^2\}, \end{aligned} \quad (22)$$

where

$$e_1 = \frac{1}{a_1^2+b_1^2}[-a_1^5+2a_1^3b_1^2+3a_1b_1^4 -(-3a_1c_1^2-6b_1c_1d_1+3a_1d_1^2)], \quad (23)$$

$$f_1 = \frac{1}{a_1^2+b_1^2}[-3a_1^4b_1-2a_1^2b_1^3+b_1^5 -(3b_1c_1^2-6a_1c_1d_1-3b_1a_1^2)], \quad (24)$$

$$e_2 = \frac{1}{a_2^2+b_2^2}[-a_2^5+2a_2^3b_2^2+3a_2b_2^4 -(-3a_2c_2^2-6b_2c_1d_2+3a_2d_2^2)], \quad (25)$$

$$f_2 = \frac{1}{a_2^2+b_2^2}[-3a_2^4b_2-2a_2^2b_2^3+b_2^5 -(3b_2c_2^2-6a_2c_2d_2-3b_2a_2^2)]. \quad (26)$$

Substituting Eqs.(19)–(26) into f in Eq.(4), we obtain

$$\begin{aligned}
f =\ & 1+[\exp(\eta_1)+\exp(\eta_2)]+[\exp(\eta_3)+\exp(\eta_4)]+\exp(A_{12})\exp(\eta_1+\eta_2)\\
&+\exp(A_{34})\exp(\eta_3+\eta_4)+[\exp(A_{13})\exp(\eta_1+\eta_3)+\exp(A_{24})\exp(\eta_2+\eta_4)]\\
&+[\exp(A_{14})\exp(\eta_1+\eta_4)+\exp(A_{23})\exp(\eta_2+\eta_3)]+\exp(A_{12})[\exp(A_{13})\exp(A_{23})\exp(\eta_1+\eta_2+\eta_3)\\
&+\exp(A_{14})\exp(A_{24})\exp(\eta_1+\eta_2+\eta_4)]+\exp(A_{34})[\exp(A_{13})\exp(A_{14})\exp(\eta_1+\eta_3+\eta_4)\\
&+\exp(A_{23})\exp(A_{24})\exp(\eta_2+\eta_3+\eta_4)]+\exp(A_{12})\exp(A_{13})\exp(A_{24})\exp(A_{14})\exp(A_{23})\exp(A_{34})\\
&\times\exp(\eta_1+\eta_2+\eta_3+\eta_4)\\
=\ & 1+2\exp(\Omega_1)\cos\theta_1+2\exp(\Omega_2)\cos\theta_2+\exp(A_{12})\exp(2\Omega_1)+\exp(A_{34})\exp(2\Omega_2)\\
&+2\exp(\Omega_1+\Omega_2)[\mathrm{Re}(\exp(A_{13}))\cos(\theta_1+\theta_2)-\mathrm{Im}(\exp(A_{13}))\sin(\theta_1+\theta_2)\\
&+\mathrm{Re}(\exp(A_{14}))\cos(\theta_1-\theta_2)-\mathrm{Im}(\exp(A_{14}))\sin(\theta_1-\theta_2)]+2\exp(A_{12})\exp(2\Omega_1+\Omega_2)\\
&\times[(\mathrm{Re}(\exp(A_{13}))\mathrm{Re}(\exp(A_{14}))+\mathrm{Im}(\exp(A_{13}))\mathrm{Im}(\exp(A_{14})))\cos\theta_2\\
&-(\mathrm{Im}(\exp(A_{13}))\mathrm{Re}(\exp(A_{14}))-\mathrm{Re}(\exp(A_{13}))\mathrm{Im}(\exp(A_{14})))\sin\theta_2]\\
&+2\exp(A_{34})\exp(\Omega_1+2\Omega_2)[(\mathrm{Re}(\exp(A_{13}))\mathrm{Re}(\exp(A_{14}))-\mathrm{Im}(\exp(A_{13}))\mathrm{Im}(\exp(A_{14})))\cos\theta_1\\
&-(\mathrm{Re}(\exp(A_{13}))\mathrm{Im}(\exp(A_{14}))+\mathrm{Im}(\exp(A_{13}))\mathrm{Re}(\exp(A_{14})))\sin\theta_1]\\
&+\exp(A_{12})\exp(A_{34})\left|\exp(A_{13})\right|^2\left|\exp(A_{14})\right|^2\exp(2(\Omega_1+\Omega_2)), \quad (27)
\end{aligned}$$

which together with Eq.(4) yields an explicit solution of the (2+1)-dimensional KP I equation

$$\begin{aligned}
u =\ & 2\{\ln(1+2\exp(\Omega_1)\cos\theta_1+2\exp(\Omega_2)\cos\theta_2+\exp(A_{12})\exp(2\Omega_1)+\exp(A_{34})\exp(2\Omega_2)\\
&+2\exp(\Omega_1+\Omega_2)[\mathrm{Re}(\exp(A_{13}))\cos(\theta_1+\theta_2)-\mathrm{Im}(\exp(A_{13}))\sin(\theta_1+\theta_2)\\
&+\mathrm{Re}(\exp(A_{14}))\cos(\theta_1-\theta_2)-\mathrm{Im}(\exp(A_{14}))\sin(\theta_1-\theta_2)]+2\exp(A_{12})\exp(2\Omega_1+\Omega_2)\\
&\times[(\mathrm{Re}(\exp(A_{13}))\mathrm{Re}(\exp(A_{14}))+\mathrm{Im}(\exp(A_{13}))\mathrm{Im}(\exp(A_{14})))\cos\theta_2-(\mathrm{Im}(\exp(A_{13}))\\
&\times\mathrm{Re}(\exp(A_{14}))-\mathrm{Re}(\exp(A_{13}))\mathrm{Im}(\exp(A_{14})))\sin\theta_2]+2\exp(A_{34})\\
&\times\exp(\Omega_1+2\Omega_2)[(\mathrm{Re}(\exp(A_{13}))\mathrm{Re}(\exp(A_{14}))-\mathrm{Im}(\exp(A_{13}))\mathrm{Im}(\exp(A_{14})))\cos\theta_1\\
&-(\mathrm{Re}(\exp(A_{13}))\mathrm{Im}(\exp(A_{14}))+\mathrm{Im}(\exp(A_{13}))\mathrm{Re}(\exp(A_{14})))\sin\theta_1]\\
&+\exp(A_{12})\exp(A_{34})\left|\exp(A_{13})\right|^2\left|\exp(A_{14})\right|^2\exp(2(\Omega_1+\Omega_2)))\}_{xx}. \quad (28)
\end{aligned}$$

The parameters are suitably chosen, when $b_1^2+d_1^2\neq 0$, $b_2^2+d_2^2\neq 0$, $a_1^2+b_1^2\neq 0$, $a_2^2+b_2^2\neq 0$, $\exp(A_{12})>1$ and $\exp(A_{34})>1$, Eq.(28) represents the interaction of two periodic-soliton solutions, when $b_1^2+d_1^2\neq 0$, $b_2=d_2=0$, $a_1^2+b_1^2\neq 0$, $a_2\neq 0$ and $\exp(A_{12})>1$, Eq.(28) denotes the interaction of one periodic-soliton and one line-soliton solutions, when $b_1=b_2=d_1=d_2=0$, $a_1\neq 0$ and $a_2\neq 0$, Eq.(28) represents the interaction of two line-soliton solutions. Plots of these solutions are given in Figs.2–4.

It is worthwhile to notice that we cannot obtain the periodic-soliton solutions for the (2+1)-dimensional KP II equation ($\sigma^2=1$) by using above method. In fact, substituting Eqs.(9)–(12) into the Eq.(6), we have

$$\exp(A_{2m-1,2m}) = \frac{(b_m c_m - a_m d_m)^2 - (b_m^6 + b_m^4 a_m^2 + b_m^2 a_m^4) - b_m^4 a_m^2}{(b_m c_m - a_m d_m)^2 + (b_m^6 + b_m^4 a_m^2 + b_m^2 a_m^4) + b_m^2 a_m^4} < 1 \tag{29}$$

($m = 1, 2, \cdots, N$). It is easy to know that the (2+1)-dimensional KP II equation has not periodic-soliton solutions on the basis of available evidence in the Section 2.

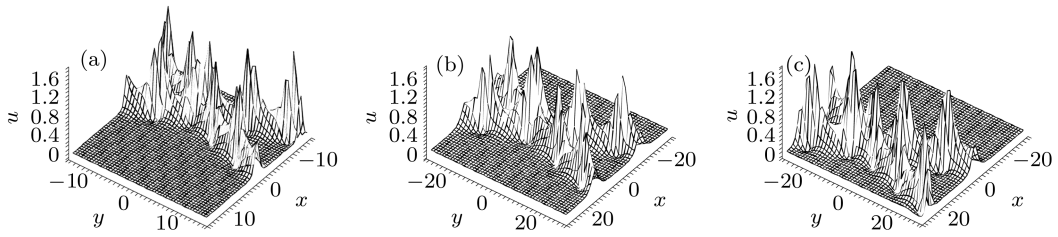


Fig.2. The interaction of two periodic-soliton solution Eq.(28) with $a_1 = -0.59$, $b_1 = -0.05$, $c_1 = 0.2$, $d_1 = 0.45$, $a_2 = 0.58$, $b_2 = 0.02$, $c_2 = 0.21$, $d_2 = 0.43$, (a) $t = -8$, (b) $t = 0$, (c) $t = 8$.

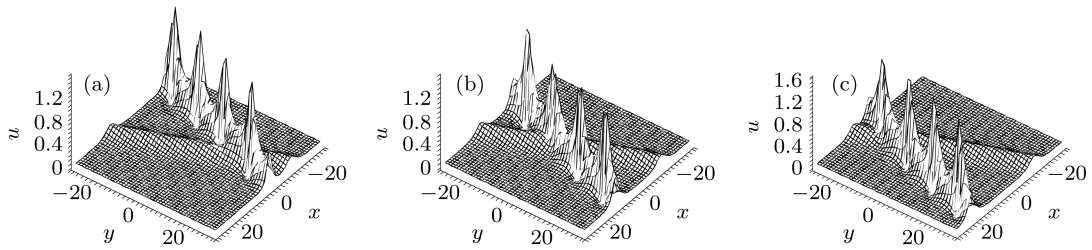


Fig.3. The interaction of one periodic-soliton and one line-soliton solution Eq.(28) with $a_1 = -0.59$, $b_1 = -0.05$, $c_1 = 0.2$, $d_1 = 0.45$, $a_2 = 0.58$, $b_2 = 0$, $c_2 = 0.21$, $d_2 = 0$, (a) $t = -8$, (b) $t = 0$, (c) $t = 8$.

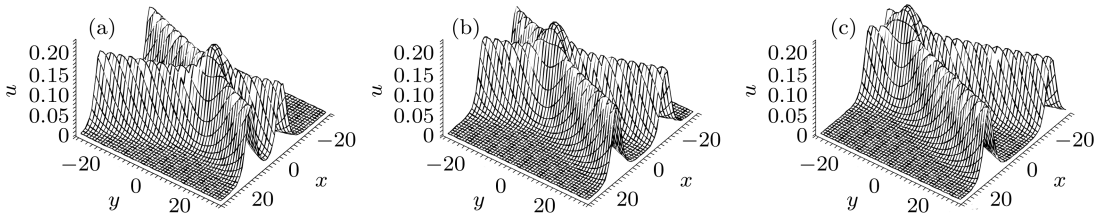


Fig.4. The interaction of two line-soliton solution Eq.(28) with $a_1 = -0.59$, $b_1 = 0$, $c_1 = 0.2$, $d_1 = 0$, $a_2 = 0.58$, $b_2 = 0$, $c_2 = 0.21$, $d_2 = 0$, (a) $t = -168$, (b) $t = 0$, (c) $t = 130$.

4. Summary

It is well known that Hirota bilinear method is an effective approach to find line-soliton solutions of nonlinear evolution equation. However, we cannot obtain the periodic-soliton solutions from standard Hirota bilinear method. In this paper, with the help of the Hirota bilinear method, we derive the periodic-soliton solutions of the (2+1)-dimensional KPI equation by suitably selecting the complex parameters. As an application, we obtain the solutions of multiple soliton interactions, including interactions between two periodic-solitons, one periodic-soliton and one line-soliton and two line-solitons. This way can be also applied to other multidimensional nonlinear evolution equations.

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