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Effect of superstrong magnetic fields on thermonuclear reaction rates on the surface of magnetars*

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A simple and efficient screening model for studying the effects of superstrong magnetic fields (such as those of magnetars) on thermonuclear reaction rates on magnetar surfaces is proposed in this paper. The most interesting thermonuclear reactions, including hydrogen burning by the CNO cycle and helium burning by the triple alpha reaction, are investigated on the surface of magnetars. We find that the superstrong magnetic fields can increase the thermonuclear reaction rates by many orders of magnitude. The enhancement may have a dramatic effect on the thermonuclear runaways and bursts on the surfaces of magnetars.

Keywords: nucleosynthesis, strong magnetic fields, neutron stars
PACC: 9710C, 2340, 9760J

1. Introduction

It is universally received that the typical estimated magnitudes of the surface magnetic fields for most of the neutron stars are about 10^8–10^9 T.[1] There probably exist magnetars with superstrong magnetic field strengths exceeding the quantum critical threshold, $B_{\rm rel}$ =4.414×10^9 T,[2−5] which is much larger than the typical magnetic field of a neutron star. Soft γ-ray repeaters (SGRs) and anomalous x-ray pulsars (AXPs) are classes of candidates for magnetars.[6] The magnetic fields of the magnetars may be so strong that they can reach two orders of magnitude higher than the quantum critical threshold.[7] The most recent observations[8−10] provide strong evidence for the validity of the old hypothesis that the surface magnetic fields of the magnetars are about 10^{10}–10^{11} T. Moreover, the observed x-ray luminosities of the AXPs may require a magnetic field strength $B \geq 10^{12}$ T.[11] And the population statistics of SGRs suggest that magnetars may comprise a significant fraction ($\geq$10%) of the neutron star population.[12] The giant magnetic field of a magnetar has a significant and observable effect on quantum electrodynamic processes occurring near the star. Hence the magnetars giving rise to many greatly interesting properties have been extensively studied.

The effects of giant magnetic fields ($B \geq 10^8$ T) on hydrogen and helium atoms have been extensively studied. Various studies are devoted to the topics such as the formation of molecules and chains (see Refs.[13, 14] for further information) as well as nuclear fusion.[15,16] The α-decay,[17] β-decay[18,19] and electron capture[20,21] processes in high magnetic fields have also been discussed. The magnetic field effects on thermonuclear reaction rates in the crusts of neutron stars were investigated, and the result indicates that the strong magnetic field ($B \approx 10^9$ T) has an evident effect on thermonuclear reaction rates, especially in the lower density environments.[16,22] Therefore, it is interesting to investigate the influence of superstrong magnetic fields ($B \approx 10^{10}$–10^{12} T) on thermonuclear reaction rates on the surface of magnetars.

2. Magnetic field effects on thermonuclear reaction rates

The binding energy of the magnetized condensed matter at zero pressure can be estimated by using the uniform electron gas model. The mean number density of electrons is $n_{\rm e} = Z/\left(4\pi r_i^3/3\right)$ in a Wigner–Seitz

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cell with a radius $r_i = Z^{1/3} r_{\rm s}$ ($r_{\rm s}$ is the mean electron spacing), and the electron Fermi momentum $p_{\rm F}$ is obtained from $n_{\rm e} = 2\left(eB/\hbar^2 c\right) p_{\rm F}$. When the Fermi energy $p_{\rm F}^2/2m_{\rm e}$ is less than the cyclotron energy $\hbar\omega_{\rm ce}$, the electrons only occupy the ground Landau level. In these cases, the energy (in atomic units) per cell can be written as

$$E_{\rm s}\left(r_i, z_j\right) = E_{\rm k}\left(r_i, z_j\right) - U_{\rm Coul}\left(r_i, z_j\right), \qquad (1)$$

where $E_{\rm k}\left(r_i, z_j\right) = \dfrac{3\pi^2 z_j^3}{8b^2 r_i^6}$ is the kinetic energy, $U_{\rm Coul}\left(r_i, z_j\right) = \dfrac{0.9 z_j^2}{r_i}$ is the Coulomb energy, z_j is the charge number of the species j, $b = B/B_0$ and $B_0 = m_{\rm e}^2 e^3 c/\hbar^3 = 2.3505 \times 10^5\,{\rm T}$. For the zero-pressure condensed matter, the r_i(in units of the Bohr radius) is given by

$$r_{i,0} \approx 1.90 z_j^{1/5} b^{-2/5} = K z_j^{1/5}, \qquad (2)$$

where $K = 1.90 b^{-2/5}$. The energy correction (in atomic units) per cell due to non-uniformity can be calculated by using the linear response theory to be[14]

$$E_{\rm TF}\left(r_i, z_j\right) = -0.0139 b^2 r_i^4 z_j. \qquad (3)$$

The uniform electron gas model can be improved by taking into consideration the Coulomb energy and Thomas–Fermi correction due to non-uniformity of the electron gas, and then the energy per cell is corrected to be

$$\begin{aligned} E_{\rm s}\left(r_i, z_j\right) =& E_{\rm k}\left(r_i, z_j\right) - U_{\rm Coul}\left(r_i, z_j\right) \\ & - E_{\rm TF}\left(r_i, z_j\right). \end{aligned} \qquad (4)$$

In expression (4), we have neglected the correction of relativistic effect and Coulomb exchange interaction to the uniform electron gas model. According to Refs.[14, 16], near the zero-pressure density, both of them are negligible relatively to the correction to the non-uniformity of the electron gas. For the two interacting nuclei, the energy required to bring two nuclei with nuclear charge numbers z_1 and z_2 so close together that they coincide with each other essentially differs from the bare Coulomb energy by an amount that, in the Wigner–Seitz approximation, is

$$U_{\rm sc} = E_{\rm s}\left(r_i, z_{12}\right) - E_{\rm s}\left(r_i, z_1\right) - E_{\rm s}\left(r_i, z_2\right), \qquad (5)$$

where $z_{12} = z_1 + z_2$. If the electron distribution is rigid, the contribution to $E_{\rm s}$ from the bulk electron energy cancels out in expression (5), and the screening potential is simply

$$U_{\rm sc} = U_{\rm Coul}\left(r_i, z_{12}\right) - U_{\rm Coul}\left(r_i, z_1\right) - U_{\rm Coul}\left(r_i, z_2\right). \qquad (6)$$

When the electron density is assumed to be uniform, the screening potential is independent of the magnetic field. However, the electrons are not rigid, and they respond to the ionic potential. Their compressibility depends on the magnetic field, and this leads in turn to a field dependence of the screening potential. Note that expression (4) for the energy contains contributions not only from the Coulomb interaction, but also from the compressibility energy of the electron in the non-uniform state.

From expressions (3) and (4), the change of the screening potential due to the compressibility of the electrons for the zero-pressure magnetized condensed matter can be obtained as

$$\delta E_{\rm TF} = -0.0139 b^2 K^4 \left(z_{12}^{9/5} - z_1^{9/5} - z_2^{9/5}\right). \qquad (7)$$

The screening potential is the sum of a screening potential with a uniform distribution and a corrected screening potential with a non-uniformity distribution. Then the screening potential in a superstrong magnetic field is given by

$$U_{\rm s} = U_{\rm sc} + \delta E_{\rm TF}. \qquad (8)$$

The total rate of the charge particle induced thermonuclear reaction is the sum of the resonant and the non-resonant reaction (also called direct capture) contributions to the nuclear reaction mechanism. Owing the thermonuclear reaction rate that is governed by the non-resonant reaction in a lower temperature range such as on the surface of magnetars, only the magnetic field effect on the non-resonant reaction rate is considered here in the present paper. We now define a screening enhancement factor (SEF) for considering the effect of magnetic field on the thermonuclear reaction rate. According to Refs.[16, 22], the SEF can be expressed as

$$f = \exp\left[\pi\eta(E)\left(U_{\rm s1} - U_{\rm s2}\right) E^{-1}\right], \qquad (9)$$

where $\eta(E) = z_1 z_2 e^2 \sqrt{\mu/2\hbar^2} E^{-1/2}$ is the Sommerfeld parameter; E is the centre-of-mass bombardment energy; $U_{\rm s1}$ and $U_{\rm s2}$ represent the screening potentials with and without magnetic field case, respectively. Then the thermonuclear reaction rate, due to the effect of magnetic field, should be corrected to be

$$N_A\langle\sigma v\rangle^{\rm m} = f \cdot N_A\langle\sigma v\rangle^{\rm um}, \qquad (10)$$

where $N_A\langle\sigma v\rangle^{\rm m}$ and $N_A\langle\sigma v\rangle^{\rm um}$ represent the thermonuclear reaction rates for the magnetized and unmagnetized cases, respectively.

3. Numerical analysis and discussion

In the model of x-ray burst source, the most significant thermonuclear reactions are hydrogen burning by the p-p chain (occurring at $\rho \sim 10^6$g·cm^{-3} and $T \sim 10^7$ K) and the CNO cycle (occurring at $\rho \sim 10^5$g·cm^{-3} and $T \sim$ a few 10^7 K) and helium burning (occurring at $\rho \sim 10^6$g·cm^{-3} and $T \sim 10^8$ K),[23] where ρ represents the density and T denotes the stellar temperature.

The screening model is obtained by assuming that the electrons populate mostly at the ground Landau levels, then this model can be applied only to the region[14]

$$\rho < \rho_{\rm B} = 7.04 \times 10^3 Y_{\rm e}^{-1} B_{12}^{3/2} {\rm g \cdot cm^{-3}}, \qquad (11)$$

and

$$T < T_{\rm B} = 1.34 \times 10^8 B_{12} \left(1 + x_{\rm F}^2\right)^{-1/2} {\rm K}, \qquad (12)$$

where B_{12} is the magnetic field strength in units of 10^{12}G, $x_{\rm F} = 1.009 \times 10^{-2} \left(Y_{\rm e}\rho\right)^{1/3}$ and $Y_{\rm e} = Z/A$ that is the number of electron per baryon. A simple calculation shows that these conditions are well satisfied in a magnetic field strength range of 10^9–10^{12} T. Then this model can be safely applied to discussing the superstrong magnetic field effects on thermonuclear reaction rates on the surface of magnetars. Moreover, according to the previous studies,[14,16] the Thomas-Fermi model can be used only for magnetic fields in a region $b < Z^3$. Then this model can be applied only to these reactions in CNO cycle and helium burning, but it cannot be applied to these reactions in p-p chain. The fusion reaction of p+p has been investigated in the previous studies,[15,16] and the result shows that the strong magnetic field could increase the thermonuclear reaction rate by roughly one order of magnitude as compared with that in the unmagnetized case. Moreover, the d+d reaction can be significantly affected by a strong magnetic field as it is already much faster than the p+p one. And the increment can be as high as two orders of magnitude. Then we can conclude that the effect of superstrong magnetic field on hydrogen burning by the p-p chain can be important on the surface of magnetars.

Figure 1 represents the SEF as a function of magnetic field strength for $\alpha + \alpha$ reaction at $\rho/\mu_{\rm e} = 10^6$mol·cm^{-3} and T=10^8K. The magnetic field can increase the reaction rates by a factor of 1.25, 1.54, 1.76, 2.93, 4.14 and 14.94 at B=10^9, 5×10^9, 10^{10}, 5×10^{10}, 10^{11} and 5×10^{11} T respectively. In other words, the giant magnetic field of magnetars can increase the rates by about one order of magnitude as compared with those in the unmagnetized case. Similarly, the superstrong magnetic field must increase the $\alpha_+{}^8$Be reaction rate by two orders of magnitude. This may further affect the reaction rate of $\alpha_+{}^{12}$C which in turn affects the nuclear reaction pathway and the energy generation rate on the surface of magnetar. The effects of superstrong magnetic field on the helium burning by the triple alpha reaction can be significant on the surface of magnetars.

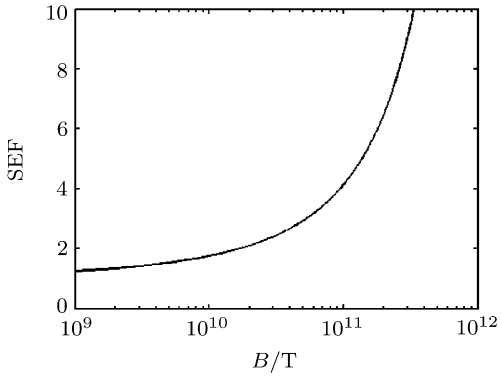


Fig.1. The SEF as a function of magnetic field strength for $\alpha + \alpha$ reaction at $\rho/\mu_{\rm e} = 10^6$mol·cm^{-3} and T=10^8K.

Figure 2 shows the SEF as a function of magnetic field strength for the reaction of ^{14}N(p,γ)^{15}O at $\rho/\mu_{\rm e} = 10^5$mol·cm^{-3} and different temperatures T=3×10^7, 5×10^7 and 8×10^7 K. The ^{14}N(p,γ)^{15}O reaction rate would increase by factors of 12.7, 127.2, 598.1, 1.9×10^5, 9.4×10^6 and 1.9×10^{13} at T=3×10^7 K, and by factors of 3.3, 9.5, 19.5, 286.2, 1.7×10^3 and 1.5×10^6 at T=5×10^7K, and by factors of 1.8, 3.0, 4.3, 16.4, 40.0 and 1.1×10^3 at T=8×10^7K and B=10^9, 5×10^9, 10^{10}, 5×10^{10}, 10^{11} and 5×10^{11} T, respectively. The effects of superstrong magnetic fields on thermonuclear reaction rates can be remarkable for the hydrogen burning by the CNO cycle, which could increase the reaction rates by many orders of magnitude, especially in a lower temperature region as compared with these in the unmagnetized case. This may influence some important magnetar evolution processes and observed phenomena. For example, ^{14}N(p,γ)^{15}O is the key reaction in the CNO cycle, which deter-

mines the cycle efficiency of the CN cycle and the hydrogen burning timescales of the CNO cycle as well as the neutrino flux reproduced by the CNO cycle in the surface of magnetars. However, the bottleneck role may disappear if the rates increase by several orders of magnitude. This could change the nuclear reaction pathway, and in turn affect the nucleosynthesis of some heavier nuclei as well as the element abundance on the surface of magnetars. As is well known, the thermonuclear ignition of the accreted material on the surface of magnetars leads to a thermonuclear explosion which is observed as an x-ray burst. Therefore the magnetic fields could potentially influence the features of x-ray bursts.

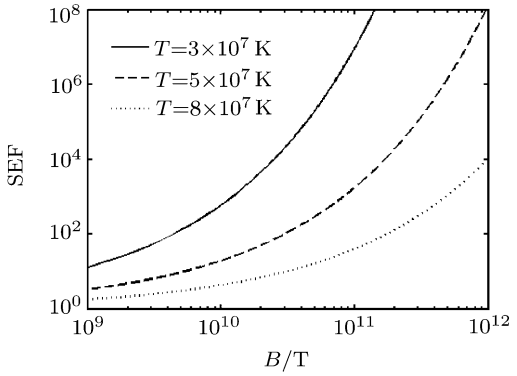


Fig.2. The SEF as a function of magnetic field strength for reaction of ^{14}N(p, γ)^{15}O at $\rho/\mu_e = 10^5$mol·cm^{-3} and different temperatures.

4. Summaries

This work proposes a simple and efficient screening model for studying the effects of superstrong magnetic fields on thermonuclear reaction rates. By calculating the SEF of the most interesting reactions on the surface of magnetars, we find that the effects of superstrong magnetic fields on thermonuclear reaction rates can be remarkable, which increases the rates by many orders of magnitude. In principle, we may obtain all screened rates on the surface of magnetars, which are multiples of corresponding SEF under their corresponding conditions. However, the effects of magnetic fields on all reactions are different due to the fact that the SEF is related to the number of nuclear charges that take part in the reaction. These discrepancies may influence the nuclear reaction pathway, which in turn influences the nucleosynthesis of some heavier nuclei and then further influences the energy generation rate and element abundance on the surface of magnetar. This will modify our understanding of both the thermal evolution and the thermal spectrum of magnetars. Certainly, quantitative predictions are expected to be made when the results of large scale stellar nuclear network calculations are available. It is concluded that the present study has a significant influence on further study of the magnetars, especially on the x-ray observations and the evolution of magnetars.

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