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Spectral feature with sub-natural linewidth due to quantum interference in a four-level system*

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This paper studies the narrow spectral feature appearing in a four-level system coupled by two strong coherent fields and probed by a weak laser field. The linewidth is examined as a function of the Rabi frequencies of coupling fields, and the result is explained by using the dressed-state formalism.

Keywords: quantum interference, linewidth, dressed-state
PACC: 4250, 3270J

1. Introduction

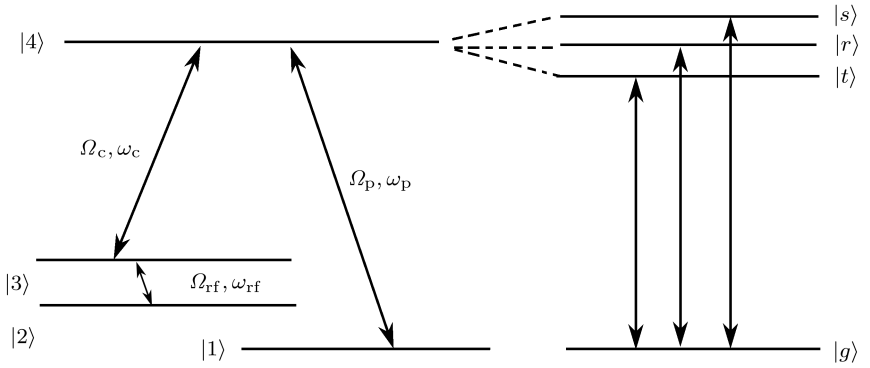
Atomic coherence and quantum interference effects have been studied extensively in recent years and in particular electromagnetically induced transparency (EIT)[1,2] in Λ three-level system with two ground levels, an excited level has attracted considerable attention due to its sub-natural width. The narrow spectral width is of great interest for potential applications such as slow light and quantum information processing[3] and hence has been a subject of many recent studies where the power broadening effect of EIT resonance has been investigated as a function of various parameters.[4−9]

The quantum interference between multiple atomic transitions pathways also gives rise to the narrowing of absorption spectral lines. It has been shown that the spectral feature with sub-natural width can be obtained in an EIT system by introducing detuning between the coupling field and the atomic transition. Vemuri *et al*[10] have predicted the possibility of obtaining sub-Doppler resolution in a Doppler broadening system or inhomogeneous broadening media by utilizing an intense control field, and this has been observed experimentally by Zhu *et al.*[11] We have studied the absorption feature with sub-natural width in an ideal Λ three-level atomic system, where the density matrix equation of motion in dressed-atom basis is derived to give an intuitive physical picture of the sub-natural width spectral feature.[12] Narrow spectral features have also been studied previously in other related systems. For example, Buzek[13] showed the existence of a very narrow line in the spectrum emitted by a system of N two-level atoms in which only one atom is initially excited, when the separation between the atoms is very small but nonzero. The presence of extremely sharp structures in the fluorescence of a three-level V system with one metastable excited level was demonstrated by Hegerfeld *et al*[14] for the resonance fluorescence spectrum and in absorption by Plenio.[15] Zhou *et al* have shown that the quantum interference can result in very narrow resonance of absorption profile for a V-type atom.[16,17] Recently, it has been shown for a variety of four level systems driven by two coupling fields that the probe absorption spectrum is characterized by a double dark resonance, consisting of two EIT windows, separated by a sharp absorption peak.[18−23]

In this paper, we study the sub-natural spectral feature in a four-level system consisting of triplet ground levels and an excited level interacting with two lasers and one radio frequency (RF) field as shown in Fig.1(a). This paper is an extension of our previous work[22] where the dynamic Stark effect of the narrow spectral dip (EIT) feature subjected to the RF driving field is studied. The new aspect of present work is the investigation of the absorption peaks with sub-natural width. We derive the density matrix equation of motion in dressed-atom basis and give the intuitive physical picture of the sub-natural spectral feature. In

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contrast to the Λ three-level EIT system where narrow spectral peaks occur in the presence of an off-resonant coupling field, the sub-natural spectral peaks in current four-level system can be obtained for resonant coupling fields. It is shown that the sub-natural spectral lines can be manipulated by controlling the relative strengths of the coupling laser and RF field.



(a) Bare state (b) Dressed-state

Fig.1. Energy-level diagram.

2. The probe absorption spectrum

We investigate a four-level atom with a triplet ground state $|1\rangle$, $|2\rangle$ and $|3\rangle$ and an excited state $|4\rangle$ as shown in Fig.1(a), where a RF field excites $|2\rangle \rightarrow |3\rangle$ transition and one laser referred as the coupling field excites the $|3\rangle \rightarrow |4\rangle$ transition. The absorption spectrum is obtained by scanning the frequency ($\omega_{\rm p}$) of the second laser, which is referred as the probe field, through the $|1\rangle \rightarrow |4\rangle$ transition. The Rabi frequencies of the coupling field, RF field and probe field are denoted as $\Omega_{\rm c}$, $\Omega_{\rm rf}$ and $\Omega_{\rm p}$, respectively. As we are interested in effects of coupling and RF field on the absorption spectrum, it is sufficient to consider a weak probe field case. The first order steady state solution for ρ_{41} can be written as (see Eq.(3) in Ref.[24]):

$$\rho_{41} = \chi_{\rm p} \frac{{\rm i}\left(d_{21}d_{31} + \chi_{\rm rf}^2\right)\left(\rho_{44}^{(0)} - \rho_{11}^{(0)}\right) - {\rm i}\chi_{\rm c}\chi_{\rm rf}\rho_{24}^{(0)} + \chi_{\rm c}d_{21}\rho_{34}^{(0)}}{d_{21}d_{31}d_{41} + \chi_{\rm c}^2 d_{21} + \chi_{\rm rf}^2 d_{41}}, \tag{1}$$

where the zero order solution is also given in Ref.[24].

The parameters $\chi_{\rm c}$, $\chi_{\rm p}$ and $\chi_{\rm rf}$ are introduced for convenience and are equal to half the Rabi frequencies $\left(\chi_{\rm c} = \dfrac{\Omega_{\rm c}}{2}, \chi_{\rm p} = \dfrac{\Omega_{\rm p}}{2} \text{ and } \chi_{\rm rf} = \dfrac{\Omega_{\rm rf}}{2}\right)$. $d_{ij} = {\rm i}\delta_{ij} - \gamma_{ij}$ are complex detunings. The detunings δ_{ij} are given by

$$\begin{aligned}
&\delta = \delta_{41} = \omega_{\rm p} - \omega_{41}, \quad \delta_{43} = \omega_{\rm c} - \omega_{43},\\
&\delta_{32} = \omega_{\rm rf} - \omega_{32}, \quad \delta_{31} = \delta_{41} - \delta_{43},\\
&\delta_{42} = \delta_{43} + \delta_{32}, \quad \delta_{21} = \delta_{41} - \delta_{32} - \delta_{43}.
\end{aligned}$$

We denote the population relaxation rate from excited state level $|4\rangle$ to ground state level $|i\rangle$ ($i = 1, 2, 3$) by W_{4i}, and $W_4 = W_{41} + W_{42} + W_{43}$. We denote the population relaxation rate between the hyperfine levels by Γ, for simplicity we have assumed that the population relaxation rates between different hyperfine levels have the same value. The coherence relaxation rate between level $|i\rangle$ and $|j\rangle$ is denoted by γ_{ij}. In this paper we consider ideal case $\gamma_{ij} = \dfrac{1}{2}(\gamma_i + \gamma_j)$, where $\gamma_4 = W_4$ and $\gamma_1 = \gamma_2 = \gamma_3 = 2\Gamma$.

The absorption profile of the probe field $\omega_{\rm p}$ probing the $|1\rangle \rightarrow |4\rangle$ transition can be obtained by plotting ${\rm Im}(\rho_{41})$ as a function of probe detuning $\delta = \omega_{\rm p} - \omega_{41}$. In the following section we present the re-

sults of numerical calculations. For simplicity we assume that $W_{41} = W_{42} = W_{43} = \dfrac{W_4}{3}$. In most real atomic systems the hyperfine transition decay rates are much smaller than those of the excited state and in this paper a ratio of $\dfrac{\Gamma}{W_4} = 10^{-4}$ is used. The Rabi frequencies ($\Omega_{\rm p}$, $\Omega_{\rm c}$ and $\Omega_{\rm rf}$) and the detunings δ_{ij} are also normalized to W_4.

Figure 2 shows probe absorption profile for various intensities of RF field and coupling field. Under resonant coupling and RF laser there is a triplet splitting of the absorption including a central one and two sideband peaks. The line narrowing effect of the two side peaks can be seen as the RF field intensity increases while keeping coupling laser intensity constant (see, Fig.2(a)). It is interesting to note that the width of the optical transition approaches the width of the hyperfine transition at the extreme cases. Likewise, the line narrowing effect of the central peak can be seen as the coupling laser intensity increases while keeping RF field intensity constant (see, Fig.2(b)). Again the width of an optical transition approaches that of a hyperfine transition. The dependence of linewidth normalized to W_4 of the narrow absorption line as a function of strength of RF field and coupling field are shown in Fig.3.

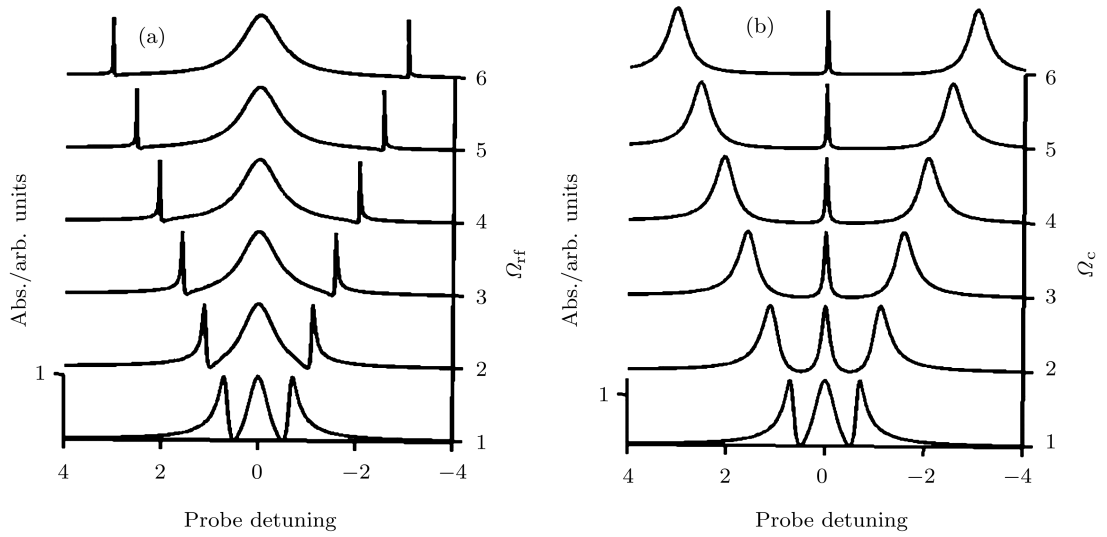


Fig.2. The probe absorption spectrum for various intensities of (a) RF field and (b) coupling field. In case (a) the coupling field has a fixed Rabi frequency $\Omega_{\rm c} = 1$ and in case (b) the RF Rabi frequency is $\Omega_{\rm rf} = 1$. The probe detuning and Rabi frequencies have been normalized to W_4.

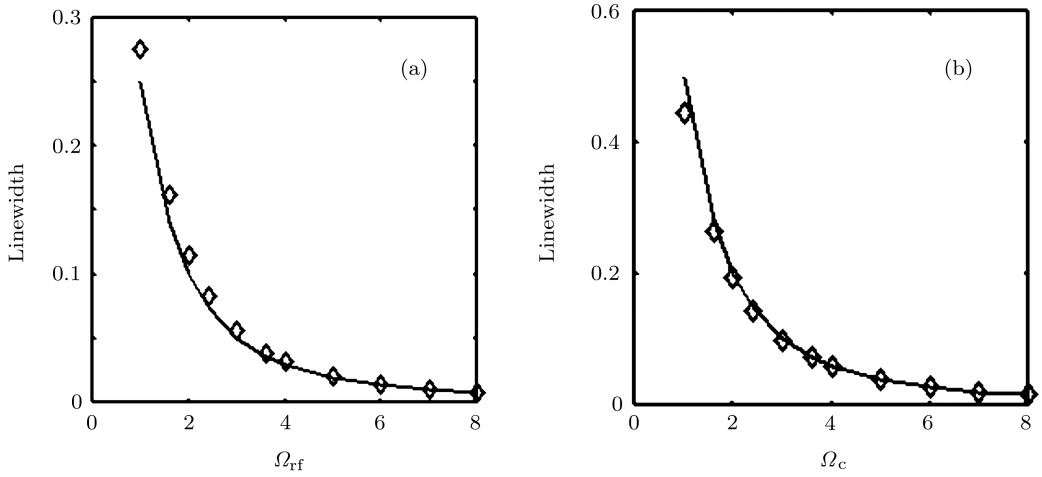


Fig.3. Dependence of the linewidth of the spectral line upon the Rabi frequency of coupling (or RF) field. (a) The linewidth of sideband peaks as a function of the Rabi frequency of RF field for $\Omega_{\rm c} = 1$. The diamonds correspond to the numerical results. Solid line is plotted according to the Eq.(8a). (b) The linewidth of central peak as a function of coupling field Rabi frequency for $\Omega_{\rm rf} = 1$. The diamonds correspond to the numerical results and the solid line is plotted according to the Eq.(8b). The linewidth of the spectral line and the Rabi frequencies of coupling and RF field have been normalized to W_4.

3. Origin of spectral line narrowing

To understand the dynamics of narrowing of absorption peaks, we introduce the dressed-state picture to reveal the mechanisms which are responsible for coupling of $|2\rangle$, $|3\rangle$, and $|4\rangle$. For the above mentioned four level system we identify two distinct coupling mechanisms. One of these mechanisms is responsible for mixing all three states, one-photon coupling, and the other for mixing $|2\rangle$ with $|4\rangle$ through $|3\rangle$, two-photon or Raman coupling.[25,26] In the case of Raman coupling the state $|3\rangle$ acts only as an intermediate state. The manifestation of these mechanisms in the probe absorption profile of the $|1\rangle - |4\rangle$ transition is the development of the triplet spectra as seen in Fig.2. The central peak here is associated with the two-photon coupling mechanism. The state associated with this coupling process can be an antisymmetric superposition of $|2\rangle$ and $|4\rangle$. Under resonant coupling condition, the dressed-state associated with two-photon coupling is given by[26]

$$|r\rangle = -(\sin\beta)|4\rangle + (\cos\beta)|2\rangle, \tag{2}$$

with

$$H'|r\rangle = 0. \tag{3}$$

Here $\tan\beta = \dfrac{\Omega_{\rm rf}}{\Omega_{\rm c}}$. H' is the total interaction Hamiltonian of the fields with the atom.

Equation (3) shows that the peak corresponding to the two-photon coupling process should occur at the same position as that of the uncoupled line centre, which is in agreement with spectra shown in Fig.2. On the other hand, the one-photon coupling process is associated with the sideband peaks. The state associated with this process is a linear superposition of all three coupled states. Under resonant coupling condition the dressed states corresponding to this process can be written as[26]

$$|s\rangle = \frac{1}{\sqrt{2}}[(\cos\beta)|4\rangle + |3\rangle + (\sin\beta)|2\rangle], \tag{4}$$

$$|t\rangle = \frac{1}{\sqrt{2}}[(\cos\beta)|4\rangle - |3\rangle + (\sin\beta)|2\rangle], \tag{5}$$

where

$$H'|s\rangle = \frac{\hbar\Omega}{2}|s\rangle, \quad \text{and} \quad H'|t\rangle = -\frac{\hbar\Omega}{2}|t\rangle, \tag{6}$$

and $\Omega = \sqrt{\Omega_{\rm c}^2 + \Omega_{\rm rf}^2}$. This shows that the peaks corresponding to the one-photon coupling process are located at $-\dfrac{\Omega}{2}$ and $+\dfrac{\Omega}{2}$ from the central peak, which is also in agreement with spectra shown in Fig.2.

In dressed state basis the two sideband peaks correspond to the transitions $|s\rangle \to |g\rangle$ and $|t\rangle \to |g\rangle$, respectively, where $|g\rangle = |1\rangle$. Above dressed state wave functions can be used to gain physical insight of the narrowing of the absorption peaks shown in Fig.2(a). We note that the ratio $\Omega_{\rm rf}/\Omega_{\rm c}$ determines the wavefunction mixing of dressed-state. When $\Omega_{\rm rf}$ increases while keeping $\Omega_{\rm c}$ constant so that $\Omega_{\rm rf} \gg \Omega_{\rm c}$ the states $|s\rangle$ and $|t\rangle$ become independent of $|4\rangle$ and are dominated by hyperfine levels $|2\rangle$ and $|3\rangle$. This gives rise to the spectral feature of sidebands whose linewidths are mainly governed by the ground state coherence relaxation rate γ_{21} and γ_{31}, which can be of many orders of magnitude smaller than that of the optical transition. Similarly the $\Omega_{\rm rf} \ll \Omega_{\rm c}$ case as shown in Fig.2(b), where the state $|r\rangle$ is dominated by hyperfine levels $|2\rangle$, gives rise to a narrow central peak whose linewidth is also mainly governed by the ground state coherence relaxation rate γ_{21}.

We derive the density matrix equations of motion in dressed state basis, and for convenience only write the associated equations of the off-diagonal elements

$$\begin{aligned}
\dot\rho_{rg} &= [-\gamma_{rg} + \mathrm{i}\delta]\cdot\rho_{rg} + \sqrt{2}\Gamma_r(\rho_{sg} + \rho_{tg}) + \mathrm{i}\alpha_B(\rho_{rr} - \rho_{gg}) - \mathrm{i}\alpha_A(\rho_{rs} + \rho_{rt}) \\
\dot\rho_{sg} &= [-\gamma_{sg} + \mathrm{i}(\delta + \Omega)]\cdot\rho_{sg} - \Gamma_{st}\cdot\rho_{tg} + \Gamma_r\cdot\rho_{rg} - \mathrm{i}\alpha_A(\rho_{ss} - \rho_{gg}) + \mathrm{i}\alpha_B\rho_{sr} - \mathrm{i}\alpha_A\rho_{st}, \\
\dot\rho_{tg} &= [-\gamma_{tg} + \mathrm{i}(\delta - \Omega)]\cdot\rho_{tg} - \Gamma_{st}\cdot\rho_{sg} + \Gamma_r\cdot\rho_{rg} - \mathrm{i}\alpha_A(\rho_{tt} - \rho_{gg}) + \mathrm{i}\alpha_B\rho_{tr} - \mathrm{i}\alpha_A\rho_{ts}.
\end{aligned} \tag{7}$$

Where

$$\Gamma_r = \frac{1}{\sqrt{2}}\sin\beta\cos\beta(\gamma_{41} - \gamma_{21}),$$

$$\Gamma_{s,t} = \frac{1}{2}(\gamma_{41}\cos^2\beta + \gamma_{21}\sin^2\beta - \gamma_{31}),$$

$$\alpha_A = \chi_{\rm p}\frac{1}{\sqrt{2}}\cos\beta,$$

$$\alpha_B = \chi_{\rm p}\sin\beta.$$

And the coherence relaxation rates between dressed state transitions are given by

$$\gamma_{sg} = \gamma_{tg} = \frac{1}{2}[\gamma_{41}\cos^2\beta + \gamma_{21}\sin^2\beta + \gamma_{31}], \quad (8a)$$

$$\gamma_{rg} = \gamma_{41}\sin^2\beta + \gamma_{21}\cos^2\beta. \quad (8b)$$

In dressed state basis, the central absorption peak corresponds to the transition $|r\rangle \rightarrow |g\rangle$, the sideband peaks correspond to the transitions $|s\rangle \rightarrow |g\rangle$ and $|t\rangle \rightarrow |g\rangle$. Therefore, Eq.(8a) describes the linewidth of the sideband absorption lines and Eq.(8b) describes the linewidth of the central absorption line. They are plotted in the Fig.3(a) and Fig.3(b), respectively. It is seen that there is a good agreement between the numerical results and the dressed state derivation. With the help of dressed state picture, it is readily seen that the coupling laser induces a wave function mixing of the ground state, which results in line narrowing of the absorption feature.

4. Conclusion

In this paper we study the absorption feature with sub-natural linewidth in a four level system driven by two strong coupling lasers. It is shown that the absorption spectrum of an optical transition can be significantly modified due to quantum interference between two transitions driven by a laser field and a RF field. The spectrum exhibits a Mollow-like triplet, the central absorption peak occurs at the same frequency as that of the uncoupled peak line centre, and the two sidebands are located at $-\sqrt{\Omega_c^2 + \Omega_{rf}^2}/2$ and $+\sqrt{\Omega_c^2 + \Omega_{rf}^2}/2$ from the central peak. The most interesting phenomenon is the spectral features with sub-natural linewidth. The mechanism of narrowing of spectral features is explained in terms of the dressed-state model. In the extreme case of the ratio of Ω_{rf}/Ω_c, the corresponding dressed states decay very slowly to the ground level $|g\rangle$. It is the slow decay that gives rise to the narrow spectral profile.

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