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Generation of entangled coherent states for two cavity modes via resonant interaction with a V-type three-level atom*

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This paper proposes a scheme for the generation of entangled coherent states for two cavity modes. In the scheme a V-type three-level atom is sent through a two-mode cavity filled with a coherent field. After the atom–cavity interaction and detection of the atomic state the cavity modes may evolve to a superposition of two-mode coherent states. As the scheme is based on resonant atom–cavity interaction, the required interaction time is short, which is important in view of the decoherence. Moreover, additional classical pulses are unnecessary before and after the atom–cavity interaction.

Keywords: entangled coherent state, resonant interaction, V-type three-level atom
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The superposition principle, which is one of the most important principles in quantum mechanics, not only successfully explains various phenomena in the microscopic level, but also plays a fundamental role in quantum information processing. However, the extension of this curious principle to the macroscopic world would cause conceptual difficulty, since effects predicted by superposition contradict our observation and measurement of macroscopic systems, as noted by the famous Schrödinger cat paradox.[1] It is widely accepted that decoherence causes the transition from quantum to classical worlds. As the decoherence rate increases as the size of the system increases, the coherence in macroscopic systems loses too fast to observe.

In quantum optics, Schrödinger cat states are defined as superpositions of coherent states, which have classical analogs. Various nonclassical properties may emerge from such states due to the quantum interference between the coherent components.[2] In the context of cavity QED, a number of theoretical schemes have been proposed for generating such states.[3−6] In Ref.[3], Brune *et al* have shown that the detection of atoms passing through a nonresonant cavity can convert an initial coherent state into a superposition of several coherent states on a circle in phase space. In Ref.[4], Garraway *et al* have proposed a scheme for generating such states based on the two-photon resonant interaction of a single-mode field in a high-Q cavity with initially excited atoms.

In the past few years, there are multi-mode generalizations of the cat states. Chai has studied superpositions of two-mode coherent states and shown that, under ceratin conditions, such superposition states can exhibit various nonclassical features such as sub-Poissonian photon number statistics, two-mode squeezing, and violations of the Cauchy–Schwarz inequalities.[7] Such superposition states are also referred to as entangled coherent states.[8] Ansari *et al*[9] and Dodonov *et al*[10] have also investigated the multi-mode even and odd coherent states. The strong correlations between these modes can be responsible for the nonclassical features especially for the two-mode case. We have presented a scheme for the generation of two-mode cat states through the dispersive cavity QED.[11]

In this paper we propose an alternative method for preparing such states for a two-mode cavity field. Unlike the previous scheme,[11] the present scheme is based on the resonant atom-field interaction and thus the required interaction time is greatly shortened, which is important for the suppression of decoherence. Furthermore, the present scheme does not require additional classical fields before and after the atom–cavity interaction.

First we send a V-type three-level atom through a two-mode cavity initially filled with a two-mode co-

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herent field. The two upper levels are denoted by $|e\rangle$ and $|f\rangle$ respectively, while the lower level is denoted by $|g\rangle$. The transition $|g\rangle \rightarrow |e\rangle$ is resonantly coupled with mode 1, and the transition $|g\rangle \rightarrow |f\rangle$ is resonantly coupled with mode 2, respectively. In the interaction picture, the atom–cavity interaction is described by[12,13]

$$H = g(a^+ |g\rangle \langle e| + b^+ |g\rangle \langle f|) + \text{h.c.}, \tag{1}$$

where a^+ and b^+ are the creation operators for the cavity modes 1 and 2, respectively. We here assume that the two transitions have the same coupling constant g with the respective cavity modes.

Assume that the atom is initially in the state $|g\rangle$. The two cavity modes are initially in the coherent states $|\alpha\rangle$ and $|\beta\rangle$, respectively. The coherent state $|\alpha\rangle$ can be expanded by

$$|\alpha\rangle = \sum_{m=0}^{\infty} C_m |m\rangle, \tag{2}$$

where

$$C_m = \mathrm{e}^{-|\alpha|^2/2} \frac{\alpha^m}{\sqrt{m!}}. \tag{3}$$

After an interaction time t the system evolves to the state

$$\begin{aligned} |\psi(t)\rangle = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \Bigg[& C_m C_n \cos(\Omega_{m,n}t) |g,m,n\rangle \\ & -\mathrm{i}\sin(\Omega_{m,n}t) \left(\frac{g\sqrt{m}}{\Omega_{m,n}} |e,m-1,n\rangle \right. \\ & \left. + \frac{g\sqrt{n}}{\Omega_{m,n}} |f,m,n-1\rangle \right) \Bigg], \end{aligned} \tag{4}$$

where

$$\Omega_{m,n} = \sqrt{m+n}g. \tag{5}$$

We can rewrite Eq.(4) as

$$\begin{aligned} & |\psi(t)\rangle \\ = & \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \Bigg\{ C_m C_n \cos(\Omega_{m,n}t) |g\rangle \\ & -\mathrm{i}\left[C_{m+1}C_n \frac{g\sqrt{m+1}}{\Omega_{m+1,n}} \sin(\Omega_{m+1,n}t) |e\rangle \right. \\ & \left. + C_m C_{n+1} \frac{g\sqrt{n+1}}{\Omega_{m,n+1}} \sin(\Omega_{m,n+1}t) |f\rangle \right) \Bigg] \Bigg\} |m,n\rangle . \end{aligned} \tag{6}$$

We assume that the initial coherent field is strong enough so that $|\alpha|, |\beta| \gg 1$. In this case the widths Δm and Δn of the photon number distributions obey the inequalities $1 \ll \Delta m \ll \bar{m}$ and $1 \ll \Delta n \ll \bar{n}$, with $\bar{m} = |\alpha|^2$ and $\bar{n} = |\beta|^2$ being the mean photon numbers. Thus we have[14]

$$C_{m+1} = \mathrm{e}^{-|\alpha|^2/2} \frac{\alpha^m |\alpha| \mathrm{e}^{\mathrm{i}\phi}}{\sqrt{m+1}\sqrt{m!}} \simeq \mathrm{e}^{\mathrm{i}\phi} C_m, \tag{7}$$

and

$$C_{n+1} = \mathrm{e}^{-|\alpha|^2/2} \frac{\beta^m |\beta| \mathrm{e}^{\mathrm{i}\varphi}}{\sqrt{n+1}\sqrt{n!}} \simeq \mathrm{e}^{\mathrm{i}\varphi} C_n, \tag{8}$$

where

$$\mathrm{e}^{\mathrm{i}\phi} = \alpha/|\alpha|, \quad \mathrm{e}^{\mathrm{i}\varphi} = \beta/|\beta|. \tag{9}$$

This leads to

$$\begin{aligned} & |\psi(t)\rangle \\ \simeq & \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_m C_n \Bigg\{ \cos(\Omega_{m,n}t) |g\rangle \\ & -\mathrm{i}\left[\mathrm{e}^{\mathrm{i}\phi} \frac{g\sqrt{m+1}}{\Omega_{m+1,n}} \sin(\Omega_{m+1,n}t) |e\rangle \right. \\ & \left. + \mathrm{e}^{\mathrm{i}\varphi} \frac{g\sqrt{n+1}}{\Omega_{m,n+1}} \sin(\Omega_{m,n+1}t) |f\rangle \right) \Bigg] \Bigg\} |m,n\rangle . \end{aligned} \tag{10}$$

Under the condition $|\alpha|, |\beta| \gg 1$, the ranges of the photon number distributions are much smaller than the corresponding mean photon numbers, thus we can also obtain

$$\frac{g\sqrt{m+1}}{\Omega_{m+1,n}} \simeq \frac{\bar{m}^{1/2}}{(\bar{m}+\bar{n})^{1/2}}, \tag{11}$$

and

$$\frac{g\sqrt{n+1}}{\Omega_{m+1,n}} \simeq \frac{\bar{n}^{1/2}}{(\bar{m}+\bar{n})^{1/2}}. \tag{12}$$

Then $|\psi(t)\rangle$ can be approximately rewritten as

$$\begin{aligned} & |\psi(t)\rangle \\ \simeq & \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_m C_n \Bigg\{ \cos(\Omega_{m,n}t) |g\rangle \\ & -\mathrm{i}\left[\mathrm{e}^{\mathrm{i}\phi} \frac{\bar{m}^{1/2}}{(\bar{m}+\bar{n})^{1/2}} \sin(\Omega_{m+1,n}t) |e\rangle \right. \\ & \left. + \mathrm{e}^{\mathrm{i}\varphi} \frac{\bar{n}^{1/2}}{(\bar{m}+\bar{n})^{1/2}} \sin(\Omega_{m,n+1}t) |f\rangle \right) \Bigg] \Bigg\} |m,n\rangle . \end{aligned} \tag{13}$$

Assuming $gt/(\bar{m}+\bar{n})^{1/2} \ll 1$ we have

$$\sqrt{m+n+1}gt \simeq \sqrt{m+n}gt, \tag{14}$$

and

$$\begin{aligned} |\psi(t)\rangle \simeq \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_m C_n \Bigg\{ & \cos(\sqrt{m+n}gt) |g\rangle \\ & -\mathrm{i}\sin(\sqrt{m+n}gt) \left[\mathrm{e}^{\mathrm{i}\phi} \frac{\bar{m}^{1/2}}{(\bar{m}+\bar{n})^{1/2}} |e\rangle \right. \\ & \left. + \mathrm{e}^{\mathrm{i}\varphi} \frac{\bar{n}^{1/2}}{(\bar{m}+\bar{n})^{1/2}} |f\rangle \right) \Bigg] \Bigg\} |m,n\rangle . \end{aligned} \tag{15}$$

The detection of the atom in the state $|g\rangle$ collapses the cavity modes to the state

$$|\phi_c(t)\rangle \simeq \mathcal{N}_c \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_m C_n \cos\sqrt{m+n}gt\,|m,n\rangle\,, \quad (16)$$

where $\mathcal{N}_c$ is a normalization factor.

Expanding $\sqrt{m+n}$ into a Taylor series around the initial mean photon number $\bar{m}+\bar{n}$, we have

$$\sqrt{m+n} = (\bar{m}+\bar{n})^{1/2} + \frac{1}{2}\frac{m+n-(\bar{m}+\bar{n})}{(\bar{m}+\bar{n})^{1/2}} - \frac{1}{8}\frac{[m+n-(\bar{m}+\bar{n})]^2}{(\bar{m}+\bar{n})^{3/2}} + \cdots . \quad (17)$$

Since the photon number distributions satisfy $m+n-(\bar{m}+\bar{n}) \sim \Delta m + \Delta n \sim \bar{m}^{1/2}+\bar{n}^{1/2}$, we obtain $[(\Delta m)^2+(\Delta n)^2]/(\bar{m}+\bar{n})^{3/2} \sim 1/(\bar{m}+\bar{n})^{1/2}$. We have assumed $gt/(\bar{m}+\bar{n})^{1/2} \ll 1$ and thus can retain the Taylor expansion only to the second term

$$\sqrt{m+n} \simeq (\bar{m}+\bar{n})^{1/2} + \frac{1}{2}\frac{m+n-(\bar{m}+\bar{n})}{(\bar{m}+\bar{n})^{1/2}}. \quad (18)$$

This leads to

$$|\phi_c(t)\rangle \simeq \frac{\mathcal{N}_c}{2}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} C_m C_n \left\{ e^{igt[(\bar{m}+\bar{n})^{1/2}/2+(m+n)/2(\bar{m}+\bar{n})^{1/2}]} + e^{-igt[(\bar{m}+\bar{n})^{1/2}/2+(m+n)/2(\bar{m}+\bar{n})^{1/2}]} \right\} |m,n\rangle\,. \quad (19)$$

We can rewrite $|\phi_c(t)\rangle$ as

$$|\phi_c(t)\rangle \simeq \frac{\mathcal{N}_c}{2}\left[e^{igt(\bar{m}+\bar{n})^{1/2}/2} \left|\alpha e^{igt/2(\bar{m}+\bar{n})^{1/2}}\right\rangle \left|\beta e^{igt/2(\bar{m}+\bar{n})^{1/2}}\right\rangle + e^{-igt(\bar{m}+\bar{n})^{1/2}/2} \left|\alpha e^{-igt/2(\bar{m}+\bar{n})^{1/2}}\right\rangle \left|\beta e^{-igt/2(\bar{m}+\bar{n})^{1/2}}\right\rangle \right]. \quad (20)$$

By this way, we obtain an entangled coherent state for the two cavity modes. It should be noted that, even though the phase difference $gt/(\bar{m}+\bar{n})^{1/2}$ between the two coherent states $\left|\alpha e^{igt/2(\bar{m}+\bar{n})^{1/2}}\right\rangle$ and $\left|\alpha e^{-igt/2(\bar{m}+\bar{n})^{1/2}}\right\rangle$ is much smaller than 1, the overlap between the two states may be very small, because the intensity of the field is strong.

On the other hand, the detection of the atom in the state $|e\rangle$ or $|f\rangle$ collapses the cavity modes to the state

$$|\phi_c'(t)\rangle \simeq \mathcal{N}_c' \sum_{n=0}^{\infty} C_m C_n \sin(\sqrt{m+n}gt)\,|m,n\rangle\,, \quad (21)$$

which can also be written as an entangled coherent state

$$|\phi_c'(t)\rangle \simeq \frac{\mathcal{N}_c'}{2}\left[e^{igt(\bar{m}+\bar{n})^{1/2}/2} \left|\alpha e^{igt/2(\bar{m}+\bar{n})^{1/2}}\right\rangle \left|\beta e^{igt/2(\bar{m}+\bar{n})^{1/2}}\right\rangle - e^{-igt(\bar{m}+\bar{n})^{1/2}/2} \left|\alpha e^{-igt/2(\bar{m}+\bar{n})^{1/2}}\right\rangle \left|\beta e^{-igt/2(\bar{m}+\bar{n})^{1/2}}\right\rangle \right]. \quad (22)$$

It is necessary to give a brief discussion on the experimental feasibility of the proposed scheme. For the Rydberg atom with principal quantum numbers around 50, the radiative time is $T_r = 3\times10^{-2}\,\mathrm{s}$, and the coupling constant is $g = 2\pi\times 25\,\mathrm{kHz}$.[15] Set $|\alpha| = |\beta| = 10$ and $gt = 2\pi$. Thus the interaction time of the atom with the cavity field is $t = 2\pi/g = 4\times10^{-5}\,\mathrm{s}$, the distance between two coherent components $\left|\alpha e^{igt/2(\bar{m}+\bar{n})^{1/2}}\right\rangle \left|\beta e^{igt/2(\bar{m}+\bar{n})^{1/2}}\right\rangle$ and $\left|\alpha e^{-igt/2(\bar{m}+\bar{n})^{1/2}}\right\rangle \left|\beta e^{-igt/2(\bar{m}+\bar{n})^{1/2}}\right\rangle$ is about $d \simeq \sqrt{2}\,|\alpha|\,gt/(\bar{m}+\bar{n})^{1/2} \simeq 6.28$. The decoherence time of the superposition of different components is $T_c' = T_c/2d^2$, where T_c is the cavity decay time. Very recently, a ultrahigh fineness Fabry–Perot resonator with a damping time $T_c = 0.13\,\mathrm{s}$ has been built.[16] The decoherence time of the cavity field is about $T_c' \simeq 1.65\times10^{-3}\,\mathrm{s}$. The infidelity induced by the decoherence is about $t/T_r + t/T_c' \simeq 2.56\times10^{-2}$.

In conclusion, we have made a proposal for the generation of entangled coherent states for a two-mode cavity field. The scheme only requires a single resonant interaction of a V-type three-level atom with the cavity modes. Therefore the scheme is simple and the required interaction time is short, which is important in view of decoherence. Furthermore, the atom does not need to be subjected to $\pi/2$ pulses before entering and after exiting the cavity.

References

[1] Schrödinger E 1935 *Naturwissenschaft* **23** p807; p823; p844

[2] Yia Y and Guo G 1989 *Phys. Lett.* A **136** 281

[3] Brune M, Haroche S, Raimond J M, Davidovich L and Zagury N 1992 *Phys. Rev.* A **45** 1992

[4] Garraway B M, Sherman B, Moya-Cessa H, Knight P L and Kurizki G 1994 *Phys. Rev.* A **49** 535

[5] Liu X and Fang M F 2000 *Acta Phys. Sin.* **49** 1707 (in Chinese)

[6] Zhou L, Song H S, Li C and Guo Y Q 2003 *Chin. Phys.* **12** 45

[7] Chai C 1992 *Phys. Rev.* A **46** 7187

[8] Sanders B C 1992 *Phys. Rev.* A **45** 6811

[9] Ansari N A and Manko V I 1994 *Phys. Rev.* A **50** 1942

[10] Dodonov V V, Manko V I and Nikonov D E 1995 *Phys. Rev.* A **51** 3328

[11] Guo G C and Zheng S B 1997 *Opt. Commun.* **133** 142

[12] Wu Y 1996 *Phys. Rev.* A **54** 1586

[13] Wu Y and Yang X X 1997 *Phys. Rev.* A **56** 2443

[14] Gea-Banacloche J 1990 *Phys. Rev. Lett.* **65** 3385

[15] Osnaghi S, Bertet P, Auffeves A, Maioli P, Brune M, Raimond J M and Haroche S 2001 *Phys. Rev. Lett.* **87** 037902

[16] Kuhr S, Gleyzes S, Guerlin C, Bernu J, Hoff U B, Deleglise S, Osnaghi S, Brune M, Raimond J M and Haroche S 2006 quant-ph/0612138