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The entanglement of two dipole–dipole coupled atoms interacting with a thermal field via a two-photon process*

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This paper studies entanglement between two dipole–dipole coupled atoms interacting with a thermal field via a two-photon process. It shows that the entanglement is dependent on the mean photon number of the thermal field and the dipole–dipole interaction. The results also show that the atom–atom entanglement through the two-photon process is larger than that through the one-photon process and a remarkable amount of entanglement between the atoms still remains at certain times even for a very highly noisy thermal field.

Keywords: entanglement, two-photon process, dipole–dipole interaction, thermal field
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1. Introduction

Quantum entanglement is one of the most striking properties of quantum mechanics and has been viewed as an elementary resource for quantum information processing. It is essential to create and manipulate an entangled state for quantum information application. Several methods of creating entanglement have been proposed involving trapped and cooled ions or neutral atoms.[1−9] Because of decoherence, which is generally related to noise, there is great difficulty in generating and keeping the integrity of a pure state. However, noise may also be associated with the formation of non-classical effects such as entanglement. In fact, it has been shown that entanglement arises due to the interaction of a large system in a mixed state with a single qubit.[10] Moreover, a thermal field may allow entanglement between two qubits.[11−13] In Ref.[14], Ficek and Tanaś have shown that spontaneous emission from two spatially separated atoms interacting via dipole–dipole interaction can lead to a transient entanglement of initially unentangled atoms. In Ref.[15], the authors have shown that the entanglement exhibits long-time oscillatory behaviour in a system of two dipole–dipole coupled atoms driven by an external coherent laser field in the dissipative process of spontaneous emission. On the other hand, a mixed state such as the thermal field can help us to understand decoherence mechanisms, simulating the interaction of the environment (thermal noise) with quantum bits. Reference [16] investigated the entanglement properties of a system of two dipole–dipole coupled atoms resonantly interacting with a thermal field in a high-Q cavity through a one-photon process. The results show that the degree of entanglement is progressively decreased with the increase of the thermal noise.

This paper studies entanglement between two dipole–dipole coupled atoms interacting with a thermal field via a two-photon process. It shows that the entanglement is dependent on the mean photon number of the thermal field and the dipole–dipole interaction. The results also show that the atom–atom entanglement through two-photon process is larger than that through one-photon process and a remarkable amount of entanglement between the atoms still remains at certain times even for a very highly noisy thermal field.

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2. The model

In this model, we consider a system composed of two two-level atoms of the same type which are coupled via a dipole–dipole interaction. Both atoms are also interacting with a single-mode thermal field through a two-photon process. The high-Q cavity mode is assumed to be resonant with the atomic transition frequency. Under the rotating wave approximation, the Hamiltonian in the interaction picture is

$$H_{\rm I} = \hbar\gamma \sum_{i=1,2} (a^{+2}\sigma_i^- + a^2\sigma_i^+) + \hbar\Omega(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+), \quad (1)$$

where the second term represents the dipole–dipole interaction, γ is the atom-field coupling constant and Ω is the coupling constant of the dipole–dipole interaction between the atoms, a^+ and a are the creation and annihilation operators of photons respectively and $\sigma_i^+ = |e\rangle_i\langle g|$, $\sigma_i^- = |g\rangle_i\langle e|$ $(i = 1, 2)$ are the atomic transition operators.

Using the above Hamiltonian and Taylor expansions of sine and cosine functions, we can obtain the exact time evolution operator $U_{\rm I}(t)$ for this system in the atomic basis $\{|ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle\}$:

$$U_{\rm I}(t) = \begin{bmatrix} U_{11} & U_{12} & U_{13} & U_{14} \\ U_{21} & U_{22} & U_{23} & U_{24} \\ U_{31} & U_{32} & U_{33} & U_{34} \\ U_{41} & U_{42} & U_{43} & U_{44} \end{bmatrix}, \quad (2)$$

where

$$\begin{aligned}
U_{11} &= 1 + 2a^2\frac{A}{\lambda}a^{+2}, \quad U_{14} = 2a^2\frac{A}{\lambda}a^2, \quad U_{41} = 2a^{+2}\frac{A}{\lambda}a^{+2}, \quad U_{44} = 1 + 2a^{+2}\frac{A}{\lambda}a^2, \\
U_{12} &= U_{13} = a^2\frac{B}{\Theta}, \quad U_{21} = U_{31} = \frac{1}{\Theta}Ba^{+2}, \quad U_{24} = U_{34} = \frac{1}{\Theta}Ba^2, \quad U_{42} = U_{43} = a^{+2}\frac{B}{\Theta}, \\
U_{22} &= U_{33} = \frac{\exp\left[-\mathrm{i}\frac{\gamma}{2}(\alpha+\Theta)t\right]}{4\Theta}\left\{[1-\exp(\mathrm{i}\gamma\Theta t)]\,\alpha + 2\Theta\exp\left[\mathrm{i}\frac{\gamma}{2}(3\alpha+\Theta)t\right] + \Theta\,[1+\exp(\mathrm{i}\gamma\Theta t)]\right\}, \\
U_{23} &= U_{32} = \frac{\exp\left[-\mathrm{i}\frac{\gamma}{2}(\alpha+\Theta)t\right]}{4\Theta}\left\{[1-\exp(\mathrm{i}\gamma\Theta t)]\,\alpha - 2\Theta\exp\left[\mathrm{i}\frac{\gamma}{2}(3\alpha+\Theta)t\right] + \Theta\,[1+\exp(\mathrm{i}\gamma\Theta t)]\right\}, \\
\alpha &= \frac{\Omega}{\gamma}, \qquad \lambda = 2(a^{+2}a^2 + a^2a^{+2}), \qquad \Theta = \sqrt{8(a^{+2}a^2 + a^2a^{+2}) + \alpha^2}
\end{aligned} \quad (3)$$

are the operators U_{ij} written in an appropriate ordering, with

$$\begin{aligned}
A &= \exp\left[-\mathrm{i}\frac{\gamma\alpha}{2}t\right]\left\{\cos\left(\frac{\gamma\Theta}{2}t\right) + \mathrm{i}\frac{\alpha}{\Theta}\sin\left(\frac{\gamma\Theta}{2}t\right)\right\} - 1, \\
B &= \exp\left[-\mathrm{i}\frac{\gamma}{2}(\alpha+\Theta)t\right][1-\exp(\mathrm{i}\gamma\Theta t)].
\end{aligned} \quad (4)$$

The calculation of the density operator follows the traditional route in quantum optics, using the evolution operator to generate the density operator at time t:

$$\rho(t) = U_{\rm I}(t)(\rho_f \otimes \rho_{\rm A})U_{\rm I}^+(t), \quad (5)$$

where $\rho_{\rm f} = \sum_n P_n|n\rangle\langle n|$ is a thermal field with mean photon number $\bar{n}$ (photon number distribution $P_n = \bar{n}^n/(1+\bar{n})^{(n+1)}$). In order to obtain the system density operator (atom–atom density operator), we trace over the field variables

$$\begin{aligned}
\rho_{\rm A}(t) &= {\rm Tr_f}[\rho(t)] \\
&= \sum_{m,n} P_n\langle m|U_{\rm I}(t)|n\rangle\rho_{\rm A}\langle n|U_{\rm I}^+(t)|m\rangle.
\end{aligned} \quad (6)$$

We are interested in the atom–atom entanglement. We analyse the negativity (ε), defined in terms of the negative eigenvalues μ_i^-, of the partial transpose of the reduced density matrix given by Eq.(6) [17,18]

$$\varepsilon = -2\sum_i \mu_i^-. \quad (7)$$

The negativity is related to the separability of the quantum states in such a way that when $\varepsilon = 0$ the system is either separable or does not become entangled and when $\varepsilon = 1$ the subsystems are maximally entangled.

3. Results

The atom–atom subsystem partial transpose matrix for initial atomic states $|eg\rangle$(or $|ge\rangle$) reads

$$\rho_{\rm A}^{\rm T_1}(t) = \begin{bmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & 0 & 0 \\ 0 & 0 & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{bmatrix}. \tag{8}$$

Thus, the measure of entanglement is

$$\varepsilon = -\rho_{11} - \rho_{44} + \sqrt{(\rho_{11} - \rho_{44})^2 + 4 \mid \rho_{14} \mid^2}, \tag{9}$$

where

$$\begin{aligned} \rho_{11} &= \sum_n \mid \sqrt{(n+2)(n+1)}\frac{B_{n+2}}{\Theta_{n+2}} \mid^2 P_{n+2}, \\ \rho_{44} &= \sum_n \mid \sqrt{n(n-1)}\frac{B_{n-2}}{\Theta_{n-2}} \mid^2 P_{n-2}, \\ \rho_{14} &= \sum_n U_{23n}U_{22n}^* P_n, \end{aligned} \tag{10}$$

with

$$\begin{aligned} \Theta_n &= \sqrt{16(n^2+n+1)+\alpha^2}, \quad B_n = \exp\left[-\mathrm{i}\frac{\gamma}{2}(\alpha+\Theta_n)t\right]\left[1-\exp(\mathrm{i}\gamma\Theta_n t)\right], \\ U_{22n} &= \frac{\exp\left[-\mathrm{i}\frac{\gamma}{2}(\alpha+\Theta_n)t\right]}{4\Theta_n}\left\{[1-\exp(\mathrm{i}\gamma\Theta_n t)]\alpha + 2\Theta_n \exp\left[\mathrm{i}\frac{\gamma}{2}(3\alpha+\Theta_n)t\right] + \Theta_n[1+\exp(\mathrm{i}\gamma\Theta_n t)]\right\}, \\ U_{23n} &= \frac{\exp\left[-\mathrm{i}\frac{\gamma}{2}(\alpha+\Theta_n)t\right]}{4\Theta_n}\left\{[1-\exp(\mathrm{i}\gamma\Theta_n t)]\,\alpha - 2\Theta_n \exp\left[\mathrm{i}\frac{\gamma}{2}(3\alpha+\Theta_n)t\right] + \Theta_n[1+\exp(\mathrm{i}\gamma\Theta_n t)]\right\}. \end{aligned} \tag{11}$$

From Eqs.(9), (10) and (11), we numerically calculate the entanglement between two dipole–dipole coupled atoms interacting with a thermal field via a two-photon process and illustrate it in Figs.1–4. For comparison, we also consider the case of entanglement between two dipole–dipole coupled atoms interacting with a thermal field via one-photon process which is shown in Ref.[16].

In Fig.1, we plot the negativity as a function of γt for the mean photon number $\bar{n} = 0$ (vacuum state) and the dipole coupling constant $\Omega = 0.1\gamma$. Figure 1(a) displays the case where the atoms interact with a single-mode field via a one-photon process. While Fig.1(b) illustrates the case where the atoms interact with a single-mode field via a two-photon process. Comparing Fig.1(a) with (b), we find that they are clearly different. In the latter, the number of the peaks is larger than that in the former. Moreover, the maximum value of ε in the case of two-photon process ($\varepsilon = 0.9970$) is greater than that in the case of one-photon process ($\varepsilon = 0.9956$). Because the two-photon process is a nonlinear one, the expressions given by Eqs.(10) and (11) are complicated. When this nonlinear process is involved, the two-atom entanglement

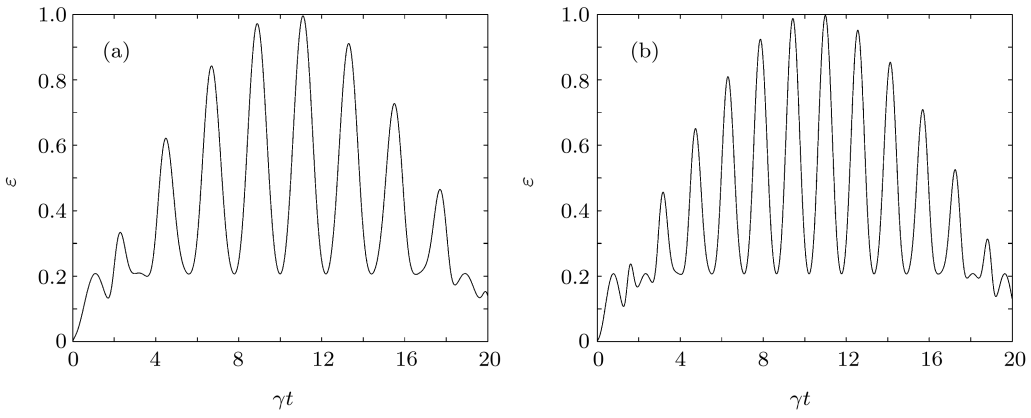


Fig.1. The negativity as a function of γt for the mean photon number $\bar{n} = 0$ and the dipole coupling constant $\Omega = 0.1\gamma$: (a) the case where the atoms interact with a single-mode field via a one-photon process; (b) the case where the atoms interact with a single-mode field via a two-photon process.

produced in the two-photon process is stronger than that produced in the one-photon process. Moreover, due to the realization of a two-photon maser oscillator,[19−21] it is of great practical significance to study entanglement via a two-photon process in experiments.

The time evolution of the negativity is shown in Fig.2 for the mean photon number $\bar{n} = 10$ and the dipole coupling constant $\Omega = 0.1\gamma$. Figure 2(a) displays the case where the atoms interact with a single-mode field via a one-photon process. While Fig.2(b) illustrates the case where the atoms interact with a single-mode field via a two-photon process. A comparison of Fig.2 with Fig.1 shows that the degree of entanglement between atoms interacting with a vacuum field ($\bar{n} = 0$) is greater than that between atoms interacting with a thermal field ($\bar{n} \neq 0$) both in the case of one-photon process and two-photon process. However, clearly, the entanglement in the case of two-photon process is larger than that in the case of one-photon process(see Figs.2(a) and (b)).

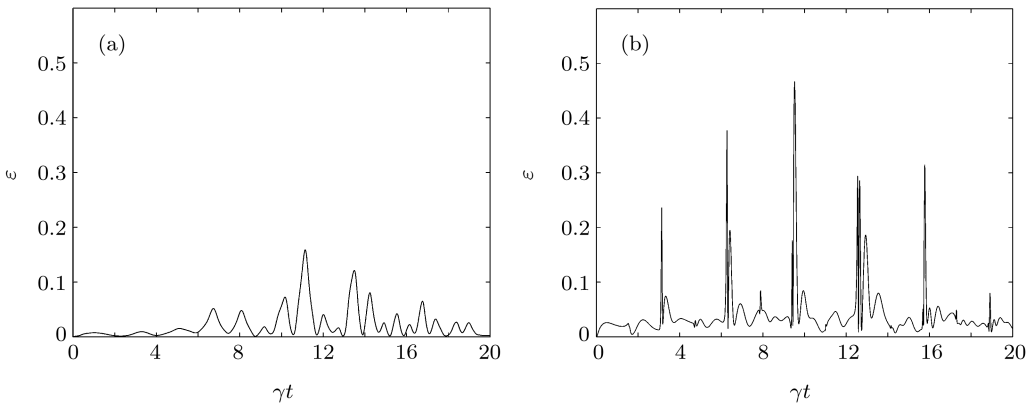


Fig.2. The negativity as a function of γt for the mean photon number $\bar{n} = 10$ and the dipole coupling constant $\Omega = 0.1\gamma$: (a) the case where the atoms interact with a single-mode field via a one-photon process; (b) the case where the atoms interact with a single-mode field via a two-photon process.

Figure 3 displays the negativity as a function of γt for the mean photon number $\bar{n} = 30$ and the dipole coupling constant $\Omega = 0.1\gamma$. Figure 3(a) displays the case where the atoms interact with a single-mode field via a one-photon process. While Fig.3(b) illustrates the case where the atoms interact with a single-mode field via a two-photon process. From Fig.3(a) and Fig.2(a), we can find that the bigger the thermal noise is, the weaker the entanglement in the case of one-photon process. However, interestingly, a remarkable amount of entanglement between the atoms still remains at certain times even for a big noisy thermal field ($\bar{n} = 30$) in the case of two-photon process (see Fig.3(b)).

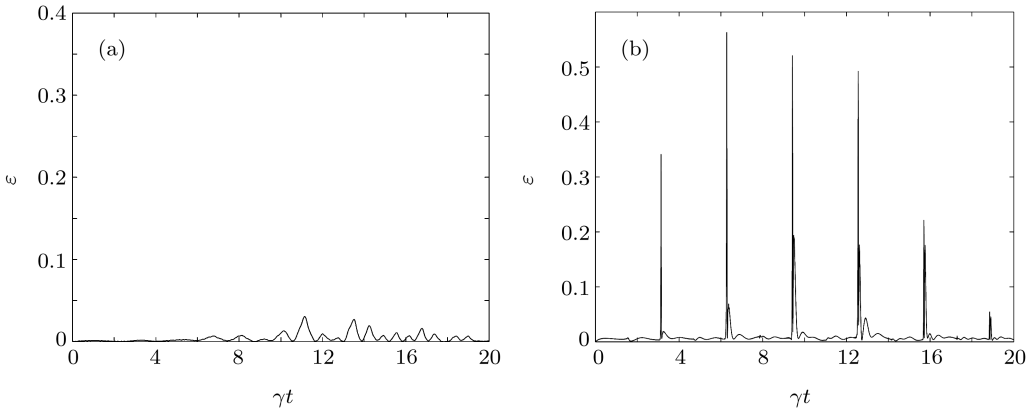


Fig.3. The negativity as a function of γt for the mean photon number $\bar{n} = 30$ and the dipole coupling constant $\Omega = 0.1\gamma$: (a) the case where the atoms interact with a single-mode field via a one-photon process; (b) the case where the atoms interact with a single-mode field via a two-photon process.

Figure 4 displays the negativity as a function of γt for the mean photon number $\bar{n} = 30$ and different values of the dipole coupling constant $\Omega = 0.3\gamma$, $\Omega = 0.1\gamma$, $\Omega = 0.01\gamma$, $\Omega = 0.001\gamma$ respectively. From these figures, we can see that the bigger the dipole coupling constant, the stronger the entanglement at some times. When $\Omega = 0.3\gamma$, the maximum value of ε is 0.78; when the dipole–dipole interaction is very small ($\Omega = 0.001\gamma$), the amount of entanglement is very weak and approximately tends to zero (see Fig.4(d)). Physically, dipole–dipole interaction can be enhanced by reducing the relative distance between two atoms or increasing the dipole polar moment for each atom.[22] The advantage of this scheme is that the relative distance between two atoms can be controlled. The present atom trapping and cooling techniques can trap two atoms at distances of order of a resonant wavelength.[23−25] In the experiment, these may provide us with a potential way to obtain the high amount of entanglement between the two atoms for some chosen system parameters under the case of big noisy thermal field.

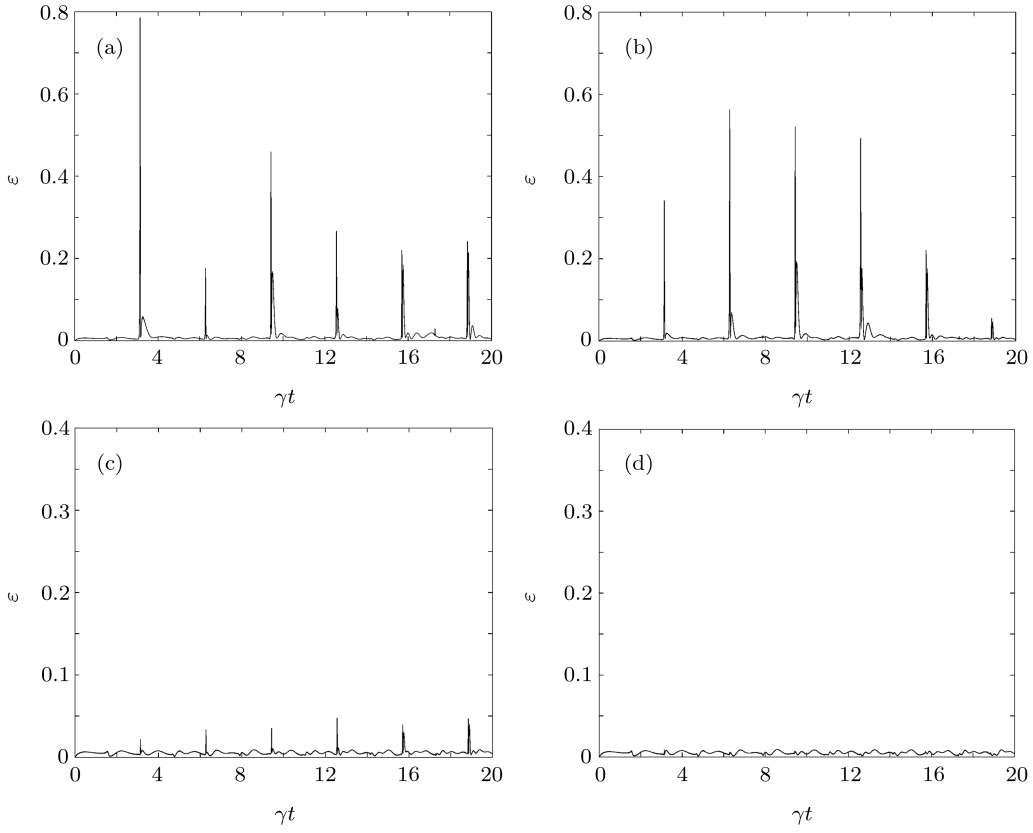


Fig.4. The negativity as a function of γt for the mean photon number $\bar{n} = 30$ and different values of the dipole coupling constant: (a) $\Omega = 0.3\gamma$; (b) $\Omega = 0.1\gamma$; (c) $\Omega = 0.01\gamma$; (d) $\Omega = 0.001\gamma$.

4. Conclusion

This paper studies entanglement between two dipole–dipole coupled atoms interacting with a thermal field via a two-photon process. It shows that the entanglement is dependent on the mean photon number of the thermal field and the dipole–dipole interaction. The results also show that the atom–atom entanglement through two-photon process is larger than that through one-photon process and a remarkable amount of entanglement between the atoms still remains at certain times even for a very highly noisy thermal field.

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