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Resonant frequencies of massless scalar field in rotating black-brane spacetime*

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This paper investigates the resonant frequencies of the massless scalar field in the near extremal Kerr-like black-brane spacetime. It is shown that the different angular quantum number will present different resonant frequencies. It is also shown that the real part of the resonant frequencies increases as the compact dimensions parameter μ_i increases, but the magnitude of the imaginary part decreases as μ_i increases.

Keywords: resonant frequency, black brane, scalar field
PACC: 0470, 0430, 0470, 9760

1. Introduction

It is believed that one can directly identify the existence of a black hole by comparing quasinormal modes (QNMs) with the gravitational waves observed in the universe because QNMs are entirely fixed by the structure of the background spacetime and irrelevant of the initial perturbations.[1,2] Meanwhile, it was argued that the study of QNMs may lead to a deeper understanding of the thermodynamic properties of black holes in loop quantum gravity,[3,4] since the real part of quasinormal frequencies with a large imaginary part for the scalar field in the Schwarzschild black hole equals to the Barbero–Immirzi parameter,[3−5] a factor introduced by hand in order that the loop quantum gravity reproduces correct entropy of the black hole. It was also argued that the QNMs of Anti-de Sitter (AdS) black holes have a direct interpretation in terms of the dual conformal field theory (CFT).[6−8] According to the AdS/CFT correspondence, a large static black hole in asymptotically AdS spacetime corresponds to a (approximately) thermal state in CFT, and the decay of the test field in the black-hole spacetime corresponds to the decay of the perturbed state in CFT. The dynamical timescale for the return to the thermal equilibrium can be done in the AdS spacetime, and then translated into the CFT by using the AdS/CFT correspondence. Thus, QNMs for various black holes have been studied extensively.[9−28]

Nowadays, it seems that the superstring theory is the most promising candidate for a consistent quantum theory of gravitation. The purpose of this paper is to study analytically the resonant frequencies of the massless scalar field in the near extremal Kerr-like black brane by using Detweiler's method[29] and present some results numerically.

The organization of this paper is as follows. In Section 2 the resonant frequencies of massless scalar field are studied. The last section is devoted to a summary.

2. Resonant frequencies of massless scalar field in rotating black-brane spacetime

In Boyer–Lindquist-type coordinates the Kerr-like black brane is described by[30]

$$\begin{aligned} \mathrm{d}s^2 = & -\frac{\Delta - a^2\sin^2\theta}{\Sigma}\mathrm{d}t^2 \\ & + \frac{(r^2+a^2)^2 - \Delta a^2\sin^2\theta}{\Sigma}\sin^2\theta\mathrm{d}\varphi^2 \\ & - \frac{2a(r^2+a^2-\Delta)\sin^2\theta}{\Sigma}\mathrm{d}t\mathrm{d}\varphi + \frac{\Sigma}{\Delta}\mathrm{d}r^2 \\ & + \Sigma\mathrm{d}\theta^2 + \mathrm{d}x^i\mathrm{d}x_i, \end{aligned} \tag{1}$$

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with

$$\Sigma = r^2 + a^2\cos^2\theta, \qquad \Delta = r^2 + a^2 - 2Mr, \quad (2)$$

where the x^i are the coordinates of the compact dimensions, and i runs from 1 to p. This metric describes a rotating black brane in an asymptotically flat, vacuum space-time with mass and angular momentum proportional to M and Ma, respectively. The event horizon is located at $r = r_+$, such that $\Delta|_{r=r_+} = 0$.

The massless scalar field Φ in the spacetime is described by

$$\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}\, g^{\mu\nu}\frac{\partial \Phi}{\partial x^\nu}\right) = 0\,, \quad (3)$$

where g is the determinant of the metric. We consider the ansatz $\Phi = \exp(-\mathrm{i}\omega t + \mathrm{i}m\varphi + \mathrm{i}\mu_i x^i)R(r)S(\theta)$, which is substituted into Eq.(3). Then we obtain the separated equations

$$\frac{1}{\sin\theta}\left(\frac{\mathrm{d}}{\mathrm{d}\theta}\sin\theta\frac{\mathrm{d}S(\theta)}{\mathrm{d}\theta}\right) + \left[a^2(\omega^2-\mu^2)\cos^2\theta - m^2\csc^2\theta + A_{lm}\right]S(\theta) = 0, \quad (4)$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta\frac{\mathrm{d}R(r)}{\mathrm{d}r}\right) + \left\{\frac{\left[\omega(r^2+a^2) - ma\right]^2}{\Delta} - \lambda - \mu^2 r^2\right\}R(r) = 0, \quad (5)$$

where A_{lm} is a constant of separation, $\lambda := A_{lm} - 2m\omega a + \omega^2 a^2$, and $\mu^2 = \Sigma\mu_i^2$. To simplify the study, introducing an usual tortoise coordinate $\mathrm{d}r_* = \dfrac{r^2+a^2}{\Delta}\mathrm{d}r$ and resolving the equation in the form

$$R(r) = \frac{Y(r)}{\sqrt{r^2+a^2}}, \quad (6)$$

we obtain the Schrödinger-like wave equation

$$\frac{\mathrm{d}^2Y(r)}{\mathrm{d}r_*^2} + (\omega^2 - V)Y(r) = 0, \quad (7)$$

with

$$V = -\frac{\mathrm{d}Z}{\mathrm{d}r_*} + Z^2 + \omega^2 - \frac{\Delta}{(r^2+a^2)^2}\left\{\frac{\left[\omega(r^2+a^2)-ma\right]^2}{\Delta} - \lambda - \mu^2 r^2\right\}, \quad (8)$$

where $Z = -\dfrac{r\Delta}{(r^2+a^2)^2}$. We will use Eqs.(4) and (7) to study the resonant frequencies of the rotating black brane.

For the rotating black brane, we have to solve Eq.(4) numerically following Leaver.[31] Defining $u = \cos\theta$, boundary condition for Eq.(4) is that $S(\theta)$ is regular at the regular singular points $u = \pm 1$ where the indices are given by $\pm m/2$ at $u = 1$ and $u = -1$. So a solution of Eq.(4) may be expressed as

$$S(u) = \exp(a\chi u)(1-u^2)^{|m|/2}\sum_{n=0}^{\infty} a_n^\theta (1+u)^n, \quad (9)$$

where the superscript θ denotes the association with the angular equation. The series coefficients are related by a three term recurrence relation and the boundary condition at $u = +1$ is satisfied only by its minimal solution sequence. The three-term recurrence relation can be written as

$$\begin{aligned} &\alpha_0^\theta a_1^\theta + \beta_0^\theta a_0^\theta = 0, \\ &\alpha_n^\theta a_{n+1}^\theta + \beta_n^\theta a_n^\theta + \gamma_n^\theta a_{n-1}^\theta = 0, \quad (n \geq 1), \end{aligned} \quad (10)$$

with

$$\begin{aligned} \alpha_n^\theta &= -2(n+1)(n+|m|+1), \\ \beta_n^\theta &= n(n-1) + 2n(|m|+1-2a\chi) \\ &\quad -(2a\chi - |m|)(|m|+1) - (a^2\chi^2 + A_{lm}), \\ \gamma_n^\theta &= 2a\chi(n+|m|), \end{aligned} \quad (11)$$

where $\chi = \sqrt{\omega^2 - \mu^2}$. We will obtain the minimal solution if the angular separation constant A_{lm} is a root of the continued fraction equation

$$0 = \beta_0^\theta - \frac{\alpha_0^\theta\gamma_1^\theta}{\beta_1^\theta -}\frac{\alpha_1^\theta\gamma_2^\theta}{\beta_2^\theta -}\frac{\alpha_2^\theta\gamma_3^\theta}{\beta_3^\theta -}\frac{\alpha_3^\theta\gamma_4^\theta}{\beta_4^\theta -}\cdots. \quad (12)$$

The QNMs of the Kerr-like black brane are defined to be the modes with purely ingoing waves at the event horizon and purely outgoing waves at infinity.[1] Then, the boundary conditions on wave function $Y(r)$ at the horizon ($r = r_+$) and infinity ($r \to +\infty$) can be expressed as

$$Y(r) \sim \begin{cases} (r-r_+)^{-\mathrm{i}\sigma_+} & r \to r_+, \\ \mathrm{e}^{\mathrm{i}\chi r} r^{\mathrm{i}\alpha} & r \to +\infty, \end{cases} \quad (13)$$

where $\sigma_\pm = \dfrac{1}{r_+ - r_-}\left[(r_\pm^2 + a^2)\omega - ma\right]$ and $\alpha = \dfrac{(\chi^2+\omega^2)(r_+^2+a^2)}{2\chi r_+}$.

By introducing

$$\begin{aligned} Y(r) &= \sqrt{r^2+a^2}(r-r_+)^{-\mathrm{i}\sigma_+}(r-r_-)^{\mathrm{i}\sigma_-} \\ &\quad \times \exp(\mathrm{i}\chi(r-r_-))\,\Psi, \\ x &= \frac{r-r_+}{r_+}, \quad d = \frac{r_+-r_-}{r_+}, \\ \eta &= -\frac{\omega^2+\chi^2}{2\chi}, \quad \Omega_H = \frac{a}{r_+^2+a^2}, \\ \tau &= \frac{r_+^2+a^2}{2r_+}(\omega - m\Omega_H), \end{aligned} \tag{14}$$

Eq.(7) can be expressed as

$$\begin{aligned} &x(x+d)\frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} - \left[\frac{2\mathrm{i}\hat{\chi}}{r_+}x^2 - (B_2 - 2\mathrm{i}\hat{\chi}d)x\right. \\ &\left. - \left(\frac{B_1}{r_+} + B_2 d\right)\right]\frac{\mathrm{d}\Psi}{\mathrm{d}x} - \left[(2\eta + \mathrm{i}B_2)\hat{\chi}x - B_3\right. \\ &\left. + \mathrm{i}\hat{\chi}\left(\frac{B_1}{r_+} + B_2 d\right)\right]\Psi = 0, \end{aligned} \tag{15}$$

where $\hat{\chi} = \chi r_+$ and

$$\begin{aligned} B_1 &= (2\mathrm{i}\omega - 1)\mathrm{d}r_+ - 4\mathrm{i}\tau, \\ B_2 &= 2(1 - \mathrm{i}\omega), \\ B_3 &= (r_+^2+a^2)\omega^2 - \mathrm{i}\omega + \hat{\chi}^2 - A_{lm}. \end{aligned} \tag{16}$$

For $|\omega| > \mu_i$, the long-lived QNMs can be obtained analytically by using an approximation for the Schrödinger-like wave equation that is valid for $a \approx M$ and $\omega \approx m\Omega_H$. Let us first study Eq.(15) for large radii, i.e., in the limit when $x \gg \max(d, \tau)$. Then, this equation is approximated by

$$\begin{aligned} &x^2\frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} - \left[\frac{2\mathrm{i}\hat{\chi}}{r_+}x^2 - (B_2 - 2\mathrm{i}\hat{\chi}d)x\right]\frac{\mathrm{d}\Psi}{\mathrm{d}x} \\ &- \left[(2\eta + \mathrm{i}B_2)\hat{\chi}x - B_3 + \mathrm{i}\hat{\chi}\left(\frac{B_1}{r_+} + B_2 d\right)\right]\Psi \\ &= 0. \end{aligned} \tag{17}$$

A solution of Eq.(17) that satisfies the boundary condition (13) can be expressed in terms of confluent hypergeometric function[32]

$$\begin{aligned} \Psi &= Ax^{-1/2+\mathrm{i}(\omega+\hat{\chi}d+\delta)}M(1/2 \\ &\quad +\mathrm{i}(\hat{\chi}d - \eta + \delta), 1 + 2\mathrm{i}\delta, 2\mathrm{i}\hat{\chi}x) \\ &\quad +B(\delta \to -\delta), \end{aligned} \tag{18}$$

where A and B are constants, $\delta = \{(1+a^2+2r_+^2)\omega^2 - 1/4 - A_{ml} - \mu^2 r_+^2 - 2[a^2 + (r_+ - 1)^2]\omega\chi + \chi[2am + (1-2r_+)^2\chi]\}^{1/2}$, and the notation $(\delta \to -\delta)$ means 'replace δ by $-\delta$ in the preceding term'.

We then try to find a solution for Eq.(15) that is valid in the region close to the black-brane's horizon, i.e. in the case when $x \ll 1$. The equation (15) is simplified to

$$\begin{aligned} &x(x+d)\frac{\mathrm{d}^2\Psi}{\mathrm{d}x^2} + \left[(B_2 - 2\mathrm{i}\hat{\chi}d)x + \left(\frac{B_1}{r_+} + B_2 d\right)\right]\frac{\mathrm{d}\Psi}{\mathrm{d}x} \\ &+ \left[B_3 - \mathrm{i}\hat{\chi}\left(\frac{B_1}{r_+} + B_2 d\right)\right]\Psi = 0. \end{aligned} \tag{19}$$

The solution of this equation can be written in terms of hypergeometric function[32]

$$\begin{aligned} \Psi &= {}_2F_1(1/2 - \mathrm{i}(\omega + \hat{\chi}d + \delta), 1/2 \\ &\quad -\mathrm{i}(\omega + \hat{\chi}d - \delta), 1 - 4\mathrm{i}\tau/d, -x/d). \end{aligned} \tag{20}$$

The solution has the desired 'purely ingoing wave' behaviour close to the event horizon since ${}_2F_1 \to 1$ as $x \to 0$.

The solutions (18) and (20) can be matched in the overlap region. Noting the fact that $M(1/2 + \mathrm{i}(\hat{\chi}d - \eta + \delta), 1 + 2\mathrm{i}\delta, 2\mathrm{i}\hat{\chi}x) = 1$ as $x \to 0$, we find that the $x \to 0$ limit of Eq.(18) is given by

$$\Psi \to Ax^{-1/2+\mathrm{i}(\omega+\hat{\chi}d+\delta)} + B(\delta \to -\delta). \tag{21}$$

Similarity, for $x/d \to \infty$, Eq.(20) becomes

$$\Psi \to \frac{\Gamma(1-4\mathrm{i}\tau/d)\Gamma(2\mathrm{i}\delta)d^{\frac{1}{2}-\mathrm{i}(\omega+\hat{\chi}d+\delta)}}{\Gamma(1/2 - \mathrm{i}(\omega+\hat{\chi}d-\delta))\Gamma(1/2+\mathrm{i}(\omega+\hat{\chi}d+\delta) - 4\mathrm{i}\tau/d)}x^{-\frac{1}{2}+\mathrm{i}(\omega+\hat{\chi}d+\delta)} + B(\delta \to -\delta). \tag{22}$$

We can extract constants A and B by matching Eqs.(21) and (22), which are given by

$$\begin{aligned} A &= \frac{\Gamma(1-4\mathrm{i}\tau/d)\Gamma(2\mathrm{i}\delta)d^{\frac{1}{2}-\mathrm{i}(\omega+\hat{\chi}d+\delta)}}{\Gamma(1/2 - \mathrm{i}(\omega+\hat{\chi}d-\delta))\Gamma(1/2+\mathrm{i}(\omega+\hat{\chi}d+\delta) - 4\mathrm{i}\tau/d)}, \\ B &= \frac{\Gamma(1-4\mathrm{i}\tau/d)\Gamma(-2\mathrm{i}\delta)d^{\frac{1}{2}-\mathrm{i}(\omega+\hat{\chi}d-\delta)}}{\Gamma(1/2 - \mathrm{i}(\omega+\hat{\chi}d+\delta))\Gamma(1/2+\mathrm{i}(\omega+\hat{\chi}d-\delta) - 4\mathrm{i}\tau/d)}. \end{aligned} \tag{23}$$

On the other hand, by approximating Eq.(18) for $x \to \infty$ we get the amplitudes

$$Z^{\rm in} = \frac{A\Gamma(1+2{\rm i}\delta)}{\Gamma(1/2-{\rm i}(\hat{\chi}d-\eta-\delta))}(-2{\rm i}\hat{\chi})^{-\frac{1}{2}-{\rm i}(\hat{\chi}d-\eta+\delta)} + B(\delta\to-\delta). \tag{24}$$

To find the resonant frequencies of the rotating black brane we now impose $Z_{\rm in}=0$, i.e.,

$$-\frac{\alpha(\hat{\chi},\delta)}{\alpha(\hat{\chi},-\delta)} = (-2{\rm i}\hat{\chi}d)^{2{\rm i}\delta}\frac{\Gamma(1/2+{\rm i}(\omega+\hat{\chi}d+\delta)-4{\rm i}\tau/d)}{\Gamma(1/2+{\rm i}(\omega+\hat{\chi}d-\delta)-4{\rm i}\tau/d)}, \tag{25}$$

where

$$\alpha(\hat{\chi},\delta) = \frac{\Gamma(2{\rm i}\delta)\Gamma(1+2{\rm i}\delta)}{\Gamma(1/2-{\rm i}(\omega+\hat{\chi}d-\delta))\Gamma(1/2-{\rm i}(\hat{\chi}-\eta-\delta))}. \tag{26}$$

The left-hand side (LHS) of the Eq.(25) has a well-defined limit as $r_+\to r_-$ and $\omega\to m\Omega_H$ for $|\omega|>\mu_i$. For the case that δ is purely real, we represent the expression of LHS by

$${\rm LHS} = q_s\exp({\rm i}\xi_s), \tag{27}$$

with

$$\begin{aligned} q_s &= \left|\frac{-\alpha(\hat{\chi},\delta)}{\alpha(\hat{\chi},-\delta)}\right| \\ &= \sqrt{\frac{\cosh[\pi(\omega+\hat{\chi}d-\delta)]}{\cosh[\pi(\omega+\hat{\chi}d+\delta)]}\frac{\cosh[\pi(\hat{\chi}d-\eta-\delta)]}{\cosh[\pi(\hat{\chi}d-\eta+\delta)]}}, \\ \xi_s &= \arg\left[\frac{-\alpha(\hat{\chi},\delta)}{\alpha(\hat{\chi},-\delta)}\right], \end{aligned} \tag{28}$$

where we used the formula $|\Gamma(1/2+{\rm i}y)|^2 = \dfrac{\pi}{\cosh\pi y}$.[32] Meanwhile, we note that we can not get a consistent solution unless $\tau/d\to\infty$ as $r_+\to r_-$. Then, in this limiting case, using Stirling's formula,[32] we find that the right-hand side (RHS) of Eq.(25) can be written as

$${\rm RHS} = (-8\hat{\chi}\tau)^{2{\rm i}\delta}. \tag{29}$$

From above discussion we know that the resonant frequencies must be the solutions of the following equation

$$f(\omega) = (-8\hat{\chi}\tau)^{2{\rm i}\delta} - q_s\exp({\rm i}\xi_s) = 0. \tag{30}$$

Letting

$$-8\hat{\chi}\tau = \varrho\exp({\rm i}\phi), \tag{31}$$

we find that the solutions of Eq.(30) can be expressed as

$$\begin{aligned} &\varrho = \exp((\xi_s-2n\pi)/2\delta), \\ &\phi = -\frac{1}{2\delta}\ln q_s, \qquad (\delta \text{ is purely real}), \end{aligned} \tag{32}$$

where n is any integer number. Since we always assume that $\omega\approx m\Omega_+$, the long lived QNMs should well approximate to

$$\begin{aligned} \omega \approx\ & m\Omega_H - \frac{\exp((\xi_s-2n\pi)/2\delta)}{2\sqrt{m^2-\mu^2}}\cos\left|\frac{\ln q_s}{2\delta}\right| \\ & -i\frac{\exp((\xi_s-2n\pi)/2\delta)}{2\sqrt{m^2-\mu^2}}\sin\left|\frac{\ln q_s}{2\delta}\right|. \end{aligned} \tag{33}$$

The imaginary part $-{\rm i}\dfrac{\exp((\xi_s-2n\pi)/2\delta)}{2\sqrt{m^2-\mu^2}}\sin\left|\dfrac{\ln q_s}{2\delta}\right|$ is always negative because $\left|\dfrac{\ln q_s}{2\delta}\right| < \pi$ for the cases that we are interesting. We present some values of $\left|\dfrac{\ln q_s}{2\delta}\right|$ and resonant frequencies for the scalar field with the purely real δ in the Table 1.

Table 1. Values of $\left|\dfrac{\ln q_s}{2\delta}\right|$ and resonant frequencies of the massless scalar field with the purely real δ and $n=0$.

	$l=m=2$		$l=m=3$		$l=m=4$	
μ_i	$\dfrac{\ln q_s}{2\delta}$	ω	$\dfrac{\ln q_s}{2\delta}$	ω	$\dfrac{\ln q_s}{2\delta}$	ω
0.0	3.14089	2.07840–0.000055i	3.14127	3.20122–0.000066i	3.14146	4.10689–0.000014i
0.1	3.14090	2.07816–0.000055i	3.14127	3.20106–0.000065i	3.14147	4.10683–0.000014i
0.2	3.14091	2.07743–0.000053i	3.14127	3.20058–0.000064i	3.14147	4.10665–0.000013i
0.3	3.14094	2.07621–0.000049i	3.14128	3.19978–0.000063i	3.14147	4.10636–0.000013i
0.4	3.14098	2.07451–0.000046i	3.14129	3.19866–0.000061i	3.14147	4.10595–0.000013i

Table 1. (*Continued*)

0.5	3.14102	2.07234–0.000041i	3.14130	3.19722–0.000058i	3.14147	4.10543–0.000013i
0.6	3.14107	2.06970–0.000036i	3.14131	3.19547–0.000055i	3.14148	4.10479–0.000012i
0.7	3.14113	2.06659–0.000031i	3.14133	3.19340–0.000051i	3.14148	4.10403–0.000012i
0.8	3.14119	2.06304–0.000026i	3.14135	3.19101–0.000047i	3.14149	4.10316–0.000011i
0.9	3.14125	2.05905–0.000020i	3.14136	3.18832–0.000043i	3.14149	4.10218–0.000010i
1.0	3.14131	2.05464–0.000015i	3.14138	3.18532–0.000039i	3.14150	4.10109–0.000010i
1.1	3.14137	2.04983–0.000011i	3.14140	3.18202–0.000035i	3.14150	4.09989–0.000009i
1.2	3.14142	2.04464–0.000008i	3.14142	3.17841–0.000030i	3.14151	4.09858–0.000008i
1.3	3.14147	2.03910–0.000005i	3.14144	3.17451–0.000026i	3.14152	4.09716–0.000008i
1.4	3.14151	2.03325–0.000003i	3.14146	3.17031–0.000022i	3.14152	4.09564–0.000007i
1.5	3.14154	2.02715–0.000002i	3.14148	3.16582–0.000019i	3.14153	4.09402–0.000006i

3. Summary

The Schrödinger-like wave equation for the massless scalar field in the Kerr-like black-brane spacetime is obtained. The resonant frequencies of the massless scalar field in the near extremal Kerr-like black brane are investigated by solving the equation with the appropriate boundary conditions, and the analytical result is expressed by Eq.(33) and some numerical results are presented in the Table 1. It is shown that the different angular quantum number will present different resonant frequencies for the case that δ is purely real. It is also shown that the real part of the frequencies increases as the compact dimensions parameter μ_i increases, but the magnitude of the imaginary part decreases as μ_i increases. It is obvious that the resonant frequencies presented in the Table 1 are the long-lived modes which shows that the Detweiler's approximation is likely to be accurate for this case.

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