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Generation of various multiatom entangled graph states via resonant interactions*

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In this paper, a scheme for generating various multiatom entangled graph states via resonant interactions is proposed. We investigate the generation of various four-atom graph states first in the ideal case and then in the case in which the cavity decay and atomic spontaneous emission are taken into consideration in the process of interaction. More importantly, we improve the possible distortion of the graph states coming from cavity decay and atomic spontaneous emission by performing appropriate unitary transforms on atoms. The generation of multiatom entangled graph states is very important for constructing quantum one-way computer in a fault-tolerant manner. The resonant interaction time is very short, which is important in the sense of decoherence. Our scheme is easy and feasible within the reach of current experimental technology.

Keywords: graph state, resonant interaction, cavity decay, spontaneous emission
PACC: 0367

1. Introduction

Entanglement is a unique phenomenon in quantum mechanics and becomes an important resource in quantum communication and quantum computation. Many applications of entanglement have been proposed, such as quantum dense coding,[1] quantum teleportation,[2] quantum key distribution,[3] etc. With the development of quantum information science, the advantages of multipartite entangled states become more and more prominent. Recently, multipartite entangled graph states attract much attention in the field of physics.[4−6] Graph states are a family of multipartite entanglement states, which comprises many important multipartite entangled states, i.e. multipartite Greenberger–Horne–Zeilinger (GHZ) states, cluster states, various error correction codes and so on. Therefore graph states have some remarkable properties and important applications in quantum communication and quantum computation: (1) they can be regarded as a useful resource for many multipartite entangled states; (2) they might violate a unique three-setting Bell inequality but others cannot violate it;[7] (3) they can be used in the one-way quantum computer[8] as graph-state model, which can offer a decomposition of a quantum algorithm in terms of alternative elementary primitives;[9] (4) all codewords in the standard quantum error correcting codes, which are very important for protecting quantum states from decohering in quantum computation, correspond to the graph states,[10] etc.

Owing to the advantages of multipartite entangled graph states, the generation of graph states is necessary for developing quantum information and constructing the quantum computer with single-qubit measurement. Many schemes for generating multipartite entangled GHZ states,[11,12] W states,[13,14] cluster states[15−20] and multipartite entangled graph states [4,6,21−23] have been proposed. In this paper, we mainly propose a scheme for generating various multiatom entangled graph states via resonant interactions. This idea is an important application of Zheng's proposal[24] for implementing quantum logical gate, where a standard controlled Z gate based on a single

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resonant interaction is designed; while we can improve the fidelity to 1.0 for the non-ideal case by using appropriate interactions between a single atom and a single cavity. Cavity quantum electrodynamics (QED) technique is a promising candidate for realizing the quantum processing. Especially, the resonant interaction time is very short, which is very important in the sense of decoherence. Meanwhile we take the cavity decay and atomic spontaneous emission into consideration, which is unavoidable in the real process of resonant interaction. Thus our scheme would be helpful for practical experiment.

The remainder of the present paper is organized as follows. In Section 2, we first briefly review the concepts of graph and graph states, and take various four-party graph states as an example in the end. Then we introduce a detailed process for generating various four-atom graph states via the QED technology with and without cavity decay and atomic spontaneous emission separately. Moreover, we generalize our scheme to the case of arbitrary graph states. In Section 3, we give the discussion of the feasibility and conclusions drawn from the present study.

2. Generation of various graph states via resonant interactions

2.1. Definition and notation for graph states

We first review the concepts and notations of graph and graph state.[4,5] A graph G is comprised of n vertices and some edges. Each graph can be represented by a diagram in a plane, where each point represents a vertex and each arc joining two vertices not necessarily distinct represents an edge. That is to say, a (undirected, finite) graph is a pair $G = (V, E)$ of a finite set $V \subset N$ and a set $E \subset [V]^2$,[4] where V represent vertices and E represent edges. The elements of G are subsets of V with two elements each. If a graph has not any edge, we will define it as empty graph.

Each graph corresponds to a graph state, which is a certain pure quantum state in a Hilbert space $H = (C^2)^{\otimes V}$. Each vertex i can be defined as an operator

$$K_G^i = \sigma_x^i \prod_{j \in N_i} \sigma_z^j,$$

where σ_x and σ_z are Pauli matrices, and N_i denotes the set of neighbours of i. A graph state $|G\rangle$ is defined as a state which has the K_G^i as a stabilizing operator, i.e.

$$K_G^i |G\rangle = |G\rangle.$$

The multipartite entangled graph states can also be written as[5]

$$|G\rangle = \prod_{(i,j) \in E} P^{(i,j)} |+\rangle^{\otimes n}, \tag{1}$$

where $P^{(i,j)} = \left(I^{(i)} \otimes I^{(j)} + \sigma_z^{(i)} \otimes I^{(j)} + I^{(i)} \otimes \sigma_z^{(j)} - \sigma_z^{(i)} \otimes \sigma_z^{(j)}\right) \Big/ 2$, which turns out to be the following operators (in the computational basis):

$$|00\rangle_{i,j} \to |00\rangle_{i,j}, \tag{2a}$$

$$|01\rangle_{i,j} \to |01\rangle_{i,j}, \tag{2b}$$

$$|10\rangle_{i,j} \to |10\rangle_{i,j}, \tag{2c}$$

$$|11\rangle_{i,j} \to -|11\rangle_{i,j}. \tag{2d}$$

$|+\rangle^{\otimes n} = ((|0\rangle + |1\rangle)/\sqrt{2})^{\otimes n}$ is an n-qubit empty graph state corresponding to an empty graph. i and j are superscripts of two vertices (two qubits).

In terms of the expression of graph states, we can classify these states by the number of vertices, n. Four-party graph states are a class of simple ones, but they can reveal some particular properties of multipartite entangled graph states. Here we take the four-vertex graph and four-party graph states as example, which are shown in Fig.1 and Table 1. For (a) we can obtain a four-party cluster state with the help of local single-qubit operation, i.e. $|\phi\rangle = (|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle)/2$. For (c), the state is a standard four-party GHZ state, i.e. $|\varphi\rangle = (|0000\rangle + |1111\rangle)/\sqrt{2}$. According to analysis, one can see that the four-party entangled graph states include cluster state, GHZ state and others.

Table 1. Four-party entangled graph states corresponding to the six graphs of Fig.1.

number	four-party graph states
(a)	$(\|0000\rangle + \|0011\rangle + \|1100\rangle - \|1111\rangle)/2$
(b)	$(\|0000\rangle + \|0111\rangle + \|1010\rangle + \|1101\rangle)/2$
(c)	$(\|0000\rangle + \|1111\rangle)/\sqrt{2}$
(d)	$(\|0000\rangle + \|0101\rangle + \|1011\rangle - \|1110\rangle)/2$
(e)	$(\|0000\rangle + \|0111\rangle + \|1011\rangle - \|1100\rangle)/2$
(f)	$(\|0000\rangle + \|0011\rangle + \|0101\rangle - \|0110\rangle +$ $\|1001\rangle - \|1010\rangle - \|1100\rangle - \|1111\rangle)/2\sqrt{2}$

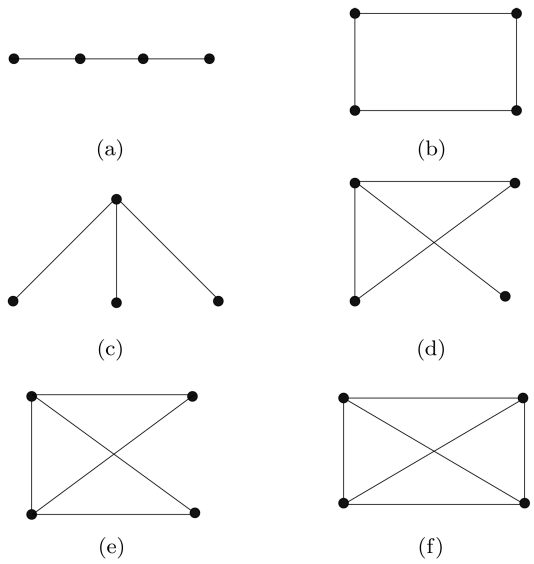


Fig.1. The sketch map of four-vertices graphs. From (a) to (f), they are in various forms and cannot overlap each other. Each graph corresponds to an entangled graph state.

2.2. Generation of four-atom graph states with ideal resonant interactions

In this section, we mainly investigate the generation of various four-atom graph states via ideal resonant interactions between two atoms and a single-mode cavity. The relevant three-level atomic structure used in this paper is shown in Fig.2. The third level $|i\rangle$ is not affected during the atom-cavity resonant interaction. The Hamiltonian of the atom-cavity resonant interaction can be expressed, in the interaction picture (assuming $\hbar = 1$), as [24]

$$H = g_1(a^\dagger S_1^- + aS_1^+) + g_2(a^\dagger S_2^- + aS_2^+), \qquad (3)$$

where g_1 and g_2 are the coupling strengths of atoms 1 and 2 with the cavity, respectively. $S^+ = |e\rangle\langle g|$, $S^- = |g\rangle\langle e|$, and $|g\rangle$ is a ground state of the atoms, $|e\rangle$ is an excited state of the atoms. $a^\dagger$ and a are the creation and annihilation operators for the cavity mode.

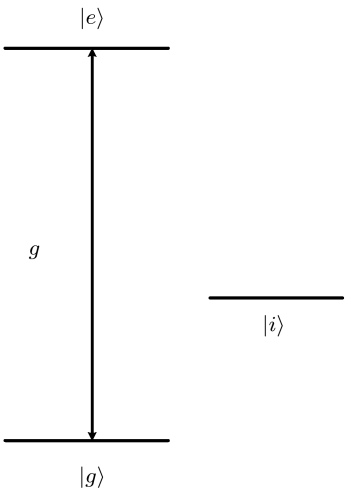


Fig.2. The level structure of the four identical atoms, where $|g\rangle$ is the ground state and $|e\rangle$ is the excited state. The cavity mode is resonantly coupled to the $|e\rangle \leftrightarrow |g\rangle$ transition. The third level $|i\rangle$ is not affected by the interaction.

To generate four-atom entangled graph states, we first prepare four identical atoms, which are all in the state $|\phi\rangle_i = (|g\rangle_i + |e\rangle_i)/\sqrt{2}$ $(i = 1, 2, 3, 4)$, and a vacuum cavity $|0\rangle_c$. So the initial state of the total system is

$$|\phi\rangle_{1234c} = [(|g\rangle + |e\rangle)/\sqrt{2}]_{1234}^{\otimes 4} \otimes |0\rangle_c. \qquad (4)$$

If one wants to generate (a) graph states (cluster state), we will send atoms 1 and 2 through the vacuum cavity simultaneously, obtaining the evolution

$$|eg\rangle_{12}|0\rangle_c \to \frac{g_1}{E}\left[\frac{1}{E}\left(g_1\cos(Et) + \frac{g_2^2}{g_1}\right)|eg\rangle_{12}|0\rangle_c + \frac{1}{E}g_2[\cos(Et) - 1]|ge\rangle_{12}|0\rangle_c - \mathrm{i}\sin(Et)|gg\rangle_{12}|1\rangle_c\right], \qquad (5a)$$

$$|ei\rangle_{12}|0\rangle_c \to [\cos(g_1t)|e\rangle_1|0\rangle_c - \mathrm{i}\sin(g_1t)|g\rangle_1|1\rangle_c]|i\rangle_2, \qquad (5b)$$

$$|gg\rangle_{12}|0\rangle_c \to |gg\rangle_{12}|0\rangle_c, \qquad (5c)$$

$$|gi\rangle_{12}|0\rangle_c \to |gi\rangle_{12}|0\rangle_c, \qquad (5d)$$

where $E = \sqrt{g_1^2 + g_2^2}$. At the same time, we make an adjustment such that $t = \pi/g_1$ and $g_2 = \sqrt{3}g_1$, which can be achieved by choosing coupling strengths and interaction time appropriately. Then we send atom 2 through a classical field tuned to the transition $|i\rangle_2 \to |e\rangle_2$. Thus, we have

$$|gg\rangle_{12} \rightarrow |gg\rangle_{12}, \tag{6a}$$
$$|ge\rangle_{12} \rightarrow |ge\rangle_{12}, \tag{6b}$$
$$|eg\rangle_{12} \rightarrow |eg\rangle_{12}, \tag{6c}$$
$$|ee\rangle_{12} \rightarrow -|ee\rangle_{12}. \tag{6d}$$

This is a standard controlled Z gate implemented by Zheng in Ref.[24]. These lead Eq.(5) to

$$|\phi\rangle_{1234} = (|gg\rangle_{12} + |ge\rangle_{12} + |eg\rangle_{12} - |ee\rangle_{12}) \otimes(|g\rangle_3 + |g\rangle_3) \otimes (|g\rangle_4 + |g\rangle_4)/4. \tag{7}$$

In succession, we will send atoms 2 and 3 through a vacuum cavity. The interaction between atoms and cavity is the same as expression (3) and the total evolution of the state for atoms 2 and 3 is the same as that for atoms 1 and 2. Finally, we send atoms 3 and 4 through a vacuum cavity and send atom 4 through a classical field as done for atoms 2 and 3. The state of atoms 1, 2, 3 and 4 becomes

$$|\varphi\rangle_{1234} = [(|gg\rangle_{12} + |eg\rangle_{12})(|gg\rangle_{34} + |ge\rangle_{34} + |eg\rangle_{34} - |ee\rangle_{34}) + (|ge\rangle_{12} - |ee\rangle_{12})(|gg\rangle_{34} + |ge\rangle_{34} - |eg\rangle_{34} + |ee\rangle_{34})]/4. \tag{8}$$

This state is a standard four-atom linear type entangled graph state. If we perform a Hadamard operation $|g\rangle \rightarrow (|g\rangle+|e\rangle)/\sqrt{2}$ and $|e\rangle \rightarrow (|g\rangle-|e\rangle)/\sqrt{2}$ on atoms 1 and 4, respectively, the state will become

$$|\psi\rangle_{1234} = (|gggg\rangle + |ggee\rangle + |eegg\rangle - |eeee\rangle)_{1234}/2, \tag{9}$$

which is just accordant with (a) graph state in Table 1.

The geometry of the experimental setting for the above interactions between atoms and a cavity is shown in Fig.3. Four atoms pass through a cavity consecutively with a certain velocity, and the distance between two nearest-neighbouring atoms is half the length of the cavity. We can control the resonant interactions between two atoms and the cavity by using the technology of "switch on" or "switch off".

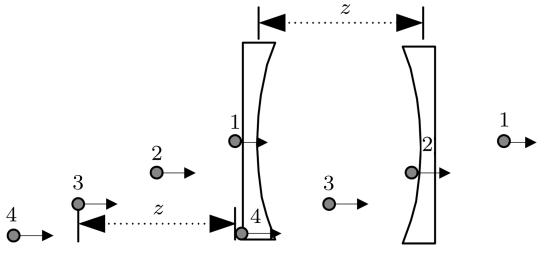


Fig.3. The geometry of the experimental setting.

Similarly, if one wants to generate (b), (c), (d), (e) or (f) atomic graph states, we can achieve them by making the above operations as expressed in Eqs.(3) to (7) on two atoms, which both connect an edge. Out of question, we can generate an arbitrary multiatom entangled graph state via the resonant interactions. Without loss of generality, here, we prepare an N-atom emanative type graph state, of which the graph is shown in Fig.4. We need send only two atoms 1 and i ($i = 2, 3, \cdots, N$) through a vacuum cavity, sequentially, and then perform appropriate single-qubit operations on them.

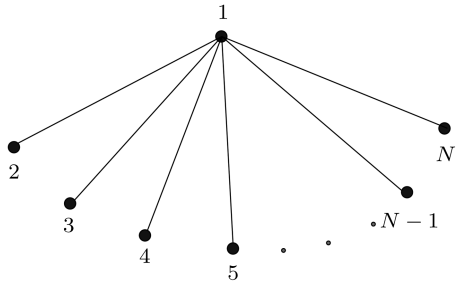


Fig.4. The graph of an N-party emanative graphs.

2.3. Generation of four-atom graph states with cavity decay and atomic spontaneous emission

In a real process of resonant interaction, the cavity decay and atomic spontaneous emission are both unavoidable. Thus we take them into consideration as done in Ref.[24], and the Hamiltonian of the atom-cavity interaction can be expressed as (under the condition that no photon is detected either from the spontaneous emission or from the leakage of a photon through the cavity mirror and assuming $\hbar = 1$)

$$H = g_1(a^+S_1^- + aS_1^+) + g_2(a^+S_2^- + aS_2^+) - \mathrm{i}\frac{\kappa}{2}a^+a - \mathrm{i}\frac{\tau}{2}\Sigma_{j=1}^2|e\rangle_j\langle e|, \tag{10}$$

where κ is the cavity decay rate and τ is the atomic spontaneous emission rate.

If we choose the coupling strength $g_2 = \sqrt{3}g_1$, and interaction time $t = \pi/g_1$, and set $\kappa = \tau = 0.1g_1$, after an appropriate operation $|i\rangle_2 \rightarrow |e\rangle_2$, we will have

$$|gg\rangle_{12} \rightarrow |gg\rangle_{12}, \tag{11a}$$
$$|ge\rangle_{12} \rightarrow |ge\rangle_{12}, \tag{11b}$$
$$|eg\rangle_{12} \rightarrow \mathrm{e}^{-\pi/20}|eg\rangle_{12}, \tag{11c}$$
$$|ee\rangle_{12} \rightarrow -\mathrm{e}^{-\pi/20}|ee\rangle_{12}. \tag{11d}$$

To generate (a) graph states, we also prepare four atoms and perform similar operations to those in the ideal case. But Eq.(8) will become

$$|\varphi\rangle_{1234} = \frac{1}{\sqrt{2+6\times \mathrm{e}^{-\pi/10}+6\times \mathrm{e}^{-\pi/5}+2\times \mathrm{e}^{-3\pi/10}}}\left[\left(|gg\rangle_{12}+\mathrm{e}^{-\pi/20}|eg\rangle_{12}\right)(|gg\rangle_{34}+|ge\rangle_{34} +\mathrm{e}^{-\pi/20}|eg\rangle_{34}-\mathrm{e}^{-\pi/20}|ee\rangle_{34}\right)+\mathrm{e}^{-\pi/20}\left(|ge\rangle_{12}-\mathrm{e}^{-\pi/20}|ee\rangle_{12}\right)(|gg\rangle_{34}+|ge\rangle_{34} -\mathrm{e}^{-\pi/20}|eg\rangle_{34}+\mathrm{e}^{-\pi/20}|ee\rangle_{34}\right)\Big]. \quad (12)$$

Then we perform a Hadamard transformation on atom 4, which leads Eq.(12) to

$$|\psi\rangle_{1234} = \frac{1}{\sqrt{1+3\times \mathrm{e}^{-\pi/10}+3\times \mathrm{e}^{-\pi/5}+\mathrm{e}^{-3\pi/10}}}\left[\left(|gg\rangle_{12}+\mathrm{e}^{-\pi/20}|eg\rangle_{12}\right)\left(|gg\rangle_{34}+\mathrm{e}^{-\pi/20}|ee\rangle_{34}\right) +\mathrm{e}^{-\pi/20}\left(|ge\rangle_{12}-\mathrm{e}^{-\pi/20}|ee\rangle_{12}\right)\left(|gg\rangle_{34}-\mathrm{e}^{-\pi/20}|ee\rangle_{34}\right)\Big]. \quad (13)$$

Here, the successful probability of obtaining Eq.(13) is $P=\dfrac{1+3(\mathrm{e}^{-\pi/10}+\mathrm{e}^{-\pi/5})+\mathrm{e}^{-3\pi/10}}{8}$. The fidelity is $F=\dfrac{[1+3(\mathrm{e}^{-\pi/20}+\mathrm{e}^{-\pi/10})+\mathrm{e}^{-3\pi/20}]^2}{8(1+3(\mathrm{e}^{-\pi/10}+\mathrm{e}^{-\pi/5})+\mathrm{e}^{-3\pi/10})}$.

Next, we discuss the process of eliminating the distortion of fidelity. We send atom 1 through a vacuum cavity. The Hamiltonian of the system can be expressed as (assuming $\hbar=1$)

$$H = \omega_{eg}S_z+\omega\left(a^\dagger a+\frac{1}{2}\right)+\frac{\Omega}{2}\left(S^+a+S^-a^\dagger\right) -\frac{\mathrm{i}\kappa}{2}a^\dagger a-\frac{\mathrm{i}\tau}{2}|e\rangle\langle e|, \quad (14)$$

where $S_z=(|e\rangle\langle e|-|g\rangle\langle g|)/2$, ω_{eg} is the transition frequency of the atom and ω is the cavity frequency. We set $\kappa=\tau$ and $\omega_{eg}=\omega$ and choose the time $\mathrm{e}^{-\frac{\kappa t}{2}}\sin\dfrac{\Omega t}{2}=\mathrm{e}^{-\frac{\pi}{20}}$, and then detect the cavity. If the cavity is in $|0\rangle$, the process is successful with a finite probability $\eta=\mathrm{e}^{-\pi/10}$. Then we send atom 3 through another vacuum cavity as done for atom 1, thus the total state of system becomes

$$|\psi\rangle_{1234} = \frac{1}{2\sqrt{1+\mathrm{e}^{-\pi/10}}}[(|gg\rangle_{12}+|eg\rangle_{12}) \times(|gg\rangle_{34}+|ee\rangle_{34}) +\mathrm{e}^{-\pi/20}(|ge\rangle_{12}-|ee\rangle_{12}) \times(|gg\rangle_{34}-|ee\rangle_{34})]. \quad (15)$$

Then we perform a Hadamard transformation on atom 1, and the state of system becomes

$$|\psi\rangle_{1234} = \frac{1}{\sqrt{2(1+\mathrm{e}^{-\pi/10})}}[|gg\rangle_{12}(|gg\rangle_{34}+|ee\rangle_{34}) +\mathrm{e}^{-\pi/20}|ee\rangle_{12}(|gg\rangle_{34}-|ee\rangle_{34})]. \quad (16)$$

Finally, we send atom 2 through a vacuum cavity as done for atom 1 (the interaction governed by expression (14)), which leads the state of all atoms to

$$|\psi\rangle_{1234} = \frac{1}{2}(|gggg\rangle+|ggee\rangle+|eegg\rangle -|eeee\rangle)_{1234}. \quad (17)$$

Obviously, we avoid the possible distortion of graph states resulting from the cavity decay and atomic spontaneous emission, i.e. the fidelity is still 1.0. However, the total successful probability is $P_{\mathrm{total}}=P\times\eta^3=\dfrac{\mathrm{e}^{-3\pi/10}[1+3(\mathrm{e}^{-\pi/10}+\mathrm{e}^{-\pi/5})+\mathrm{e}^{-3\pi/10}]}{8}$. The total successful probability is decreased as compared with the successful probability of obtaining a four-atom graph state with $F\neq 1$. Similarly, we can generate other n-atom graph states by using the above method.

3. Discussion and conclusions

For a resonant cavity, to generate multiatom graph states successfully, the relationship between the interaction time and the excited atom lifetime should be taken into consideration. The interaction time should be much shorter than that of atom radiation. Hence, atom with a sufficiently long excited lifetime should be chosen. For Rydberg atoms with principal quantum numbers 50 and 51, the radiative time is $T_1\simeq 3\times 10^{-2}$s.[25] From the analysis in Ref.[26], the interaction time T is about 2×10^{-4} s, which is much shorter than the atomic radiative time. So the condition can be satisfied by choosing Rydberg atoms. Our scheme requires that two atoms be simultaneously sent through a cavity, otherwise there will appear an error. Assume that during the generation, then one atom will enter the cavity $0.01t$ earlier than another atom where t is the time for which each atom stays in the cavity. We can obtain the error of fidelity to be

about 10^{-7} for generating (a) four-party graph state. Obviously in this case the operation is almost unaffected.

Furthermore one needs to reach the Lamb–Dicke regime in order to generate multiatom entangled graph states successfully. For the initial state of expression (4), in the Lamb–Dicke regime, the infidelity due to the spatial extension of the atomic wave function is about $\Delta \simeq (ka)^2\pi$, where k is the wave vector of the cavity mode and a is the spread of the atomic wave function. Setting $\Delta \simeq 0.01$, we have $a \simeq 0.01\lambda$, where λ is the wavelength of the cavity mode. If the atom trajectories cross the cavity with a deviation of less 0.1 degrees from its pre-determined direction, we can ensure that the fidelity is about 0.98 for generating (a) graph state. In order to maintain $g_2 = \sqrt{3}g_1$ in the process of atomic motion in the cavity, we can choose the parameter of cavity $z \leq 0.5z_0$, where $z_0 = \dfrac{\pi\omega^2}{\lambda}$ and $2z$ is the length of the cavity. We can obtain the error to be only about 10^{-3}. According to analysis and calculation, for generating other multiatom entangled graph states, the fidelities are all over 0.98. On the other hand, the efficiency and the dark counts of detector should be considered in current paper, for our scheme uses a single photon detector. The finite efficiency of the detector will reduce the successful probability, and the dark count will reduce the fidelity. However, the influence of dark count is about $5\% - 10\%$, which only reduces the fidelity slightly.[27] Therefore our scheme is feasible with the current cavity QED technology.

In conclusion, the scheme for generating multiatom graph states via resonant interactions is proposed. The interaction time is very short, which is very important in the sense of decoherence. For the ideal case, the successful probability and the fidelity are both perfect (equal to 1.0). For the real case, we can avoid the possible distortion of graph states resulting from the cavity decay and atomic spontaneous emission by using resonant interactions between one atom and a cavity and single-qubit transformations, that is to say, the fidelity is still 1.0. However, the successful probability of generation $P_{\text{total}} = \dfrac{\mathrm{e}^{-3\pi/10}[1 + 3(\mathrm{e}^{-\pi/10} + \mathrm{e}^{-\pi/5}) + \mathrm{e}^{-3\pi/10}]}{8}$. Our idea is simple and can be generalized to ion trap systems.

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