Vol 17 No 6, June 2008
1674-1056/2008/17(06)/1972-07

Chinese Physics B

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Generation of multipartite entangled states by cavity QED*

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(Received 18 September 2007; revised manuscript received 5 November 2007)

We propose the methods of generating multipartite entanglement by considering the interaction of a system of N two-level atoms in M cavities of high quality factor with a strong classical driving field. It is shown that, with the cavity detuning, the applied driving field detuning and vacuum Rabi coupling, we can produce an entangled coherent state in two single-mode cavities and generate the entangled coherent cluster states in two bimodal vacuum cavities. Tuning these parameters also allows us to acquire the anti-Jaynes–Cummings (AJC) interaction, with which we can generate the maximally two-photon entangled states, and the two-atom and the two-photon entangled cluster states.

Keywords: Cavity QED, entangled state, anti-Jaynes–Cummings, cluster state
PACC: 0365, 4250

1. Introduction

Entanglement plays a crucial role in quantum information processing.[1] Quantum algorithms (particularly, Shor's algorithm of finding the prime factors of an n-bit integer) exploit entanglement to speed up computation. In addition, quantum communication protocols use entangled states as a medium to send information. However, creating entangled states is a great challenge to both theoretical and experimental physicists. Recently, there is much interest in the quantum entanglement of many qubits. It has become clear for there to be several inequivalent classes of entangled states in a system shared by three and more parties.[2] In Ref.[3], Briegel and Raussendorf[3] introduced a class of entangled states, i.e. the cluster states, which can be regarded as a resource for Greenberger–Horne–Zeilinger (GHZ) states and are more immune to decoherence than the GHZ states.[4] In Ref.[5], the proof of Bell's theorem without the inequalities was given for cluster states, and a new Bell inequality was considered, which is maximally violated by the four-qubit cluster state, but it was not violated by the four-qubit GHZ state. On the other hand, cluster states have been shown to constitute a universal resource for quantum computation assisted by local measurement only.[6]

Some schemes for preparing the four-qubit cluster state[7−12] have been proposed in optical systems. In Ref.[7], Zou and Mathis proposed a scheme to generate a four-photon polarization-entangled cluster state only by using linear optical elements and a four-photon coincidence detection, but the successful probability is less than $\frac{1}{4}$. Because the coherent state can be used to encode quantum information on continuous variables,[13] in this paper we propose a scheme to generate the entangled coherent state in the following form:

$$|\Phi_{\pm}\rangle = N_{\pm}(|\alpha\rangle|\alpha\rangle \pm |-\alpha\rangle|-\alpha\rangle),$$

and

$$|\Psi_{\pm}\rangle = N_{\pm}(|\alpha\rangle|-\alpha\rangle \pm |-\alpha\rangle|\alpha\rangle), \qquad (1)$$

where $N_{\pm}$ is the normalization constants, and $|\alpha\rangle$ and $|-\alpha\rangle$ are two coherent states with the same amplitudes but a phase difference of π. We also generate the four-mode coherent cluster states as mentioned before.[14] In this paper, we propose a cluster-type four-qubit coherent state that is generated by using two bimodal QED cavities, two two-level Rydberg atoms that are driven by an external classical field respectively, a single-mode cavity, a Hadamard gate and an atomic ionization detector. Favourable features of our scheme are that the two bimodal QED cavities are vacuum,

*Project supported by the National Natural Science Foundation of China (Grant No 10774088), the Key Program of National Natural Science Foundation of China (Grant No 10534030) and the Funds from Qufu Normal University, China (Grant No XJ0621).

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and in the third cavity the photon-number-dependent part in the effective Hamiltonian is cancelled. Therefore, the system is insensitive to cavity decay and thermal field; our scheme is easily realizable by using the cavity QED techniques.

If the dressed-state basis is chosen, the purely anti-Jaynes–Cummings (AJC) interaction is demonstrated. In this system, we can generate the cluster state between two atoms and two photons by the AJC interaction in the following form:

$$|\text{cluster}\rangle = \frac{1}{2}(|e,e,1,1\rangle + |e,g,1,0\rangle + |g,e,0,1\rangle - |g,g,0,0\rangle), \quad (2)$$

where $|e\rangle$ and $|g\rangle$ are the excited and the ground states of the two atoms, while $|1\rangle$ and $|0\rangle$ present the single-photon cavity and the vacuum cavity, respectively. A number of other interesting situations can arise when the AJC interaction is chosen.

2. Theoretical model

We consider a spatially narrow bunch of N identical two-level atoms (transition frequency ω_0) in a single-photon interaction with M modes (frequency ω_{ci}) of a high-quality cavity, driven additionally by an external classical field with a frequency ω_L. The Hamiltonian of the system can be expressed as ($\hbar = 1$)[15]

$$H = \omega_0 \sum_{j=1}^{N} S_{z,j} + \sum_{i=1}^{M} \omega_{ci} a_i^+ a_i + \sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}(a_i^+ S_j^- + a_i S_j^+) + \Omega \sum_{j=1}^{N}(\mathrm{e}^{-\mathrm{i}\omega_L t} S_j^+ + \mathrm{e}^{\mathrm{i}\omega_L t} S_j^-), \quad (3)$$

where $S_{z,j} = \frac{1}{2}(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$, $S_j^+ = |e_j\rangle\langle g_j|$, and $S_j^- = |g_j\rangle\langle e_j|$ are the jth atomic spin operators for population, raising, and lowering, respectively; $a_i^+$and a_i are the creation and annihilation operators for the cavity mode; and g_{ij} and Ω are the coupling constants of each atom to the cavity modes and to the driving field, respectively. In the rotating frame with respect to the driving field frequency ω_L, the Hamiltonian is given by

$$H^L = \sum_{j=1}^{N} \Delta S_{z,j} + \sum_{i=1}^{M} \delta_i a_i^+ a_i + \Omega \sum_{j=1}^{N}(S_j^+ + S_j^-) + \sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}(a_i^+ S_j^- + a_i S_j^+), \quad (4)$$

where $\Delta = \omega_0 - \omega_L$ and $\delta_i = \omega_{ci} - \omega_L$. Assume that $\Delta = 0$ is satisfied, then in the interaction picture we will have

$$\begin{aligned} H_i =& \mathrm{e}^{\mathrm{i}H_0^L t} H_I^L \mathrm{e}^{-\mathrm{i}H_0^L t} \\ =& \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}[|+_j\rangle\langle+_j| - |-_j\rangle\langle-_j| \\ &+ \mathrm{e}^{\mathrm{i}2\Omega t}|+_j\rangle\langle-_j| - \mathrm{e}^{-\mathrm{i}2\Omega t}|-_j\rangle\langle+_j|]a_i \mathrm{e}^{-\mathrm{i}\delta_i t} \\ &+ \text{H.c.}, \end{aligned} \quad (5)$$

where

$$H_0^L = \sum_{i=1}^{M} \delta_i a_i^+ a_i + \Omega \sum_{j=1}^{N}(S_j^+ + S_j^-), \quad (6)$$

$$H_I^L = \sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}(a_i^+ S_j^- + a_i S_j^+), \quad (7)$$

$|\pm_j\rangle = (|g_j\rangle \pm |e_j\rangle)/\sqrt{2}$, and $S_{j,x}|\pm_j\rangle = (S_j^+ + S_j^-)|\pm_j\rangle = \pm|\pm_j\rangle$. In a strong driving regime in which $\Omega \gg \delta_i, g_{ij}$, the effective Hamiltonian in the rotating-wave approximation is obtained as

$$\begin{aligned} H_{\text{ieff}} &= \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}(|+_j\rangle\langle+_j| - |-_j\rangle\langle-_j|) \\ &\quad \times (a_i \mathrm{e}^{-\mathrm{i}\delta_i t} + a_i^+ \mathrm{e}^{\mathrm{i}\delta_i t}) \\ &= \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} S_{j,x} g_{ij}(a_i^+ \mathrm{e}^{-\mathrm{i}\delta_i t} + a_i \mathrm{e}^{\mathrm{i}\delta_i t}). \end{aligned} \quad (8)$$

A noticeable feature of expression (8), more evident if $\delta_i = 0$, $N = 1$, and $M = 1$ are chosen, is the simultaneous realization of Jaynes–Cummings (JC) and anti-Jaynes–Cummings (AJC) interactions. Some examples of the possible application of the interaction described by expression (8) are given in the following.

3. Generation of entangled coherent states and cluster-type of four-qubit coherent states

We consider that two identical two-level atoms that are driven by a strong classical field enter into a single-mode cavity separately. At $t = 0$, the initial state of the system is $|g_A\rangle|0_A\rangle \otimes |g_B\rangle|0_B\rangle$, and the evolved state given by expression (8) in time t will be

$$
\begin{aligned}
|\Psi(t)\rangle =&\frac{1}{\sqrt{2}}(|+_A\rangle|\alpha_A\rangle + |-_A\rangle|-\alpha_A\rangle) \otimes \frac{1}{\sqrt{2}}(|+_B\rangle|\alpha_B\rangle + |-_B\rangle|-\alpha_B\rangle) \\
=&\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|+_A\rangle|+_B\rangle + |-_A\rangle|-_B\rangle)\right](|\alpha_A\rangle|\alpha_B\rangle + |-\alpha_A\rangle|-\alpha_B\rangle) \\
&+\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}(|+_A\rangle|-_B\rangle + |-_A\rangle|+_B\rangle)\right](|-\alpha_A\rangle|\alpha_B\rangle + |\alpha_A\rangle|-\alpha_B\rangle), \qquad (9)
\end{aligned}
$$

with $\alpha_A = g_A(\mathrm{e}^{\mathrm{i}\delta_A t} - 1)/2\delta_A$, and $\alpha_B = g_B(\mathrm{e}^{\mathrm{i}\delta_B t} - 1)/2\delta_B$. For the simpler case, $\delta_A = \delta_B = 0$ and $g_A = g_B = g$, we have $\alpha_A = \alpha_B = -\mathrm{i}gt/2$. On the basis of the states $|e\rangle$ and $|g\rangle$, we can write expression (9) in the following form:

$$
\begin{aligned}
|\Psi(t)\rangle = \frac{\tilde{N}}{2\sqrt{2}}[&|\Phi^+\rangle(|\alpha_A, \alpha_B\rangle + |-\alpha_A, -\alpha_B\rangle) \\
&+|\Psi^+\rangle(|\alpha_A, \alpha_B\rangle - |-\alpha_A, -\alpha_B\rangle) \\
&+|\Psi^-\rangle(|\alpha_A, -\alpha_B\rangle + |-\alpha_A, \alpha_B\rangle) \\
&+|\Phi^-\rangle(|-\alpha_A, \alpha_B\rangle - |\alpha_A, -\alpha_B\rangle)], \qquad (10)
\end{aligned}
$$

where

$$
|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|g_A, g_B\rangle \pm |e_A, e_B\rangle),
$$

and

$$
|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|g_A, e_B\rangle \pm |e_A, g_B\rangle), \qquad (11)
$$

Expression (10) shows that we can acquire the four entangled coherent states by measuring the four Bell-states, respectively. These states have recently been proposed as an important tool in the theory and experiments relating to the field of quantum information.[16,17] We also find that these states can be generated by simultaneously implementing a displacement in each of the two vacuum cavity modes, and we also can achieve a successful probability of 1 for our scheme.

To generate the entangled cluster states for two optical cavities, we concentrate on the case in which a two-level atom interacts with an optical cavity, with a strong external driving field assisted. From expression (8), we can acquire the associated Hamiltonian

$$
\begin{aligned}
H_{\text{eff}} =&\frac{1}{2}(S^+ + S^-)[g_R(a_R\mathrm{e}^{-\mathrm{i}\delta_R t} + a_R^+\mathrm{e}^{\mathrm{i}\delta_R t}) \\
&+ g_L(a_L\mathrm{e}^{-\mathrm{i}\delta_L t} + a_L^+\mathrm{e}^{\mathrm{i}\delta_L t})], \qquad (12)
\end{aligned}
$$

where g_R and g_L are the coupling strengths of the atom, respectively, to the right-circularly and the left-circularly polarized modes of the cavity field, S^+ and S^- are the atomic spin operators for raising and lowering; a_R and a_L (a_R^+ and a_L^+) are the annihilation (creation) operators of the right-circularly and the left-circularly polarized modes of the cavity field, respectively. Considering two atoms and two optical cavities, i.e. the atom A and the optical cavity A, the atom B and the optical cavity B, let them interact with each other as expressed by the Hamiltonian. For the simpler case, we have $g_R = g_L = g$ and $\delta_R = \delta_L = 0$. And we prepare the system initially in the state $|g_A\rangle|0_{AR}\rangle|0_{AL}\rangle \otimes |g_B\rangle|0_{BR}\rangle|0_{BL}\rangle$, the state evolution of the whole system will be

$$
\begin{aligned}
|\Psi(t)\rangle = \prod_{i=A,B}\frac{1}{\sqrt{2}}&(|+_i\rangle|\alpha_{iR}\rangle|\alpha_{iL}\rangle \\
&+|-_i\rangle|-\alpha_{iR}\rangle|-\alpha_{iL}\rangle), \qquad (13)
\end{aligned}
$$

with $\alpha_{AR} = \alpha_{AL} = \alpha_{BR} = \alpha_{BL} = \alpha = -\mathrm{i}gt/2$. After the two atoms A and B have left from the cavities, if we implement a Hadamard gate operation

$$
|+_i\rangle \longrightarrow |e_i\rangle, |-_i\rangle \longrightarrow |g_i\rangle, \qquad (14)
$$

the state evolution of the whole system can be written as

$$
\begin{aligned}
|\Psi(t)\rangle = \prod_{i=A,B}\frac{1}{\sqrt{2}}&(|e_i\rangle|\alpha_{iR}\rangle|\alpha_{iL}\rangle \\
&+|g_i\rangle|-\alpha_{iR}\rangle|-\alpha_{iL}\rangle). \qquad (15)
\end{aligned}
$$

Having completed the Hadamard gate operation, we send them into the cavity C simultaneously and drive them by a classical field, then the Hamiltonian is

$$
\begin{aligned}
H =&\omega_0 \sum_{i=A,B} S_{z,i} + \omega_c c^+ c \\
&+ \sum_{i=A,B}[g'(c^+ S_i^- + cS_i^+) \\
&+ \Omega(S_i^+\mathrm{e}^{-\mathrm{i}\omega' t} + S_i^-\mathrm{e}^{\mathrm{i}\omega' t})], \qquad (16)
\end{aligned}
$$

where c^+ and c are the creation and the annihilation operators for cavity C; ω_c and ω' are the frequencies of cavity C mode and classical field respectively; g' is the coupling constant between the atom and the cavity field; Ω is the coupling constant between the atom and the classical field. Assume $\omega_0 = \omega'$ and strong driving regime $\Omega \gg \delta, g'$, the interaction between the

atoms and the cavity C will be a large detuning interaction. Then the evolution operator of the system is[18]

$$U(t) = \exp(-\mathrm{i}H_0 t)\exp(-\mathrm{i}H_e t), \tag{17}$$

where

$$H_0 = \Omega \sum_{i=A,B} (S_i^+ + S_i^-), \tag{18}$$

and

$$H_e = \frac{g'^2}{2\delta}(1 + S_A^+ S_B^+ + S_A^+ S_B^- + S_A^- S_B^+ + S_A^- S_B^-). \tag{19}$$

According to expressions (17)–(19), we choose the interaction time t and Rabi-frequency Ω such that they appropriately satisfy

$$\frac{g'^2 t}{2\delta} = \frac{\pi}{4}, \quad \Omega t = \left(2k + \frac{1}{4}\right)\pi, \quad k = 1, 2, 3, \ldots. \tag{20}$$

Then we can obtain

$$\begin{aligned}
U(t)|g_A\rangle|g_B\rangle =& \frac{1}{2}(|g_A\rangle|g_B\rangle - |g_A\rangle|e_B\rangle - |e_A\rangle|g_B\rangle - |e_A\rangle|e_B\rangle),\\
U(t)|g_A\rangle|e_B\rangle =& \frac{1}{2}(-|g_A\rangle|g_B\rangle + |g_A\rangle|e_B\rangle - |e_A\rangle|g_B\rangle - e_A\rangle|e_B\rangle),\\
U(t)|e_A\rangle|g_B\rangle =& \frac{1}{2}(-|g_A\rangle|g_B\rangle - |g_A\rangle|e_B\rangle + |e_A\rangle|g_B\rangle - e_A\rangle|e_B\rangle),\\
U(t)|e_A\rangle|e_B\rangle =& \frac{1}{2}(-|g_A\rangle|g_B\rangle - |g_A\rangle|e_B\rangle - |e_A\rangle|g_B\rangle + |e_A\rangle|e_B\rangle).
\end{aligned} \tag{21}$$

Transforming expression (21) leads the state of the system, expression (15), to evolve into

$$|\Psi(t)\rangle = \frac{1}{2}(|g_3, g_4\rangle|\mathrm{cluter}_\alpha^+\rangle + |g_3, e_4\rangle|C_\alpha^+\rangle + |e_3, g_4\rangle|C_\alpha^-\rangle + |e_3, e_4\rangle|\mathrm{cluter}_\alpha^-\rangle), \tag{22}$$

where[14]

$$|\mathrm{cluter}_\alpha^\pm\rangle = \frac{1}{2}(\pm|\alpha,\alpha,\alpha,\alpha\rangle + |\alpha,\alpha,-\alpha,-\alpha\rangle + |-\alpha,-\alpha,\alpha,\alpha\rangle \mp |-\alpha,-\alpha,-\alpha,-\alpha\rangle),$$

and

$$|C_\alpha^\pm\rangle = \frac{1}{2}(|\alpha,\alpha,\alpha,\alpha\rangle \pm |\alpha,\alpha,-\alpha,-\alpha\rangle \mp |-\alpha,-\alpha,\alpha,\alpha\rangle + |-\alpha,-\alpha,-\alpha,-\alpha\rangle). \tag{23}$$

It is worthy to notice that for the outcome of the atomic measurement, one always ends up with a spatially-separated four-cavity cluster-type entangled coherent state. For instance, in the events in which atoms are detected in a state $|g_A\rangle|g_B\rangle$, one ends up with the state

$$|\mathrm{cluter}_\alpha^+\rangle = \frac{1}{2}(|\alpha,\alpha,\alpha,\alpha\rangle + |\alpha,\alpha,-\alpha,-\alpha\rangle + |-\alpha,-\alpha,\alpha,\alpha\rangle - |-\alpha,-\alpha,-\alpha,-\alpha\rangle). \tag{24}$$

4. Generation of maximally two-photon entangled states and two-atom two-photon entangled cluster state

As mentioned before, if the limit $\delta_i = \pm 2\Omega$ is taken in expression (5), the resulting Hamiltonians are

$$H_{\mathrm{JC}}^{(+)} = \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}[|+_j\rangle\langle -_j|a_i + |-_j\rangle\langle +_j|a_i^+],$$

and

$$H_{\mathrm{AJC}}^{(-)} = \frac{1}{2}\sum_{j=1}^{N}\sum_{i=1}^{M} g_{ij}[|-_j\rangle\langle +_j|a_i + |+_j\rangle\langle -_j|a_i^+], \tag{25}$$

which represent an effective implementation of a JC interaction and an AJC interaction in the $|\pm_j\rangle$ atomic dressed basis. As is known, the anti-JC interaction does not appear naturally in the context of cavity QED, while the JC model discards the so-called non-conserving energy terms, corresponding to exciting (de-exciting) the internal atomic state while creating

(annihilating) an intracavity photon. Another noticeable feature of the interactions described in expression (25) is that they produce, in a controlled manner, the absorption and the emission of an intracavity photon while preserving the mean value of energy for the atomic state. Clearly, the imbalance in the energy terms originates from the external driving field.

We consider that two identical two-level atoms each dispersively interact with two single-mode cavities, the two cavity modes have the same frequencies and coupling strengths with the atoms, and the two atoms are each driven by a strong classical field. At $t = 0$, and satisfying $\delta_i = -2\Omega$ $(i = A, B)$, the atom–field state is $|-_A\rangle|0_A\rangle \otimes |-_B\rangle|0_B\rangle$. After $H_{\rm AJC}^{(-)}$ has acted for time t, the atom-field state will be

$$\begin{aligned}|\varphi(t)\rangle =& \prod_{i=A,B}\left(-\mathrm{i}\sin\frac{gt}{2}|+_i\rangle|1_i\rangle+\cos\frac{gt}{2}|-_i\rangle|0_i\rangle\right)\\ =&-\sin\frac{gt}{2}^2|+_A,+_B,1_A,1_B\rangle+\mathrm{i}\sin\frac{gt}{2}\cos\frac{gt}{2}|+_A,-_B,1_A,0_B\rangle\\ &-\mathrm{i}\sin\frac{gt}{2}\cos\frac{gt}{2}|-_A,+_B,0_A,1_B\rangle+\cos\frac{gt}{2}^2|-_A,-_B,0_A,0_B\rangle. \end{aligned} \tag{26}$$

According to the basis $|e\rangle$ and $|g\rangle$, we choose the interaction time such that it satisfies $gt/2 = \dfrac{\pi}{4}$, and write expression (26) as the following form:

$$\begin{aligned}|\varphi(t)\rangle =&\frac{1}{2}\Big[\frac{1}{\sqrt{2}}(|g_A,g_B\rangle+|e_A,e_B\rangle)\otimes\frac{1}{\sqrt{2}}(|1_A,1_B\rangle+|0_A,0_B\rangle)\\ &+\frac{1}{\sqrt{2}}(|g_A,e_B\rangle+|e_A,g_B\rangle)\otimes\frac{1}{\sqrt{2}}(|1_A,1_B\rangle-|0_A,0_B\rangle)\\ &+\frac{\mathrm{i}}{\sqrt{2}}(|g_A,g_B\rangle-|e_A,e_B\rangle)\otimes\frac{1}{\sqrt{2}}(|1_A,0_B\rangle-|0_A,1_B\rangle)\\ &-\frac{\mathrm{i}}{\sqrt{2}}(|g_A,g_B\rangle-|e_A,e_B\rangle)\otimes\frac{1}{\sqrt{2}}(|1_A,0_B\rangle+|0_A,1_B\rangle)\Big]. \end{aligned} \tag{27}$$

From expression (27), we can acquire the maximally entangled states between the two cavity modes in the $(|1\rangle, |0\rangle)$ subspace through measuring the atomic Bell-state, respectively. Then the maximally entangled states are in the forms, i.e.

$$|\phi_\pm\rangle = \frac{1}{\sqrt{2}}(|1_A,1_B\rangle \pm |0_A,0_B\rangle),$$

and

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|1_A,0_B\rangle \pm |0_A,1_B\rangle)]. \tag{28}$$

These entangled states were recently produced in the cavity QED with a sequence of differently tuned interactions of a single atom with two cavity modes.[19]

In the following, we propose a scheme of preparing the two-atom and two-photon entangled cluster states by three cavities. The operation processing can be divided into the following four steps.

Firstly, we prepare the atoms and fields in an initial state

$$|-_A\rangle|0_A\rangle \otimes |-_B\rangle|0_B\rangle. \tag{29}$$

Secondly, we send two atoms through a single-mode cavity separately, and at the same time the two atoms are driven by a classical field. In the case of $\delta_i = -2\Omega$ $(i = A, B)$, after $H_{\rm AJC}^{(-)}$ has acted for time t, the atoms-fields state will be

$$|\psi\rangle = \prod_{i=A,B}\left(-\mathrm{i}\sin\frac{gt_1}{2}|+_i\rangle|1_i\rangle+\cos\frac{gt_1}{2}|-_i\rangle|0_i\rangle\right), \tag{30}$$

we choose the interaction time such that it satisfies $gt_1/2 = \dfrac{\pi}{4}$, then we can write expression (30) as the form of

$$\begin{aligned}|\psi\rangle =&\frac{1}{2}(|+_A\rangle|1_A\rangle+\mathrm{i}|-_A\rangle|0_A\rangle)\\ &\otimes(|+_B\rangle|1_B\rangle+\mathrm{i}|-_B\rangle|0_B\rangle). \end{aligned} \tag{31}$$

Thirdly, after the two atoms A and B have left

the cavities, we send them into the cavity C simultaneously and drive them by a classical field. According to expressions (17)–(19), if we choose the interaction time t and Rabi-frequency Ω appropriately such that they satisfy

$$\frac{g'^2 t_2}{2\delta} = \frac{\pi}{4}, \quad \Omega t_2 = \left(k + \frac{3}{4}\right)\pi, \quad k = 0, 1, 2, \ldots, \tag{32}$$

we can obtain

$$\begin{aligned} |+_A\rangle|+_B\rangle &\rightarrow |+_A\rangle|+_B\rangle, \\ |+_A\rangle|-_B\rangle &\rightarrow |+_A\rangle|-_B\rangle, \\ |-_A\rangle|+_B\rangle &\rightarrow |-_A\rangle|+_B\rangle, \\ |-_A\rangle|-_B\rangle &\rightarrow -|-_A\rangle|-_B\rangle, \end{aligned} \tag{33}$$

with $|\pm\rangle = \frac{1}{\sqrt{2}}(|g\rangle \pm |e\rangle)$. Using this controlled phase gate described by expression (33), the system state of expression (31) will become

$$\begin{aligned} |\psi\rangle = \frac{1}{2}(&|+_A, 1_A, +_B, 1_B\rangle + \mathrm{i}|+_A, 1_A, -_B, 0_B\rangle \\ &+ \mathrm{i}|-_A, 0_A, +_B, 1_B\rangle - |-_A, 0_A, -_B, 0_B\rangle). \end{aligned} \tag{34}$$

Lastly, when the two atoms A and B leave from the cavity C, the atoms are driven through a classical field, separately, to implement a Hadamard gate operation

$$|e\rangle \longrightarrow |+\rangle, |g\rangle \longrightarrow |-\rangle, \tag{35}$$

and we can transform the state expression (34) into

$$\begin{aligned} |\psi\rangle = \frac{1}{2}(&|e_A, e_B, 1_A, 1_B\rangle + \mathrm{i}|e_A, g_B, 1_A, 0_B\rangle \\ &+ \mathrm{i}|g_A, e_B, 0_A, 1_B\rangle - |g_A, g_B, 0_A, 0_B\rangle). \end{aligned} \tag{36}$$

Using the local operation on the atoms yields the standard entangled cluster state in the form of

$$\begin{aligned} |\psi\rangle = \frac{1}{2}(&|e_A, e_B, 1_A, 1_B\rangle + |e_A, g_B, 1_A, 0_B\rangle \\ &+ |g_A, e_B, 0_A, 1_B\rangle - |g_A, g_B, 0_A, 0_B\rangle). \end{aligned} \tag{37}$$

In the scheme, firstly, the cavity C field is only virtually excited and the photon-number-dependent part in the effective Hamiltonian is cancelled, so the system is insensitive to the cavity decay and the thermal field, and secondly, we have noticed that two atoms are sent into a microwave cavity simultaneously, and the achieved experiment[20] was done by using van der Waals collision of the atoms in the central area of the microwave cavity, instead of sending the atoms through the microwave cavity strictly simultaneously. At the same time, the scheme can prepare a cluster state with a successful probability 1. That is to say, our scheme is closer to the current techniques of cavity QED.

5. Conclusion

In conclusion, we have proposed the schemes that are experimentally feasible for preparing multipartite entangled states in the cavity QED. The first scheme is for generating the entangled coherent states in two single-mode cavities and the four-coherent-mode cluster states. And the second one can be used to acquire the two-atom and two-photon entangled cluster states and the maximally entangled states between the two photons through two atoms interaction each with two single-mode cavities. In these two schemes we must use strong classical fields to drive the atoms. Our schemes have shown that an additional driving field in the usual cavity QED experiments and also in trapped ion systems can be an important tool for generating multi-partite nonclassical states. Therefore, the schemes may be realized by using the current technology.

It is not surprising that entanglement can be generated by coupling between atoms and photons. But what comes out in our scheme is different: the local atom-cavity coupling not only creates the entanglement between an atom and its cavity, but also generates non-local entanglement between the cavities if we make the measurement on the atoms at the same time.

References

[1] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (New York: Cambridge University Press)

[2] Duer W, Vidal G and Cirac J I 2000 *Phys. Rev.* A **62** 062314

[3] Briegel H J and Raussendorf R 2001 *Phys. Rev. Lett.* **86** 910

[4] Duer W and Briegel H J 2004 *Phys. Rev. Lett.* **92** 180403

[5] Scarani V, Acin A, Schenck E and Aspelmeyer M quant-ph/0405119

[6] Raussendorf R and Briegel H J 2001 *Phys. Rev. Lett.* **86** 5188

[7] Zou X B and Mathis W 2005 *Phys. Rev.* A **71** 032308

[8] Tokunaga Y, Yamamoto T, Koashi M and Imoto N 2005 *Phys. Rev.* A **71** 030301
[9] Walther P, Resch K J, Rudolph T, Schenck E, Weinfurter H, Vedral V, Aspelmeyer M and Zeilinger A 2005 *Nature* **434** 169
[10] Jin X R, Zhang Y Q, Zhang S and Jin D Z 2007 *Chin. Phys.* **16** 1220
[11] Yang R C and Li H C 2006 *Chin. Phys.* **15** 2315
[12] Huang Z P, Li H C, Lin X and Yang R C 2007 *Chin. Phys.* **16** 2219
[13] Lloy S and Braunstein S L 1999 *Phys. Rev. Lett.* **82** 1784
[14] Munhoz P P, Vidiella-Barranco A and Roversi J A quant-ph/07051549
[15] Solano E, Agarwal G S and Walther H 2003 *Phys. Rev. Lett.* **90** 027903
[16] Munro W, Milburn G J and Sanders B C 2000 *Phys. Rev.* A **62** 052108
[17] Xia Y J and Gao D Y 2007 *Acta Phys. Sin.* **56** 3703 (in Chinese)
[18] Zheng S B 2003 *Phys. Rev.* A **68** 035801
[19] Rauschenbeutel A, Bertet P, Osnaghi S, Nogues G, Brune M, Raimond J M and Haroche S 2001 *Phys. Rev.* A **64** 050301
[20] Osnaghi S, Bertet P, Auffeves A, Maiodi P, Brune M, Raimond J M and Haroche S 2001 *Phys. Rev. Lett.* **87** 037902