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Perturbation to Mei symmetry and Mei adiabatic invariants for mechanical systems in phase space

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For a perturbed mechanical system in phase space, considering $\tilde{\mathrm{d}}/\mathrm{d}t$ in the structure equation and process of proof including infinitesimal parameter ε obviously, this paper studies the perturbation to Mei symmetry and adiabatic invariants. Firstly, the exact invariant induced directly from the Mei symmetry of the system without perturbation is given. Secondly, based on the concept of high-order adiabatic invariant, the determining equations of the perturbation to Mei symmetry are established, the condition of existence of the Mei adiabatic invariant led by the perturbation to Mei symmetry is obtained, and its form is presented. Lastly, an example is given to illustrate the application of the results.

Keywords: Mei symmetry, perturbation, adiabatic invariant, mechanical system
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1. Introduction

More and more attention has been paid to the research on the perturbation to symmetries and adiabatic invariants for mechanical systems recently. The adiabatic invariants mean that they are almost unchanged when the parameter varies very slowly.[1,2] In fact, the parameter varying very slowly is equivalent to the action of a small disturbance. The adiabatic invariants play a very important role in the research on the quasi-integrability of mechanical systems, so the studies on the perturbation to symmetries and adiabatic invariants for mechanical systems have become very important, and many important results have been obtained.[3−24] But the adiabatic invariants obtained by all these studies belong to the types of Noether[3−15] and Hojman,[16−24] and the adiabatic invariants of Mei type have never been studied so far. Moreover, $\tilde{\mathrm{d}}/\mathrm{d}t$ in the determining equations and processes of proofs did not include infinitesimal parameter ε obviously after the systems being perturbed in these studies. Recently, infinitesimal parameter ε included in $\tilde{\mathrm{d}}/\mathrm{d}t$ obviously after the systems being perturbed was first considered in Ref.[25], which is a new approach to study the perturbation to symmetries and adiabatic invariants for mechanical systems.

In the beginning of this century, Mei presented a new symmetry—the form invariance.[26,27] Mei's form invariance is also called Mei symmetry,[28,29] which means that the dynamical function in the equation of motion still satisfies the equation's primary form after some infinitesimal transformations. The Mei exact invariant can be induced directly by Mei symmetry. As we know, Mei symmetry, Noether symmetry, and Lie symmetry are three modern methods of finding conserved quantities of mechanical systems by using symmetries. Noether symmetry can induce Noether exact invariant directly, and the perturbation to Noether symmetry can lead to Noether adiabatic invariant directly. Also, Lie symmetry can induce Hojman exact invariant directly, and the perturbation to Lie symmetry can lead to Hojman adiabatic invariant directly. In order to perfect the symmetrical theories, we try to seek for the adiabatic invariant of Mei type led by the perturbation to Mei symmetry directly.

In this paper, considering $\tilde{\mathrm{d}}/\mathrm{d}t$ in the structure equation and process of proof including infinitesimal parameter ε obviously after the system is perturbed, we study the perturbation to Mei symmetry and Mei adiabatic invariant for a mechanical system in phase space. The Mei adiabatic invariant is a new type of adiabatic invariant, which has important theoretical and practical significance and perfects the symmetrical theories.

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2. Mei symmetry and exact invariant

The differential equations of motion for a mechanical system in phase space can be written as

$$\dot{q}_s = \frac{\partial H}{\partial p_s}, \quad \dot{p}_s = -\frac{\partial H}{\partial q_s} + Q_s, \quad (s = 1, 2, \cdots, n), \quad (1)$$

where $H = H(t, \boldsymbol{q}, \boldsymbol{p})$ is the Hamiltonian of the system, and $Q_s = Q_s(t, \boldsymbol{q}, \boldsymbol{p})$ are the non-potential generalized forces. Expanding Eq.(1), we get

$$\begin{aligned} &\dot{q}_s = g_s(t, \boldsymbol{q}, \boldsymbol{p}), \\ &\dot{p}_s = h_s(t, \boldsymbol{q}, \boldsymbol{p}), \ (s = 1, 2, \cdots, n). \end{aligned} \quad (2)$$

Introduce the infinitesimal transformations

$$\begin{aligned} &t^* = t + \varepsilon\tau^0(t, \boldsymbol{q}, \boldsymbol{p}), \\ &q_s^*(t^*) = q_s(t) + \varepsilon\xi_s^0(t, \boldsymbol{q}, \boldsymbol{p}), \\ &p_s^*(t^*) = p_s(t) + \varepsilon\eta_s^0(t, \boldsymbol{q}, \boldsymbol{p}) \end{aligned} \quad (3)$$

where ε is an infinitesimal parameter, τ^0, ξ_s^0 and η_s^0 are infinitesimal generators.

According to the Mei symmetry theory, the determining equations of the Mei symmetry for the system are

$$\begin{aligned} &\frac{\partial}{\partial p_s}\left[X_0^{(0)}(H)\right] = 0, \\ &\frac{\partial}{\partial q_s}\left[X_0^{(0)}(H)\right] - X_0^{(0)}(Q_s) = 0, \end{aligned} \quad (4)$$

where

$$X_0^{(0)} = \tau^0\frac{\partial}{\partial t} + \xi_s^0\frac{\partial}{\partial q_s} + \eta_s^0\frac{\partial}{\partial p_s}. \quad (5)$$

For the mechanical system in phase space Eq.(1), if the infinitesimal generator τ^0, ξ_s^0 and η_s^0 satisfy Eq.(4), the symmetry is the Mei symmetry of the system.

Proposition 1[30] For the mechanical system in phase space Eq.(1), if the infinitesimal generators τ^0, ξ_s^0 and η_s^0 of the Mei symmetry and the gauge function $G_{\rm M}^0 = G_{\rm M}^0(t, \boldsymbol{q}, \boldsymbol{p})$ satisfy the following structure equation

$$\begin{aligned} &X_0^{(0)}(p_s)\frac{\bar{\rm d}\xi_s^0}{{\rm d}t} + \frac{\bar{\rm d}}{{\rm d}t}\left[X_0^{(0)}(p_s)\right]\xi_s^0 - X_0^{(0)}\left[X_0^{(0)}(H)\right] \\ &-X_0^{(0)}(H)\frac{\bar{\rm d}\tau^0}{{\rm d}t} + X_0^{(0)}(Q_s)\left(\xi_s^0 - \dot{q}_s\tau^0\right) + \frac{\bar{\rm d}G_{\rm M}^0}{{\rm d}t} \\ &= 0, \end{aligned} \quad (6)$$

where

$$\frac{\bar{\rm d}}{{\rm d}t} = \frac{\partial}{\partial t} + g_k\frac{\partial}{\partial q_k} + h_k\frac{\partial}{\partial p_k}, \quad (k = 1, 2, \cdots, n), \quad (7)$$

the Mei symmetry of the system can lead to the Mei exact invariant

$$I_{\rm M0} = X_0^{(0)}(p_s)\xi_s^0 - X_0^{(0)}(H)\tau^0 + G_{\rm M}^0 = \text{const}. \quad (8)$$

3. Perturbation to Mei symmetry and adiabatic invariant

Firstly, we present the concept of high-order adiabatic invariant.

Definition[2] If $I_z(t, \boldsymbol{q}, \boldsymbol{p}, \varepsilon)$ is a physical quantity including ε in which the highest power is z in a mechanical system, and its derivative with respect to time t is directly proportional to ε^{z+1}, I_z is called a zth order adiabatic invariant of the mechanical system in phase space.

Suppose the mechanical system in phase space Eq.(1) is perturbed by small quantities $\varepsilon W_s = \varepsilon W_s(t, \boldsymbol{q}, \boldsymbol{p})$, the equations of motion for the system become

$$\dot{q}_s = \frac{\partial H}{\partial p_s}, \quad \dot{p}_s = -\frac{\partial H}{\partial q_s} + Q_s + \varepsilon W_s. \quad (9)$$

From Eqs.(1) and (2), Eq.(9) can be also written in the form

$$\begin{aligned} &\dot{q}_s = g_s(t, \boldsymbol{q}, \boldsymbol{p}), \\ &\dot{p}_s = h_s(t, \boldsymbol{q}, \boldsymbol{p}) + \varepsilon W_s(t, \boldsymbol{q}, \boldsymbol{p}), \\ &(s = 1, 2, \cdots, n). \end{aligned} \quad (10)$$

Because of the action of εW_s, the original symmetries and invariants of the system may vary. Assume that the variation is a small perturbation based on the symmetrical transformations of the system without perturbation, and $\tau(t, \boldsymbol{q}, \boldsymbol{p}), \xi_s(t, \boldsymbol{q}, \boldsymbol{p})$ and $\eta_s(t, \boldsymbol{q}, \boldsymbol{p})$ express the generators of infinitesimal transformations after being perturbed, then

$$\begin{aligned} &\tau = \tau^0 + \varepsilon\tau^1 + \varepsilon^2\tau^2 + \cdots, \\ &\xi_s = \xi_s^0 + \varepsilon\xi_s^1 + \varepsilon^2\xi_s^2 + \cdots, \\ &\eta_s = \eta_s^0 + \varepsilon\eta_s^1 + \varepsilon^2\eta_s^2 + \cdots. \end{aligned} \quad (11)$$

According to the Mei symmetry theory, the determining equations of the Mei symmetry for the mechanical system in phase space perturbed by small quantities εW_s become

$$\begin{aligned} &\frac{\partial}{\partial p_s}\left[X^{(0)}(H)\right] = 0, \\ &\frac{\partial}{\partial q_s}\left[X^{(0)}(H)\right] - X^{(0)}(Q_s) - \varepsilon X^{(0)}(W_s) = 0, \end{aligned} \quad (12)$$

where

$$X^{(0)} = \tau\frac{\partial}{\partial t} + \xi_s\frac{\partial}{\partial q_s} + \eta_s\frac{\partial}{\partial p_s}. \tag{13}$$

At the same time, after being perturbed, contrasting with Eq.(6), we construct the following structure equation

$$\begin{aligned} &X^{(0)}(p_s)\frac{\tilde{\mathrm{d}}\xi_s^0}{\mathrm{d}t} + \frac{\tilde{\mathrm{d}}}{\mathrm{d}t}\left[X^{(0)}(p_s)\right]\xi_s^0 - X_0^{(0)}\left[X^{(0)}(H)\right] \\ &-X^{(0)}(H)\frac{\tilde{\mathrm{d}}\tau^0}{\mathrm{d}t} + X^{(0)}(Q_s)(\xi_s^0 - \dot{q}_s\tau^0) \\ &+\varepsilon X^{(0)}(W_s)\left(\xi_s^0 - \dot{q}_s\tau^0\right) + \frac{\tilde{\mathrm{d}}G_{\mathrm{M}}}{\mathrm{d}t} = 0, \end{aligned} \tag{14}$$

where

$$\begin{aligned} \frac{\tilde{\mathrm{d}}}{\mathrm{d}t} &= \frac{\partial}{\partial t} + g_k\frac{\partial}{\partial q_k} + (h_k + \varepsilon W_k)\frac{\partial}{\partial p_k} \\ &= \frac{\bar{\mathrm{d}}}{\mathrm{d}t} + \varepsilon W_k\frac{\partial}{\partial p_k}, \end{aligned} \tag{15}$$

and G_{M} is a gauge function. After being perturbed, the gauge function comes into

$$G_{\mathrm{M}} = G_{\mathrm{M}}^0 + \varepsilon G_{\mathrm{M}}^1 + \varepsilon^2 G_{\mathrm{M}}^2 + \cdots. \tag{16}$$

Substituting Eq.(11) into Eq.(13), we have

$$X^{(0)} = \varepsilon^m X_m^{(0)}, \quad (m = 0, 1, \cdots, z), \tag{17}$$

where

$$\begin{aligned} X_m^{(0)} &= \tau^m\frac{\partial}{\partial t} + \xi_s^m\frac{\partial}{\partial q_s} + \eta_s^m\frac{\partial}{\partial p_s}, \\ &(m = 0, 1, \cdots, z). \end{aligned} \tag{18}$$

Substituting Eqs.(11) and (15) into Eqs.(12) and (14), noticing Eqs.(16)–(18), and making the coefficients of ε^m equal, we get

$$\begin{aligned} &\frac{\partial}{\partial p_s}\left[X_m^{(0)}(H)\right] = 0, \\ &\frac{\partial}{\partial q_s}\left[X_m^{(0)}(H)\right] - X_m^{(0)}(Q_s) - X_{m-1}^{(0)}(W_s) = 0, \end{aligned} \tag{19}$$

$$\begin{aligned} &X_m^{(0)}(p_s)\frac{\bar{\mathrm{d}}\xi_s^0}{\mathrm{d}t} + X_{m-1}^{(0)}(p_s)W_k\frac{\partial\xi_s^0}{\partial p_k} + \frac{\bar{\mathrm{d}}}{\mathrm{d}t}\left[X_m^{(0)}(p_s)\right]\xi_s^0 + W_k\frac{\partial}{\partial p_k}\left[X_{m-1}^{(0)}(p_s)\right]\xi_s^0 - X_0^{(0)}\left[X_m^{(0)}(H)\right] \\ &-X_m^{(0)}(H)\frac{\bar{\mathrm{d}}\tau^0}{\mathrm{d}t} - X_{m-1}^{(0)}(H)W_k\frac{\partial\tau^0}{\partial p_k} + X_m^{(0)}(Q_s)\left(\xi_s^0 - \dot{q}_s\tau^0\right) + X_{m-1}^{(0)}(W_s)\left(\xi_s^0 - \dot{q}_s\tau^0\right) \\ &+\frac{\bar{\mathrm{d}}G_{\mathrm{M}}^m}{\mathrm{d}t} + W_k\frac{\partial G_{\mathrm{M}}^{m-1}}{\partial p_k} = 0, \quad (s, k = 1, 2, \cdots, n), \end{aligned} \tag{20}$$

When $m = 0$, we note that $\tau^{m-1} = \xi_s^{m-1} = \eta_s^{m-1} = G_{\mathrm{M}}^{m-1} = 0$ holds, then Eqs.(19) and (20) turn into Eqs.(4) and (6) respectively.

For the mechanical system in phase space perturbed by small quantities εW_s, if the infinitesimal generators τ^m, ξ_s^m and η_s^m satisfy Eq.(19), the corresponding variety of the symmetry is called the perturbation to Mei symmetry. And Eq.(19) can be called the determining equations of the perturbation to Mei symmetry.

Proposition 2 For the mechanical system in phase space perturbed by small quantities εW_s, if the infinitesimal generators τ^m, ξ_s^m and η_s^m of the Mei symmetry and the gauge function $G_{\mathrm{M}}^m = G_{\mathrm{M}}^m(t, \boldsymbol{q}, \boldsymbol{p})$ satisfy the structure equation (20), the perturbation to Mei symmetry of the system can lead to the zth order adiabatic invariant

$$\begin{aligned} &I_{\mathrm{M}z} = \varepsilon^m\left[X_m^{(0)}(p_s)\xi_s^0 - X_m^{(0)}(H)\tau^0 + G_{\mathrm{M}}^m\right], \\ &(m = 0, 1, \ldots, z). \end{aligned} \tag{21}$$

Proof

$$\begin{aligned} &\frac{\tilde{\mathrm{d}}I_{\mathrm{M}z}}{\mathrm{d}t} \\ &= \varepsilon^m\left\{X_m^{(0)}(p_s)\frac{\tilde{\mathrm{d}}\xi_s^0}{\mathrm{d}t} + \frac{\tilde{\mathrm{d}}}{\mathrm{d}t}\left[X_m^{(0)}(p_s)\right]\xi_s^0\right. \\ &\left.-X_m^{(0)}(H)\frac{\tilde{\mathrm{d}}\tau^0}{\mathrm{d}t} - \frac{\tilde{\mathrm{d}}}{\mathrm{d}t}\left[X_m^{(0)}(H)\right]\tau^0 + \frac{\tilde{\mathrm{d}}G_{\mathrm{M}}^m}{\mathrm{d}t}\right\}. \end{aligned} \tag{22}$$

Substituting Eq.(15) into Eq.(22), and using Eqs.(10), (18), (19) and (20), we have

$$\begin{aligned}\frac{\tilde{\mathrm{d}}I_{\mathrm{M}z}}{\mathrm{d}t} &= \varepsilon^m\Bigg\{X_m^{(0)}(p_s)\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}+\varepsilon W_k\frac{\partial}{\partial p_k}\right)\xi_s^0+\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}+\varepsilon W_k\frac{\partial}{\partial p_k}\right)\left[X_m^{(0)}(p_s)\right]\xi_s^0-X_m^{(0)}(H)\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}+\varepsilon W_k\frac{\partial}{\partial p_k}\right)\tau^0\\ &\quad-\left[\frac{\partial X_m^{(0)}(H)}{\partial t}+g_k\frac{\partial X_m^{(0)}(H)}{\partial q_k}+(h_k+\varepsilon W_k)\frac{\partial X_m^{(0)}(H)}{\partial p_k}\right]\tau^0+\frac{\bar{\mathrm{d}}G_{\mathrm{M}}^m}{\mathrm{d}t}+\varepsilon W_k\frac{\partial G_{\mathrm{M}}^m}{\partial p_k}\Bigg\}\\ &= \varepsilon^m\Bigg\{X_m^{(0)}(p_s)\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}+\varepsilon W_k\frac{\partial}{\partial p_k}\right)\xi_s^0+\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}+\varepsilon W_k\frac{\partial}{\partial p_k}\right)\left[X_m^{(0)}(p_s)\right]\xi_s^0-X_m^{(0)}(H)\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}+\varepsilon W_k\frac{\partial}{\partial p_k}\right)\tau^0\\ &\quad-\left[\frac{\partial X_m^{(0)}(H)}{\partial t}\tau^0+\frac{\partial X_m^{(0)}(H)}{\partial q_k}\xi_k^0+\frac{\partial X_m^{(0)}(H)}{\partial p_k}\eta_k^0\right]+\frac{\partial X_m^{(0)}(H)}{\partial q_k}(\xi_k^0-g_k\tau^0)\\ &\quad+\frac{\partial X_m^{(0)}(H)}{\partial p_k}\left[\eta_k^0-(h_k+\varepsilon W_k)\tau^0\right]+\frac{\bar{\mathrm{d}}G_{\mathrm{M}}^m}{\mathrm{d}t}+\varepsilon W_k\frac{\partial G_{\mathrm{M}}^m}{\partial p_k}\Bigg\}\\ &= \varepsilon^m\Bigg\{\varepsilon X_m^{(0)}(p_s)W_k\frac{\partial\xi_s^0}{\partial p_k}-X_{m-1}^{(0)}(p_s)W_k\frac{\partial\xi_s^0}{\partial p_k}+\varepsilon W_k\frac{\partial}{\partial p_k}\left[X_m^{(0)}(p_s)\right]\xi_s^0-W_k\frac{\partial}{\partial p_k}\left[X_{m-1}^{(0)}(p_s)\right]\xi_s^0\\ &\quad-\varepsilon X_m^{(0)}(H)W_k\frac{\partial\tau^0}{\partial p_k}+X_{m-1}^{(0)}(H)W_k\frac{\partial\tau^0}{\partial p_k}+\varepsilon W_k\frac{\partial G_{\mathrm{M}}^m}{\partial p_k}-W_k\frac{\partial G_{\mathrm{M}}^{m-1}}{\partial p_k}\Bigg\}\\ &= \varepsilon^{z+1}\left\{X_z^{(0)}(p_s)W_k\frac{\partial\xi_s^0}{\partial p_k}+W_k\frac{\partial}{\partial p_k}\left[X_z^{(0)}(p_s)\right]\xi_s^0-X_z^{(0)}(H)W_k\frac{\partial\tau^0}{\partial p_k}+W_k\frac{\partial G_{\mathrm{M}}^z}{\partial p_k}\right\}. \end{aligned}\tag{23}$$

It shows that $\tilde{\mathrm{d}}I_{\mathrm{M}z}/\mathrm{d}t$ is directly proportional to ε^{z+1}, so $I_{\mathrm{M}z}$ is a zth order adiabatic invariant of the mechanical system in phase space. The adiabatic invariant Eq.(21) is a new adiabatic invariant, which can be called the Mei adiabatic invariant. When $z=0$, which mean εW_s vanish, the system becomes the one without perturbation Eq.(1). Then the adiabatic invariant Eq.(21) turns into the exact invariant Eq.(8).

4. An example

The Hamiltonian of a mechanical system is

$$H=\frac{1}{2}\left(p_1^2+p_2^2\right)+q_2,\tag{24}$$

the non-potential generalized forces are $Q_1=t+p_1+p_2$, $Q_2=0$, and the system is perturbed by the small quantities

$$\varepsilon W_1=\varepsilon q_1,\quad \varepsilon W_2=\varepsilon p_2.\tag{25}$$

Let us study the exact invariant, the perturbation and the first order adiabatic invariant of the Mei symmetry for the system.

Firstly, we seek the exact invariant. The equations of motion for the system are

$$\begin{aligned}&g_1=\dot{q}_1=p_1,\quad g_2=\dot{q}_2=p_2,\\ &h_1=\dot{p}_1=t+p_1+p_2,\quad h_2=\dot{p}_2=-1.\end{aligned}\tag{26}$$

Choosing the infinitesimal generators as

$$\tau^0=1,\quad \xi_1^0=1,\quad \xi_2^0=p_2,\quad \eta_1^0=0,\quad \eta_2^0=-1,\tag{27}$$

we can easily verify that they satisfy Eq.(4), so the infinitesimal generators Eq.(27) are Mei symmetrical for the system Eq.(24).

From Eq.(27), when taking

$$G_{\mathrm{M}}^0=-t,\tag{28}$$

the structure equation (6) is satisfied.

According to Proposition 1, Eqs.(27) and (28) give the following Mei exact invariant

$$I_{\mathrm{M}0}=-p_2-t=\mathrm{const.}\tag{29}$$

Secondly, we seek the first order adiabatic invariant. Taking

$$\begin{aligned}&\tau^1=1,\quad \xi_1^1=p_1,\quad \xi_2^1=p_1+p_2-q_2,\\ &\eta_1^1=-1,\quad \eta_2^1=-1,\end{aligned}\tag{30}$$

we find that the infinitesimal generators Eq.(30) satisfy the determining equations (19) of the perturbation to Mei symmetry. The structure equation (20) gives

$$\begin{aligned}&X_1^{(0)}(p_s)\frac{\bar{\mathrm{d}}\xi_s^0}{\mathrm{d}t}+X_0^{(0)}(p_s)W_k\frac{\partial\xi_s^0}{\partial p_k}+\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\left[X_1^{(0)}(p_s)\right]\xi_s^0\\ &+W_k\frac{\partial}{\partial p_k}\left[X_0^{(0)}(p_s)\right]\xi_s^0-X_0^{(0)}\left[X_1^{(0)}(H)\right]\\ &-X_1^{(0)}(H)\frac{\bar{\mathrm{d}}\tau^0}{\mathrm{d}t}-X_0^{(0)}(H)W_k\frac{\partial\tau^0}{\partial p_k}\\ &+X_1^{(0)}(Q_s)(\xi_s^0-\dot{q}_s\tau^0)+X_0^{(0)}(W_s)(\xi_s^0-\dot{q}_s\tau^0)\\ &+\frac{\bar{\mathrm{d}}G_{\mathrm{M}}^1}{\mathrm{d}t}+W_k\frac{\partial G_{\mathrm{M}}^0}{\partial p_k}=0,\end{aligned}\tag{31}$$

it has a solution

$$G_{\rm M}^{1} = -t. \tag{32}$$

According to Proposition 2, we obtain a first order adiabatic invariant of the system

$$I_{\rm M1} = -p_2 - t + \varepsilon(-p_2 + q_2 - t - 1). \tag{33}$$

5. Conclusion

For a perturbed mechanical system in phase space, considering $\tilde{\rm d}/{\rm d}t$ in the structure equation and process of proof including infinitesimal parameter ε obviously, the perturbation to Mei symmetry and Mei adiabatic invariants are studied in this paper. We obtain the condition of existence of the Mei adiabatic invariant led by the perturbation to Mei symmetry, and present the form of the Mei adiabatic invariant.

In this paper we consider that $\tilde{\rm d}/{\rm d}t$ include infinitesimal parameter ε obviously after the system being perturbed, such consideration is a new approach to study the perturbation to symmetries and adiabatic invariants for mechanical systems. Meanwhile, the adiabatic invariants obtained by all the previous studies belong to the types of Noether and Hojman. The Mei adiabatic invariant shown in this paper is a new type of adiabatic invariant, which is different from the Noether adiabatic invariant and the Hojman adiabatic invariant. The study on the perturbation to Mei symmetry and Mei adiabatic invariants perfects the symmetrical theories and has important theoretical and practical significance.

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