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Zeeman effects on Josephson current in d-wave superconductor/d-wave superconductor junctions*

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This paper solves a self-consistent equation for the d-wave superconducting gap and the effective exchange field in the mean-field approximation, and studies the Zeeman effects on the d-wave superconducting gap and thermodynamic potential. The Josephson currents in the d-wave superconductor(S)/insulating layer(I)/d-wave S junctions are calculated as a function of the temperature, exchange field, and insulating barrier strength under a Zeeman magnetic field on the two d-wave Ss. It is found that the Josephson critical currents in d-wave S/d-wave S junction to a great extent depend on the relative orientation of the effective exchange field of the two S electrodes, and the crystal orientation of the d-wave S. The exchange field under certain conditions can enhance the Josephson critical current in a d-wave S/I/d-wave S junction.

Keywords: Zeeman effect, d-wave S/d-wave S, ferromagnetic d-wave superconductor, Josephson current

PACC: 7450, 7475

1. Introduction

Recently, the coexistence of superconductivity and ferromagnetism has been an interesting subject in condensed matter physics.[1−4] Following the original work of Clogston[5] and Chandrasekhar,[6] the modification of superconductivity by the Zeeman coupling between the spins of the electrons and applied magnetic field has attracted intermittent attention.[7] A more unusual aspect of this physics was uncovered by Fulde and Ferrell[8] and Larkin and Ovchinnikov[9](FFLO) that a finite momentum pairing state can occur under magnetic fields when electron momenta at the Fermi energy are different for the two spin directions, the FFLO state is predicted for a clean spin-singlet superconductor (S) as a result of the competition between pairing correlations, favouring antiparallel spin alignment and the Zeeman effect, favouring parallel spin alignment along the field. Experimentally, the classic predictions on the nature of the *s*-wave phase boundary have been confirmed by the work on thin Al films.[10] A number of layered organic Ss have been suggested as candidate for FFLO phase.[11−13] However, recent interest has focussed on the heavy fermion material $CeCoIn_5$, because several measurements have led to a renewed discussion of a possible high field FFLO state.[14−17] $CeCoIn_5$ has quasi-two-dimensional metallic planes, and shows evidence of unconventional d-wave superconductivity[18−23] with a transition temperature of $T_c = 2.3\,\mathrm{K}$. Unlike the conventional Cooper pair in which two electrons have opposite spins and momenta ($\boldsymbol{k}\uparrow, -\boldsymbol{k}\downarrow$), the Cooper pair in the FFLO state has a finite centre-of-mass momentum of the order of $2h_0/\hbar v_F$, and consequently leads to a spatially modulated superconducting order parameter, where h_0 is the magnetic exchange energy corresponding to the half of the difference in the energy between the spin-up and spin-down bands, and v_F is the Fermi velocity. For a two-dimensional d-wave S, the superconducting state responds non-trivially at arbitrarily small values of the magnetic field due to the existence of gap nodes, so that electron spins may be polarized with little cost in energy. The response of a d-wave S to a Zeeman magnetic field should provide a possible candidate to study the coexistence of

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superconductivity and ferromagnetism.

The Josephson effect has attracted much attention since it was discovered in S/S junction, and there has been a continuously growing interest in the fundamental physics and the application of this effect. In recent years, the ferromagnet (F) has been introduced to the Josephson tunnel structure, giving rise to some new physical effects. As an example, if very thin insulating layers (I) in a S/I (insulating layer)/S junction are replaced by thin ferromagnetic layers, a new type of S/F/S Josephson junction is formed.[24−28] At the same time, Bergeret *et al*[29] and Li *et al*[30,31] proposed a Josephson tunnel junction of two F/S bilayers separated by a thin insulating film. On the assumption that a thin F/S bilayer is equivalent to a homogeneous ferromagnetic superconductor (FS), the S/F/I/F/S structure may be simplified as a FS/FS junction. They found that the presence of an exchange field may increase the critical current in the FS/FS junction in the case of an antiparallel alignment of the magnetization in the ferromagnets at low temperatures and strong barrier strength. These above theories dealt with isotropic ferromagnetic s-wave S(FsS), but the Josephson current in ferromagnetic d-wave superconductor (FdS)/FdS has not yet been studied. It is known that the transport properties in d-wave S junctions are quite different from the s-wave S junctions. In the *a*-*b* plane junction of the d-wave S, not only the magnitude of the pair potential, but also the orientation of the S crystal with respect to the interface normal influence significantly the Josephson current in the junctions. It is highly desirable to clarify that the Josephson currents depend on the relative orientation of the effective exchange field of the two FdS electrodes, and the crystal orientation of the d-wave S in such a FdS/FdS junction, as well as its dependence on the temperature, barrier strength.

The purpose of this paper is twofold. The first one is to study the response of a d-wave S to a Zeeman magnetic field. We solve self-consistent equations for the d-wave superconducting gap and the magnetization in the mean-field approximation, and calculate the difference in thermodynamic potential between the FdS and normal states. The other purpose of this paper is to present the Josephson current in FdS/FdS junction by applying an extended Furusaki and Tsukada approach.[32] The analytic expressions for the Josephson current in parallel and antiparallel alignments of the magnetizations in two FdSs are obtained as functions of the temperature, exchange field, and insulating barrier strength. It is found that the Josephson critical currents in FdS/FdS junction to a great extent depend on the crystal orientation of the d-wave S, and the relative orientation of the effective exchange field of the two FdS electrodes. The presence of an exchange field usually suppresses the Josephson critical current in FdS/FdS junction. The only exception that the critical current increases with exchange field $\alpha = 0$ (α is the angle between the a axis of the d-wave S crystal and the interface normal, as shown in Fig.1), one needs low temperatures, strong barrier strength, and an antiparallel configuration. In the case of $\alpha \neq 0$, however, the enhancement of Josephson critical current only on condition of weak barrier strength, low temperatures, and the parallel configuration of the two FdS electrodes.

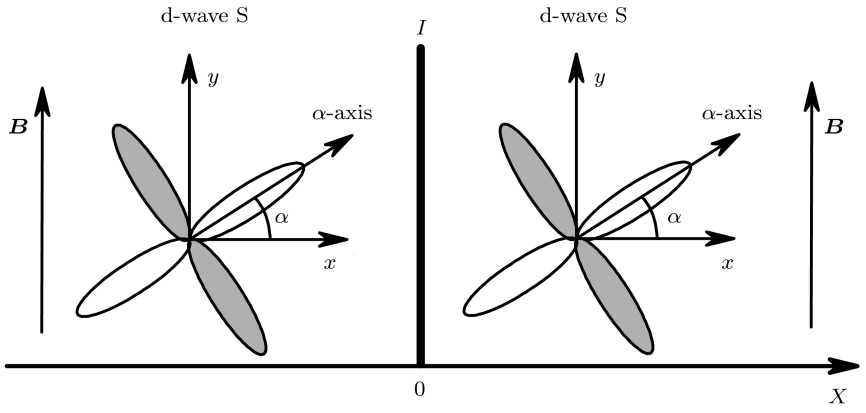


Fig.1. Schematic illustration of the model for d-wave S/I/ d-wave S junction, in which α the angle between the a axis of the d-wave S crystal and the interface normal, $\boldsymbol{B}$ is the applied magnetic field along the y-axis direction.

2. Zeeman effects on d-wave S

Consider a two-dimensional d-wave S/I/ d-wave S junction structure, in which the left and right electrodes are made of the same d-wave S, and they are separated by very thin insulating layer. The thin insulating layer at $x = 0$ can be modelled to be a δ-type barrier potential $U(x) = U_0\delta(x)$, where U_0 depends on the product of barrier height and width. As in the previous works,[33,34] we neglect for simplicity the self-consistency of spatial distribution of the pair potential in S and take them as a step function $\Delta(x) = \Delta[e^{i\phi_L}\Theta(-x) + e^{i\phi_R}\Theta(x)]$, where Δ is the pair potential for a d-wave S, ϕ_L and ϕ_R stand for the macroscopic phase of the left and right S, respectively. There is an uniform magnetic field $\boldsymbol{B}$ applied to d-wave S along the y-axis direction, as shown in Fig.1, the Zeeman energy $h_0 = \mu B$, where $\mu = g\mu_B/2$ is the magnetic moment of an electron, μ_B is the Bohr magneton, g is the factor of the paramagnetic centre. The Hamiltonian of the system is expressed as

$$H = \sum_{\boldsymbol{k}\sigma}\varepsilon_k c^+_{\boldsymbol{k}\sigma}c_{\boldsymbol{k}\sigma} + \sum_{\boldsymbol{k}} h_0\left(c^+_{\boldsymbol{k}\uparrow}c_{\boldsymbol{k}\uparrow} - c^+_{\boldsymbol{k}\downarrow}c_{\boldsymbol{k}\downarrow}\right) + \sum_{\boldsymbol{k}\boldsymbol{k}'} V_{\boldsymbol{k}\boldsymbol{k}'}c^+_{\boldsymbol{k}\uparrow}c^+_{-\boldsymbol{k}\downarrow}c_{-\boldsymbol{k}'\downarrow}c_{\boldsymbol{k}'\uparrow}, \quad (1)$$

where $\varepsilon_k = \hbar^2k^2/(2m) - E_F$ is the kinetic energy measured relative to the Fermi energy E_F, $V_{\boldsymbol{k}\boldsymbol{k}'}$ is the attractive potential between the conducting electrons, and $V_{\boldsymbol{k}\boldsymbol{k}'} = -2V_0\cos[2(\phi_k - \phi_{k'})]$, where V_0 is the strength of the effective attractive potential between the conducting electrons, $\phi = \arctan\left(\widehat{\boldsymbol{k}}_x/\widehat{\boldsymbol{k}}_y\right)$ is the azimuthal angle of $\boldsymbol{k}$. Under the self-consistent field and the mean-field approximation, by means of a Bogoliubov transformation:

$$\gamma_{\boldsymbol{k}\sigma} = u_k c_{\boldsymbol{k}\sigma} - \eta_\sigma v_k c^+_{-\boldsymbol{k}\bar{\sigma}}, \quad (2)$$

$$u_k^2 = (1 + \varepsilon_k/\xi_k)/2, \quad (3)$$

$$v_k^2 = (1 - \varepsilon_k/\xi_k)/2. \quad (4)$$

Hamiltonian (1) can be diagonalized as

$$\bar{H} = \sum_{\boldsymbol{k}} E_{k\uparrow}\gamma^+_{\boldsymbol{k}\uparrow}\gamma_{\boldsymbol{k}\uparrow} + \sum_{\boldsymbol{k}} E_{k\downarrow}\gamma^+_{\boldsymbol{k}\downarrow}\gamma_{\boldsymbol{k}\downarrow} + E_s(0) \quad (5)$$

with

$$E_s(0) = \sum_{\boldsymbol{k}}(\varepsilon_k - \xi_k) + \Delta^2/V_0, \quad (6)$$

and $E_{k\sigma} = \xi_k + \eta_\sigma h_0$, $\xi_k = \sqrt{\varepsilon_k^2 + \Delta^2}$, Δ is taken as $\Delta = \Delta_0(T, h_0)\cos(2\phi)$. In Eq.(2), $\eta_\sigma = 1$ for $\sigma = \uparrow$, $\eta_\sigma = -1$ for $\sigma = \downarrow$, where $\bar{\sigma}$ stands for the spin opposite to σ. The gap equation is given by[35]

$$1 = \frac{N(0)V_0}{2\pi}\int_0^{2\pi}d\phi\int_0^{\varepsilon_c}d\varepsilon_k\frac{\cos^2(2\phi)}{\xi_k} \times\left\{\tanh\left[\frac{\xi_k + h_0}{2k_BT}\right] + \tanh\left[\frac{\xi_k - h_0}{2k_BT}\right]\right\}, \quad (7)$$

where $N(0)$ is the single particle density of states at E_F for each spin, and ε_c is the Debye energy. For $h_0 = 0$, from Eq.(7) we can obtain the zero temperature gap $\Delta_{00}(00)$ and the critical temperature T_{c0}. Solving Eq.(7), we obtain the dependence of the effective order parameter Δ_0 on the temperature and Zeeman energy. At finite temperature, it is found that for a Zeeman energy, the order parameter decreases with increasing temperature and has a sudden drop from a finite value to zero at the critical temperature $T_c(h_0)$, as shown in Fig.2, such a first-order phase transition arises from the presence of the Zeeman energy. When the Zeeman energy $h_0 = 0$, the superconducting transition remains second-order phase transition as shown by solid line in Fig.2. With increasing Zeeman energy, the critical temperature $T_{c0}(h_0)$ also decreases and is always lower than $T_{c0}(h_0 = 0)$. At zero temperature, the order parameter Δ_0 is gradually lowered with increasing h_0. However, in a s-wave S, $\Delta_0(0, h_0)$ remains unchanged for $h_0 < \Delta_0(0, h_0)$, independent of the increase of h_0,[4] as h_0 is increased to $\Delta_0(0, h_0)/\sqrt{2}$, Δ_0 suddenly drop to zero, exhibiting a first-order phase transition from the superconducting state to normal state. This feature may be used to distinguish between d-wave S and s-wave S.

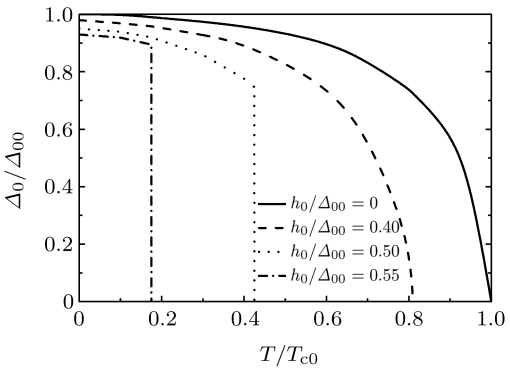


Fig.2. Dependence of the effective order parameter in d-wave S on the temperature for different Zeeman energies.

The thermodynamic potential of FdS is given by

$$\Omega_s(T, h_0) = -\frac{1}{4}N(0)\Delta^2\left(1 + 2\ln\left(\frac{\Delta_{00}}{\Delta}\right)\right)$$

$$-\frac{N(0)k_{\mathrm{B}}T}{\pi}\int_0^{2\pi}\mathrm{d}\phi\int_0^{\varepsilon_{\mathrm{c}}}\mathrm{d}\varepsilon_k \times\left\{\ln\left[1+\mathrm{e}^{-(\xi_k+h_0)/(k_{\mathrm{B}}T)}\right]+\ln\left[1+\mathrm{e}^{-(\xi_k-h_0)/(k_{\mathrm{B}}T)}\right]\right\}. \quad (8)$$

If Δ is taken to be zero in Eq.(8), we obtain the normal state thermodynamic potential $\Omega_{\mathrm{n}}(T,h_0)$. Figure 3 shows the $\Omega_{\mathrm{s}}(T,h_0)-\Omega_{\mathrm{n}}(T,0)$ as a function of Zeeman energy h_0 for different temperatures. The calculated result indicates that $\Omega_{\mathrm{s}}(T,h_0)$ is smaller than $\Omega_{\mathrm{n}}(T,h_0)$ at small h_0 and at low temperature, with the increase of h_0 or T, the difference between them decreases gradually. As h_0 and/or T are increased to a set of critical values $h_0=h_{\mathrm{c}}$ and/or $T=T_{\mathrm{c}}$, there will be $\Omega_{\mathrm{s}}(T,h_0)=\Omega_{\mathrm{n}}(T,h_0)$. The system has a stable ferromagnetic superconducting state for $\Omega_{\mathrm{s}}(T,h_0)<\Omega_{\mathrm{n}}(T,h_0)$, and normal state for $\Omega_{\mathrm{s}}(T,h_0)>\Omega_{\mathrm{n}}(T,h_0)$. It is worth mentioning that we get $[\Omega_{\mathrm{s}}(0,h_0)-\Omega_{\mathrm{n}}(0,h_0)]/N(0)=h_0^2-(0.56)^2\Delta_{00}^2$ at $T=0$, yielding $h_{\mathrm{c}}=0.56\Delta_{00}$, as represented by the cross point between the solid line and the dashed line in Fig.3. At $T=0$, one finds that, for $h_0<0.56\Delta_{00}$, the ferromagnetic superconducting state has lower energy than the normal state and therefore is the stable state. For $h_0>0.56\Delta_{00}$, however, the ferromagnetic superconducting state has higher energy than the normal state, the stable state becomes the normal state.

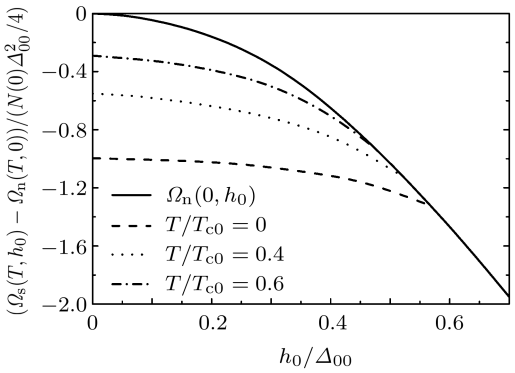


Fig.3. Thermodynamic potential in d-wave S as a function of the Zeeman energy for different temperatures.

Thus far we have only considered zero momentum pairing. In the FdS state with finite momentum pairing $\boldsymbol{q}$, the excitation energy of the quasiparticle is given by[36]

$$E_{kq\sigma}=\xi_{kq}+\eta_\sigma h_0+\eta_\sigma Q, \quad (9)$$

where $\xi_{kq}=\sqrt{\varepsilon_{kq}^2+\Delta^2}$ and $\varepsilon_{kq}=\hbar^2k^2/(2m)+q^2/(2m)-E_{\mathrm{F}}$. $Q=(v_{\mathrm{F}}q/2)\cos(\theta-\theta_{\boldsymbol{q}})$, where $\theta_{\boldsymbol{q}}$ is the polar angle of the pairing momentum $\boldsymbol{q}$, which is determined by the minimization of the thermodynamic potential. The gap equation and the thermodynamic potential in the finite momentum pairing are given respectively by

$$1=\frac{N(0)V_0}{2\pi}\int_0^{2\pi}\mathrm{d}\phi\int_0^{\varepsilon_{\mathrm{c}}}\mathrm{d}\varepsilon_k\frac{\cos^2(2\phi)}{\xi_{kq}}\left\{\tanh\left[\frac{\xi_{kq}+h_0+Q}{2k_{\mathrm{B}}T}\right]+\tanh\left[\frac{\xi_{kq}-h_0-Q}{2k_{\mathrm{B}}T}\right]\right\} \quad (10)$$

and

$$\Omega_{\mathrm{s}}(T,h_0,q)=-\frac{1}{4}N(0)\Delta^2\left(1+2\left(\ln\frac{\Delta_{00}}{\Delta}\right)\right)-\frac{N(0)k_{\mathrm{B}}T}{\pi}\int_0^{2\pi}\mathrm{d}\phi\int_0^{\varepsilon_{\mathrm{c}}}\mathrm{d}\varepsilon_k \times\left\{\ln\left[1+\mathrm{e}^{-(\xi_{kq}+h_0+Q)/(k_{\mathrm{B}}T)}\right]+\ln\left[1+\mathrm{e}^{-(\xi_{kq}-h_0-Q)/(k_{\mathrm{B}}T)}\right]\right\}. \quad (11)$$

In order to determine the phase boundary between the normal and FFLO states, one needs to find the solutions of Eqs.(10) and (11) with $\Delta=0$ and the largest h_0. At $T=0$, we find that the largest value is taken as $h_0=1.06\Delta_{00}$, it leads to the window $0.56\Delta_{00}<h_0<1.06\Delta_{00}$ for the stability of the FFLO state, which is considerably larger than the window $0.707\Delta_{00}<h_0<0.754\Delta_{00}$ for the s-wave case.[8,9]

3. Josephson current in FdS/FdS junction

We adopt the Bogoliubov–de Gennes (BdG) approach[37] to study the transport coefficients of the quasiparticles in the d-wave S/ d-wave S structures under a Zeeman magnetic field on the d-wave S. In the absence of spin-flip scattering, the four-component

BdG equations may be decoupled into two sets of two-component equations: one for spin-up electronlike and spin-down holelike quasiparticle wave functions $(u_\uparrow, v_\downarrow)$, the other for $(u_\downarrow, v_\uparrow)$. The decoupled BdG equation has the form

$$\begin{pmatrix} H_0(\boldsymbol{r}) - \eta_\sigma h_0 & \Delta(T, h_0) \\ \Delta^*(T, h_0) & -H_0(\boldsymbol{r}) + \eta_{\bar\sigma} h_0 \end{pmatrix} \begin{pmatrix} u_\sigma(\boldsymbol{r}) \\ v_{\bar\sigma}(\boldsymbol{r}) \end{pmatrix} = E \begin{pmatrix} u_\sigma(\boldsymbol{r}) \\ v_{\bar\sigma}(\boldsymbol{r}) \end{pmatrix}, \tag{12}$$

where $H_0(\boldsymbol{r}) = -\hbar^2\nabla_{\boldsymbol{r}}^2/(2m) + V(\boldsymbol{r}) - E_{\mathrm{F}}$ with $V(\boldsymbol{r})$ the usual static potential, and the excitation energy E is measured relative to Fermi energy E_{F}. $\Delta(T, h_0)$ is the superconducting pair potential of d-wave S, and taken as $\Delta(T, h_0) = \Delta_\pm = \Delta_0(T, h_0)\cos(2\theta \mp 2\alpha)$ [33] corresponding to the pair potentials for electronlike and holelike quasiparticles, respectively, with θ the angle between the quasiparticle momentum and the S/S interface normal.

Considering an electron for spin σ incident on the interface at $x = 0$ from left-hand S at angle θ to the interface normal, we find that there are four possible trajectories: normal reflection (b_σ), Andreev reflection(AR) $(a_{\bar\sigma})$,[38] transmission to the right-hand electrode as an electronlike quasiparticle (c_σ) and as a holelike quasiparticle $(d_{\bar\sigma})$. For the parallel configuration, from the Eq.(12), the wave functions in two regions have the following form:

$$\Psi_{\mathrm{L}}(x) = \begin{pmatrix} u_{\mathrm{L}+}\mathrm{e}^{\mathrm{i}\phi_{\mathrm{L}+}} \\ v_{\mathrm{L}+} \end{pmatrix} \mathrm{e}^{\mathrm{i}k_+^{\mathrm{e}} x\cos\theta} + a_{\bar\sigma} \begin{pmatrix} v_{\mathrm{L}+}\mathrm{e}^{\mathrm{i}\phi_{\mathrm{L}+}} \\ u_{\mathrm{L}+} \end{pmatrix} \mathrm{e}^{\mathrm{i}\bar{k}_+^{\mathrm{h}} x\cos\theta} + b_\sigma \begin{pmatrix} u_{\mathrm{L}-}\mathrm{e}^{\mathrm{i}\phi_{\mathrm{L}-}} \\ v_{\mathrm{L}-} \end{pmatrix} \mathrm{e}^{-\mathrm{i}\bar{k}_-^{\mathrm{e}} x\cos\theta} \tag{13}$$

for $x < 0$; and

$$\Psi_{\mathrm{R}}(x) = c_\sigma \begin{pmatrix} u_{\mathrm{R}+}\mathrm{e}^{\mathrm{i}\phi_{\mathrm{R}+}} \\ v_{\mathrm{R}+} \end{pmatrix} \mathrm{e}^{\mathrm{i}k_+^{\mathrm{e}} x\cos\theta} + d_{\bar\sigma} \begin{pmatrix} v_{\mathrm{R}-}\mathrm{e}^{\mathrm{i}\phi_{\mathrm{R}-}} \\ u_{\mathrm{R}-} \end{pmatrix} \mathrm{e}^{-\mathrm{i}k_-^{\mathrm{h}} x\cos\theta} \tag{14}$$

for $x > 0$, where $a_{\bar\sigma}$, b_σ, c_σ and $d_{\bar\sigma}$ corresponding, respectively, to coefficients for the AR, normal reflection, transmission to the right FdS as electronlike quasiparticles and transmission as holelike quasiparticles. In the Eqs.(13) and (14),

$$k_+^{\mathrm{e}} = \sqrt{2m\left(E_{\mathrm{F}} + \sqrt{(E + \eta_\sigma h_0)^2 - |\Delta_+(T, h_0)|^2}\right)}/\hbar \tag{15}$$

and

$$\bar{k}_-^{\mathrm{e}} = \sqrt{2m\left(E_{\mathrm{F}} + \sqrt{(E + \eta_\sigma h_0)^2 - |\Delta_-(T, h_0)|^2}\right)}/\hbar \tag{16}$$

are the wave vectors for the electronlike quasiparticles corresponding to two different transport directions,

$$k_-^{\mathrm{h}} = \sqrt{2m\left(E_{\mathrm{F}} - \sqrt{(E - \eta_{\bar\sigma} h_0)^2 - |\Delta_-(T, h_0)|^2}\right)}/\hbar \tag{17}$$

and

$$\bar{k}_+^{\mathrm{h}} = \sqrt{2m\left(E_{\mathrm{F}} - \sqrt{(E - \eta_{\bar\sigma} h_0)^2 - |\Delta_+(T, h_0)|^2}\right)}/\hbar \tag{18}$$

are the wave vectors for the holelike quasiparticles corresponding to two different transport directions.

$$\left(u_{\mathrm{L(R)}\pm}\right)^2 = \left\{1 + \sqrt{1 - |\Delta_\pm|^2/\left(E + \eta_\sigma h_0\right)^2}\right\}/2, \tag{19}$$

$$\left(v_{\mathrm{L(R)}\pm}\right)^2 = \left\{1 - \sqrt{1 - |\Delta_\pm|^2/\left(E - \eta_{\bar\sigma} h_0\right)^2}\right\}/2, \tag{20}$$

$$\phi_\pm = \cos^{-1}[\cos 2(\theta \mp \alpha)/|\cos 2(\theta \mp \alpha)|], \tag{21}$$

$$\phi_{\mathrm{L(R)}\pm} = \phi_{\mathrm{L(R)}} + \phi_\pm. \tag{22}$$

All the coefficients in Eqs.(13) and (14) can be determined by the boundary conditions at $x = 0$. They are $\Psi_{\mathrm{II}}(0) = \Psi_{\mathrm{I}}(0)$, $(\mathrm{d}\Psi_{\mathrm{II}}/\mathrm{d}x)_{x=0} - (\mathrm{d}\Psi_{\mathrm{I}}/\mathrm{d}x)_{x=0} = 2mU_0\Psi_{\mathrm{I}}(0)/\hbar^2$. Using the boundary conditions on the wave functions, we find

$$a_{\bar\sigma} = Q/W, \tag{23}$$

$$\begin{aligned} Q = {} & z^2\Big[u_{\mathrm{R}+}u_{\mathrm{R}-}u_{\mathrm{L}+}v_{\mathrm{L}-}\mathrm{e}^{\mathrm{i}(\phi_+ - \phi_-)} \\ & + v_{\mathrm{R}+}v_{\mathrm{R}-}v_{\mathrm{L}+}u_{\mathrm{L}-}\mathrm{e}^{\mathrm{i}(\phi_- - \phi_+)}\Big] \\ & -(1+z^2)(u_{\mathrm{R}+}u_{\mathrm{R}-}v_{\mathrm{L}+}u_{\mathrm{L}-} + u_{\mathrm{L}+}v_{\mathrm{L}-}v_{\mathrm{R}+}v_{\mathrm{R}-}) \\ & + u_{\mathrm{L}+}u_{\mathrm{L}-}u_{\mathrm{R}-}v_{\mathrm{R}+}\mathrm{e}^{-\mathrm{i}\phi} \\ & + v_{\mathrm{L}+}v_{\mathrm{L}-}v_{\mathrm{R}-}u_{\mathrm{R}+}\mathrm{e}^{\mathrm{i}\phi}, \end{aligned} \tag{24}$$

$$\begin{aligned} W = {} & \left(1+z^2\right)\left(u_{\mathrm{R}+}u_{\mathrm{R}-}u_{\mathrm{L}+}u_{\mathrm{L}-} + v_{\mathrm{R}+}v_{\mathrm{R}-}v_{\mathrm{L}+}v_{\mathrm{L}-}\right) \\ & -z^2\Big[u_{\mathrm{R}+}u_{\mathrm{R}-}v_{\mathrm{L}+}v_{\mathrm{L}-}\mathrm{e}^{\mathrm{i}(\phi_+ - \phi_-)} \\ & + v_{\mathrm{R}+}v_{\mathrm{R}-}u_{\mathrm{L}+}u_{\mathrm{L}-}\mathrm{e}^{\mathrm{i}(\phi_- - \phi_+)}\Big] \\ & -v_{\mathrm{R}+}u_{\mathrm{R}-}u_{\mathrm{L}-}v_{\mathrm{L}+}\mathrm{e}^{-\mathrm{i}\phi} \\ & -u_{\mathrm{R}+}v_{\mathrm{R}-}u_{\mathrm{L}+}v_{\mathrm{L}-}\mathrm{e}^{\mathrm{i}\phi}, \end{aligned} \tag{25}$$

where $\phi = \phi_{\mathrm{R}} - \phi_{\mathrm{L}}$, $z = z_0/\cos\theta$, $z_0 = mU_0/\left(\hbar^2 k_{\mathrm{F}}\right)$. Since h_0 is much smaller than E_{F}, we have made the approximation of $\bar{k}^{\mathrm{e(h)}}_{\mathrm{L(R)}\pm} = k^{\mathrm{e(h)}}_{\mathrm{L(R)}\mp} \approx k_{\mathrm{F}}$. For the antiparallel configuration, the coefficient of AR $a_{\bar\sigma}(E,\phi,\theta)$ will be obtained by changing η_σ to $\eta_{\bar\sigma}$ in Eqs.(23)–(25) from $u_{\mathrm{R}\pm}$ and $v_{\mathrm{R}\pm}$. Following obtained $a_{\bar\sigma}(E,\phi,\theta)$, the Josephson current can be calculated by using the generalized coefficient of AR $a_{\bar\sigma}(E,\phi,\theta)$, which is obtained by analytic continuation of E to $\mathrm{i}w_{\mathrm{n}}$, yielding[31,32]

$$\begin{aligned} I = {} & \frac{e\Delta_{\mathrm{L}}k_{\mathrm{B}}T}{2\hbar}\mathrm{Re}\sum_{w_{\mathrm{n}}}\sum_{\sigma=\uparrow\downarrow}\int_{-\pi/2}^{\pi/2} \\ & \times\frac{a_\sigma(\mathrm{i}w_{\mathrm{n}},\phi,\theta) - a_\sigma(\mathrm{i}w_n,-\phi,\theta)}{\Omega_{n\sigma}}\cos\theta\mathrm{d}\theta, \end{aligned} \tag{26}$$

where $\Omega_{n\uparrow\downarrow} = \sqrt{\left(w_{\mathrm{n}} \mp \mathrm{i}h_0\right)^2 + \Delta^2(T,h_0)}$, and the Matsubara frequency $w_{\mathrm{n}} = (2n+1)\pi k_{\mathrm{B}}T$ with $n = 0, \pm1, \pm2, \cdots$. Substituting Eqs.(23)–(25) into Eq.(26), we obtain the Josephson current for the parallel configuration as for $\alpha = 0$

$$I_{\mathrm{P}} = \frac{\pi k_{\mathrm{B}}T\sin\phi}{2e}\mathrm{Re}\left\{\sum_{w_{\mathrm{n}}}\int_{-\pi/2}^{\pi/2}(f_1+f_2)\cos\theta\mathrm{d}\theta\right\}, \tag{27}$$

$$f_1 = \frac{2\Delta_0^2\cos^2(2\theta)}{\Delta_0^2\cos^2(2\theta)\cos\phi + z^2\Delta_0^2\cos^2(2\theta) + (1+z^2)\left[2\left(w_{\mathrm{n}} + \mathrm{i}h_0\right)^2 + \Delta_0^2\cos^2(2\theta)\right]}, \tag{28}$$

$$f_2 = \frac{2\Delta_0^2\cos^2(2\theta)}{\Delta_0^2\cos^2(2\theta)\cos\phi + z^2\Delta_0^2\cos^2(2\theta) + (1+z^2)\left[2\left(w_{\mathrm{n}} - \mathrm{i}h_0\right)^2 + \Delta_0^2\cos^2(2\theta)\right]}, \tag{29}$$

and for $\alpha = \pi/4$

$$I_{\mathrm{P}} = \frac{\pi k_{\mathrm{B}}T\sin\phi}{2e}\mathrm{Re}\left\{\sum_{w_{\mathrm{n}}}\int_{-\pi/2}^{\pi/2}(f_3+f_4)\cos\theta\mathrm{d}\theta\right\}, \tag{30}$$

$$f_3 = \frac{2\Delta_0^2\sin^2(2\theta)}{\Delta_0^2\sin^2(2\theta)\cos\phi - z^2\Delta_0^2\cos^2(2\theta) + (1+z^2)\left[2\left(w_{\mathrm{n}} + \mathrm{i}h_0\right)^2 + \Delta_0^2\sin^2(2\theta)\right]}, \tag{31}$$

$$f_4 = \frac{2\Delta_0^2\sin^2(2\theta)}{\Delta_0^2\sin^2(2\theta)\cos\phi - z^2\Delta_0^2\sin^2(2\theta) + (1+z^2)\left[2\left(w_{\mathrm{n}} - \mathrm{i}h_0\right)^2 + \Delta_0^2\sin^2(2\theta)\right]}. \tag{32}$$

Similarly, we obtain the Josephson current for the antiparallel configuration as for $\alpha = 0$

$$I_{\mathrm{AP}} = \frac{\pi k_{\mathrm{B}} T \sin\phi}{2e} \mathrm{Re}\left\{ \sum \int_{-\pi/2}^{\pi/2} 2 f_5 \cos\theta \mathrm{d}\theta \right\}, \tag{33}$$

$$f_5 = \frac{2\Delta_0^2 \cos^2(2\theta)}{(1+z^2)\left[w_{\mathrm{n}}^2 + h_0^2 + \xi_1\right] - \Delta_0^2 \cos^2(2\theta)\cos\phi - z^2\left[w_{\mathrm{n}}^2 + h_0^2 - \xi_1\right]}, \tag{34}$$

$$\xi_1 = \sqrt{\left(w_{\mathrm{n}} + \mathrm{i}h_0\right)^2 + \Delta_0^2 \cos^2(2\theta)} \sqrt{\left(w_{\mathrm{n}} - \mathrm{i}h_0\right)^2 + \Delta_0^2 \cos^2(2\theta)}, \tag{35}$$

and for $\alpha = \pi/4$

$$I_{\mathrm{AP}} = \frac{\pi k_{\mathrm{B}} T \sin\phi}{2e} \mathrm{Re}\left\{ \sum_{w_{\mathrm{n}}} \int_{-\pi/2}^{\pi/2} 2 f_6 \cos\theta \mathrm{d}\theta \right\}, \tag{36}$$

$$f_6 = \frac{2\Delta_0^2 \sin^2(2\theta)}{(1+z^2)\left[w_{\mathrm{n}}^2 + h_0^2 + \xi_2\right] - \Delta_0^2 \sin^2(2\theta)\cos\phi + z^2\left[w_{\mathrm{n}}^2 + h_0^2 - \xi_2\right]}, \tag{37}$$

$$\xi_2 = \sqrt{\left(w_n + \mathrm{i}h_0\right)^2 + \Delta_0^2 \sin^2(2\theta)} \sqrt{\left(w_{\mathrm{n}} - \mathrm{i}h_0\right)^2 + \Delta_0^2 \sin^2(2\theta)}. \tag{38}$$

If h_0 is taken to be zero in above equations, we obtain the Josephson current $I = I_{\mathrm{P}} = I_{\mathrm{AP}}$. However, in the presence of h_0, the Josephson currents of the junction for the parallel and antiparallel configurations are different. Figures 4 and 5 show numerical results for the I_{P} and I_{AP}, respectively, as functions of the temperature at different magnetic exchange energy h_0 and different barrier strength z_0. With increasing temperature, I_{P} and I_{AP} decrease and drop to zero at $T_{\mathrm{c}}(h_0)$, which is lower than $T_{\mathrm{c}}(h_0 = 0)$. Such a sudden drop in I_{P} and I_{AP} in the presence of h_0 results due to a drop in $\Delta(T, h_0)$ from finite value to zero at $T_{\mathrm{c}}(h_0)$, at which there is a first-order phase transition from a superconducting state with finite $\Delta(T, h_0)$ to a normal state of $\Delta = 0$. $T_{\mathrm{c}}(h_0)$, at which the Josephson current drops to zero, shown by Fig.2, depending neither on the magnitude of z_0 nor on the magnetic configuration of two FdSs, as seen in Figs.4 and 5. It's shown that the presence of h_0 always suppresses the Josephson critical current at higher temperature for either parallel or antiparallel configuration. However, at lower temperature, the exchange field can under certain conditions enhance the Josephson critical current in a d-wave S/I/d-wave S junction, the conditions are: for $\alpha = \pi/4$, one needs weak barrier strength [$z_0 = 0.5$, in Fig.4(c)] and a parallel configuration. In the case of $\alpha = 0$, the enhancement of Josephson critical current only on condition of strong barrier strength [$z_0 = 5$, in Fig.5(b)] and an antiparallel configuration of the two FdS electrodes. The interesting effect for I_{AP} increasing with h_0 is very similar to that presented in Ref.[29]. It is worth pointing out here that this effect holds only for strong barrier strength, low temperature, and the antiparallel configuration. It is easily seen that, in the large z_0 limit ($z_0 \to \infty$) , and $\alpha = \beta = \theta = 0$, Eq.(26) in this paper is reduced just to Eqs.(7) and (8) of Ref.[29].

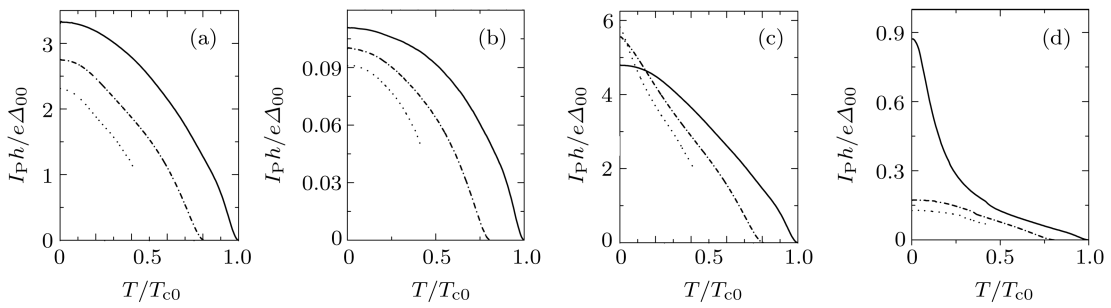


Fig.4. Dependence of the normalized critical current on T for different Zeeman energies: $h_0/\Delta_{00} = 0$ (solid line), $h_0/\Delta_{00} = 0.4$ (dash dot line), $h_0/\Delta_{00} = 0.5$ (dot line), in the case of a parallel orientation. Here $\phi = \pi/2$, in which $\alpha = 0$ and $z_0 = 0.5$ (a); $\alpha = 0$ and $z_0 = 5$ (b); $\alpha = \pi/4$ and $z_0 = 0.5$ (c); and $\alpha = \pi/4$ and $z_0 = 5$ (d).

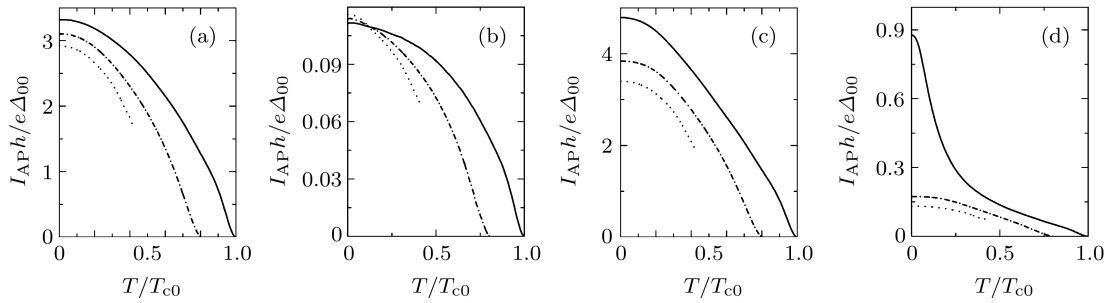


Fig.5. Dependence of the normalized critical current on T for different Zeeman energies: $h_0/\Delta_{00} = 0$ (solid line), $h_0/\Delta_{00} = 0.4$ (dash dot line), $h_0/\Delta_{00} = 0.5$ (dot line), in the case of a antiparallel orientation. Here $\phi = \pi/2$, in which $\alpha = 0$ and $z_0 = 0.5$ (a); $\alpha = 0$ and $z_0 = 5$ (b); $\alpha = \pi/4$ and $z_0 = 0.5$ (c); and $\alpha = \pi/4$ and $z_0 = 5$ (d).

4. Conclusion

We have studied the Zeeman effects on d-wave S and the Josephson current in a d-wave S/I/d-wave S junction under a Zeeman magnetic field on two Ss. In the mean-field approximation, the self-consistent equations for the ferromagnetic d-wave superconducting gap and the exchange energy are considered simultaneously, and the thermodynamic potential in FdS is obtained. Two necessary conditions are required for the coexistence of superconductivity and ferromagnetism; one is that there is a nonzero solution for the superconducting order parameter in the energy gap equation; the other is that the thermodynamic potential in the superconducting state should be lower than that in the normal state. The Josephson currents in a FdS/I/FdS tunnel junction have been analytically obtained as a function of the temperature, exchange field, and the insulating barrier strength. It is found that an increase in temperature or insulating barrier strength always gives rise a suppression of the Josephson critical current for either parallel or antiparallel configuration. Also, it is shown that the presence of exchange field always suppresses the Josephson critical current at higher temperature for either parallel or antiparallel configuration. However, at lower temperature, the critical current I_P increases with exchange field for strong barrier strength in the case of $\alpha = 0$, and the critical current I_{AP} increases with exchange field for weak barrier strength in the case of $\alpha \neq 0$, the feature may be used to distinguish between FdS and FsS. It is expected that the theoretical results obtained may be further confirmed in the future experiment.

References

[1] Combescot R and Mora C 2005 *Phys. Rev.* B **71** 144517
[2] Mora C and Combescot R 2005 *Phys. Rev.* B **71** 214504
[3] Voronsov B A, Sauls J A and Graf M M 2005 *Phys. Rev.* B **72** 184501
[4] Dong Z C 2005 *Chin. Phys.* **14** 1209
[5] Clogston A M 1962 *Phys. Rev. Lett.* **9** 266
[6] Chandrasekhar B S 1962 *Appl. Phys. Lett.* **1** 7
[7] Fulde P 1973 *Adv. Phys.* **22** 667
[8] Fulde P and Ferrell R A 1964 *Phys. Rev.* A **135** 550
[9] Larkin A I and Ovchinnikov Y N 1964 *Zh. Eksp. Teor. Fiz.* **47** 1136
[10] Wu W and Adams P W 1994 *Phys. Rev. Lett.* **73** 1412
[11] Bulaevskii L, Buzdin A and Maley M 2003 *Phys. Rev. Lett.* **90** 067003
[12] Konoike T, Uji S, Terashima T, Nishimura M, Yasuzuka S, Enomoto K, Fujiwara H, Zhang S and Kobayashi H 2004 *Phys. Rev.* B **70** 094514
[13] Tanatar M A Suzuki M Ishiguro T, Tanaka H, Fujiwara H, Koboyashi H, Toito T and Yamada J 2003 *Synth. Met.* **137** 1291
[14] Radovan H, Fortune N A, Murphy T P, Hannahs S T, Palm E C, Tozer S W and Hall D 2003 *Nature* (London) 425 51
[15] Won H, Maki K, Haas S, Oeschler N, Weickert F and Gegenwart P 2004 *Phys. Rev.* B **69** 180504(R)
[16] Kakuyanagi K, Saitoh M, Kumagai K, Takashima S, Nohara M, Takagi H and Matsuda Y 2005 *Phys. Rev. Lett.* **94** 047602
[17] Kumagai K, Saitoh M, Oyaizu T, Furukawa Y, Takashima S, Nohara M, Takagi H and Matsuda Y 2006 *Phys. Rev. Lett.* **97** 227002
[18] Rourke P M C, Tanatar M A, Turel C S, Berdeklis J, Petrovic C and Wei J Y T 2005 *Phys. Rev. Lett.* **94** 107005
[19] Park W K, Greene L H, Sarrao J L and Thompson J D 2005 *Phys. Rev.* B **72** 052509
[20] Kasahara Y, Nakajima Y, Izawa K, Matsuda Y, Behnia K, Shishido H, Settai R and Onuki Y 2005 *Phys. Rev.* B **72** 214515
[21] Cui Q H, Hu C R, Wei J Y T and Yang K 2006 *Phys. Rev.* B **73** 214514

[22] Wang Q, Hu C R and Ting C S 2006 *Phys. Rev.* B **74** 214501
[23] Tanaka Y, Asano Y, Ichioka M and Kashiwawa S 2007 *Phys. Rev. Lett.* **98** 077001
[24] Ryazanov V V, Oboznov V A, Rusanov A Y, Veretennikov A V, Golubov A A and Aarts J 2001 *Phys. Rev. Lett.* **86** 2427
[25] Kontos T, Aprili M, Lesueur J, Genet F, Stephanidis B and Boursier R 2002 *Phys. Rev. Lett.* **89** 137007
[26] Radovic Z, Lazarides N and Flytzanis N 2003 *Phys. Rev.* B **68** 14501
[27] Volkov A F and Anishchanka A 2005 *Phys. Rev.* B **71** 24501
[28] Bergeret F S, Volkov A F and Efetov K B 2005 *Rev. Mod. Phys.* **77** 1321
[29] Bergeret F S, Volkov A F and Efetov K B 2001 *Phys. Rev. Lett.* **86** 3140
[30] Li X W, Zheng Z M, Xing D Y, Sun G Y and Dong Z C 2002 *Phys. Rev.* B **65** 134507
[31] Li X W 2002 *Acta Phys. Sin.* **51** 1821 (in Chinese)
[32] Furusaki A and Tsukada M 1991 *Solid State Commun.* **78** 299
[33] Tanaka Y and Kashiwaya S 1995 *Phys. Rev. Lett.* **74** 3451
[34] Hu C R 1994 *Phys. Rev. Lett.* **72** 1526
[35] Yang K and Sondhi S L 1988 *Phys. Rev.* B **57** 8566
[36] Abrikosov A A 1988 *Fundamentals of the Theory of Metals* (New York: Elsevier)
[37] de Gennes P G 1966 *Superconductivity of Metals and Aollys* (New York: Benjamin)
[38] Andreev A F 1964 *Zh. Eksp. Teor. Fiz.* **46** 1823