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Cellular automaton model considering headway-distance effect*

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This paper presents a cellular automaton model for single-lane traffic flow. On the basis of the Nagel–Schreckenberg (NS) model, it further considers the effect of headway-distance between two successive cars on the randomization of the latter one. In numerical simulations, this model shows the following characteristics. (1) With a simple structure, this model succeeds in reproducing the hysteresis effect, which is absent in the NS model. (2) Compared with the slow-to-start models, this model exhibits a local fundamental diagram which is more consistent to empirical observations. (3) This model has much higher efficiency in dissolving congestions compared with the so-called NS model with velocity-dependent randomization (VDR model). (4) This model is more robust when facing traffic obstructions. It can resist much longer shock times and has much shorter relaxation times on the other hand. To summarize, compared with the existing models, this model is quite simple in structure, but has good characteristics.

Keywords: traffic flow, headway-distance, phase-transition, fundamental diagram
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1. Introduction

Traffic phenomena have attracted much interest from physicists since the 1950s. Several traffic models have been established to simulate traffic systems in order to understand the traffic characters.[1−16] Among them, cellular automaton (CA) models are broadly used owing to their simpleness and practicality in computer simulations. In 1992, Nagel and Schreckenberg (NS) suggested a 'minimal' CA model called the NS model which later becomes the basis of this field.[17] In the NS model, due to the variables of discrete time and space, the cars update their positions and velocities according to the following rules:

1) acceleration, $v_j(t) \to v_j\left(t+\frac{1}{3}\right) = \min(v_j(t)+1, v_{\max})$;

2) braking, $v_j\left(t+\frac{1}{3}\right) \to v_j\left(t+\frac{2}{3}\right) = \min\left(v_j\left(t+\frac{1}{3}\right), d_j(t)\right)$;

3) randomization, $v_j\left(t+\frac{2}{3}\right) \to v_j(t+1) = \max\left(0, v_j\left(t+\frac{2}{3}\right)-1\right)$ with probability p;

4) driving, car j moves $v_j(t+1)$ cells ahead.

The $v_{\max}$ above is the maximal velocity of a car, and $d_j(t)$ is the number of empty cells in front of car j. In their simulations, the road is divided into cells of length 7.5m, i.e., the normal space headway of two successive cars in real traffic. Each cell can either be empty or occupied by just one vehicle at a certain time. Meanwhile, the time is divided into time steps of one second. Thus, a velocity of n means $n \times 7.5$m/s.

The NS model can reproduce phenomena such as phase transition, phase separation, traffic congestion, etc., which have been observed in real traffic. It is a minimal model because each of the four updating steps is necessary for producing realistic results. With only one parameter p to induce random process, the NS model simulates single-lane traffic quite successfully. It has great importance in CA field. But just because of its simpleness, it does not carry out some details in traffic. So some important characters of real traffic cannot be reproduced yet, such as metastability and

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hysteresis. Moreover, the flow flux simulated by the NS model is 40% lower than the flux measured in real traffic. In order to make up these shortages, several mended CA models are proposed. In these models, detail motions in traffic are considered in the updating process. Such as slow-to-start actions,[18−20] velocity effect,[21] brake light effect[22] etc. Most of these models are derived from the NS model but added new updating rules to it. With these models, we can simulate hysteresis and other phenomena which are absent in the NS model.

In this paper, we consider the effect of headway-distance between two successive cars on the randomization of the latter one. Being only slightly different to the NS model, we obtain several expected simulation results.

2. A CA model with headway-distance effect

In the NS model, p is the only parameter inducing random process. The randomization action reflects the fluctuations at maximum velocity, the retarded acceleration and the over-reactions at braking in real traffic.[23] In the NS model, p is a constant in spite of varying traffic conditions. However the traffic situations influence the drivers' actions seriously in reality. In the experiences of real-driving, the braking motions always depend on the relative speed to the neighbouring vehicles, and the headway-distance to the frontier. However, the latter factor has not been particularly studied. We believe that when the headway-distance is large enough for safety, the driver would like to maintain his velocity rather than over-deceleration. On the other hand, smaller headway-distances will make the driver more cautious thus induce more probability to decelerate. Considering this factor, we suggest a simple model with headway-distance (HD) effect and call it HD model for short. In fact, we do not expect to describe the traffic character so comprehensively or accurately with such a simple attempt. So in this paper, we are just trying to understand the influence of the HD effect. In our model, the randomization probability

$$p = \begin{cases} (v_{\max} - d) \times p_0, & d < v_{\max}, \\ 0, & d \geq v_{\max}. \end{cases}$$

In this paper, $p_0 = 0.18$, and the road length is 10000, with periodical boundary conditions. The other updating rules of NS model are kept unchanged.

3. Fundamental diagrams

The fundamental diagram is an important approach in the study of traffic flow. It can reflect the primary character of the flux–velocity–density system and is often used to evaluate the rationality of traffic models. Figure 1 shows the hysteresis curves in the fundamental diagram of the HD model. In the diagram, the curve of 'Jam' is obtained from a jamming initial state, in which all the vehicles congest together tightly. After evolution for a long time, we plot the averaged flow over several time steps. At the same time, we obtain the other curve of 'Hom' from homogeneous initial states, in which vehicles are equal-distance arranged.[18] In Fig.1, we can see hysteresis curves between ρ_1 and ρ_2, with similar characteristics to those of the slow-to-start (s2s) models.[18−20] ρ_1 and ρ_2 in the figure show the positions of the two phase-transition points, which are boundaries of phases having different macroscopic characters of traffic flow. In the region $\rho < \rho_1$, homogeneous states are steady. Jams will dissolve soon and the traffic will turn into a homogeneous free flow over evolvement. In the region $\rho > \rho_2$, jamming states are steady. Little jams will assemble to large ones and the traffic will be steady on a phase-separated state in the end. There are one or more large jams and several free flow regimes in the final system. In the region between ρ_1 and ρ_2, metastable states can live long and finally evolve to different steady states determined by the initial conditions.

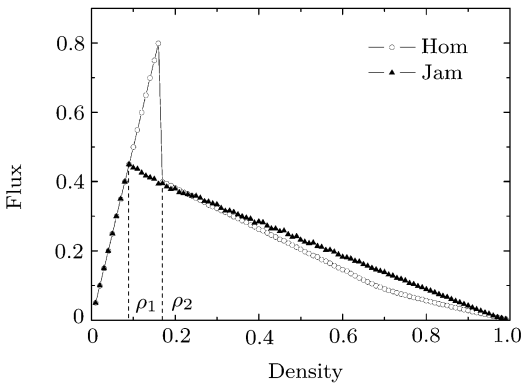


Fig.1. Hysteresis curves in the fundamental diagrams, obtained by the HD model. The x-axis is the density of cars and the y-axis is the global flux. The upper branch is obtained from a homogeneous initial state while the lower branch from a completely jamming initial state.

The so-called s2s models[18−20] have similar fundamental diagrams to our model. They can reproduce hysteresis effect but have two fatal shortcomings. Firstly, the maximum flows simulated by the s2s models are even lower than the simulated results of the NS model which has been lower than real measurements already. Secondly, the s2s models reproduce a worse local fundamental diagram, which should have reflected the relationships between local densities and local fluxes and distinguished different traffic clearly.[24,25] Figure 2(b) shows the local fundamental diagram of the s2s models. We can hardly distinguish the free flow and the congested flow in the diagram. Thus the s2s models reproduce the empirically observed congested phase unfitted. In contrast, Fig.2(a) shows the local fundamental diagram of the HD model. Although only coarse variables are adopted so that detailed regime of the congested flow cannot be described, the free flow phase and the congested phase can be roughly distinguished. So the HD model shows better results than the s2s models.

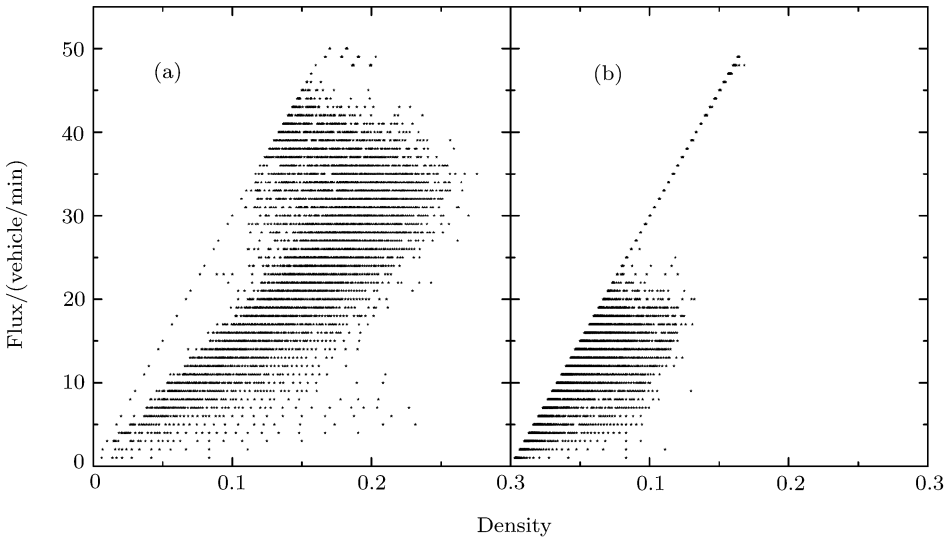


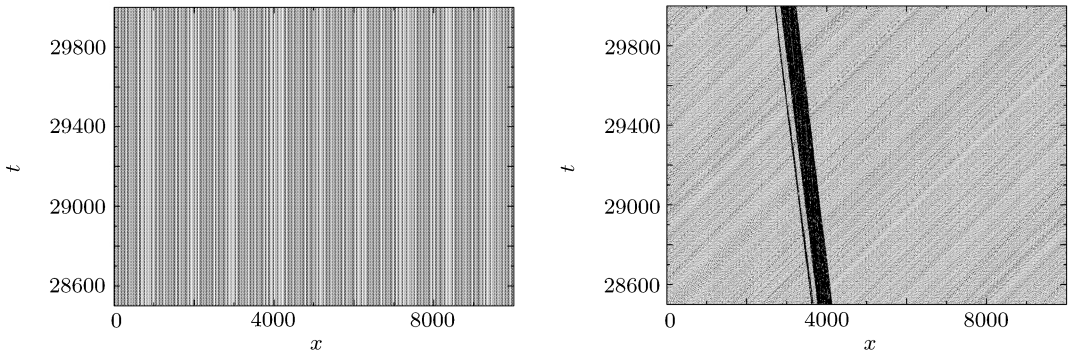
Fig.2. Local fundamental diagrams of the HD model (a) and the s2s models (b).

Therefore, the HD model is an obvious improvement on the bases of the NS model and the s2s models. It has quite simple and realistic updating rules. It can reproduce hysteresis curves and realistic local fundamental diagrams. It has flow fluxes adjacent to the real measurements. So it is worth studying on the model further. More simulation results will be shown in the following part of this paper.

4. Characters near the phase-transition points

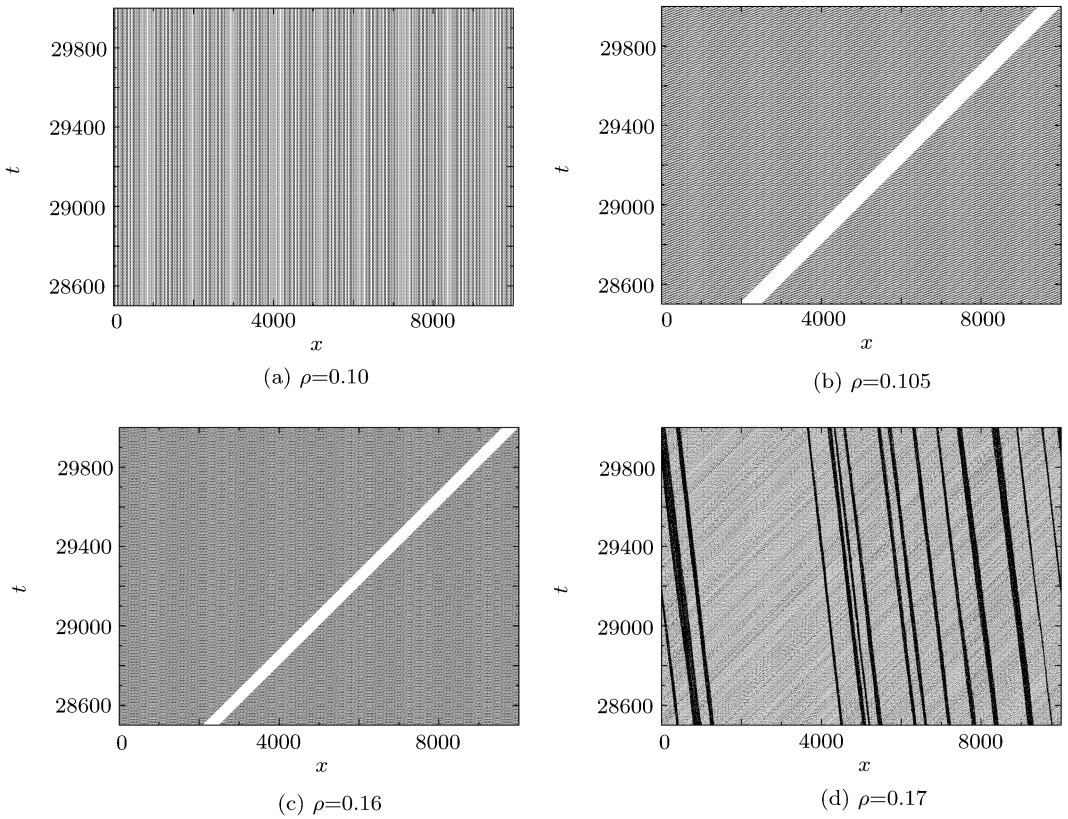
Figure 3 shows the sensitivity to initial states on the spatial-temporal diagrams. It can give us intuitive impressions. Figure 3(a) shows the diagram from a homogeneous initial state while Fig.3(b) from a jamming initial state. We can see that the former is symmetrical everywhere, however the latter shows a road which consists of a jamming part and a homogeneous part. The global densities are both $\rho = 0.10$, near ρ_1. Only with different initial states, we obtain totally different outcomes.

Figure 4 shows the phase-transition phenomena in the spatial-temporal diagrams at the two phase-transition points signed in Fig.1. Figures 4(a) and 4(b) show the phase-transition near ρ_1; Figs.4(c) and 4(d) show the phase-transition near ρ_2. The diagrams tells us that only a tiny difference in densities near the phase-transition points can produce totally different steady states in the end. The two phase-transition points divide the whole density scale into three phases. In the $\rho < \rho_1$ region, the global prospect of the steady states are reflected in Fig.4(a). In the $\rho_1 < \rho < \rho_2$ region, the global prospect is shown in Figs.4(b) and 4(c). The road is covered with homogeneous flow but has a narrow gap of sparseness in it. Phase separation phenomenon begins to occur. In the $\rho > \rho_2$ region, the global prospect is shown in Fig.4(d). The road is covered with several jamming regimes and homogeneous areas.



(a) From homogeneous initial state (b) From jamming initial state

Fig.3. Sensitivity to initial states shown on the spatial-temporal diagrams. (a) From a homogeneous initial state. (b) From a completely jamming initial state. The global density are both $\rho = 0.10$.



(a) ρ=0.10 (b) ρ=0.105

(c) ρ=0.16 (d) ρ=0.17

Fig.4. Phase transitions shown in the spatial-temporal diagrams. (a) $\rho = 0.10$. (b) $\rho = 0.105$. (c) $\rho = 0.16$. (d) $\rho = 0.17$. All from homogeneous initial states.

Following the time-dependence study on the VDR model,[18] we find that Fig.5 shows the process of dissolving initial jams in the system. Beginning from a complete jamming state, we plot the lengthes of the largest jam in the system with time, which reflects the relaxation process of traffic congestion. Under the density near $\rho = \rho_1$, homogeneous states are steady. The length of the biggest jam will sooner or later decay to 1. In the VDR model, it takes more than 80000 time steps to accomplish this process, and the time-dependence shows an exponential form. On the other hand, Fig.6 shows a linear decay. It takes only about 1200 time steps to reach the steady state. So the HD model has a much higher efficiency in dissolving jams, which is a remarkable virtue and will benefit the traffic capacity greatly. Figure 5 shows the cluster-size dis-

tributions in the system near $\rho = \rho_2$. It characters like exponential form approximately. As in the NS model, the cluster-size distribution near the phase-transition point has an exponential form. The HD model has similar character near ρ_2.

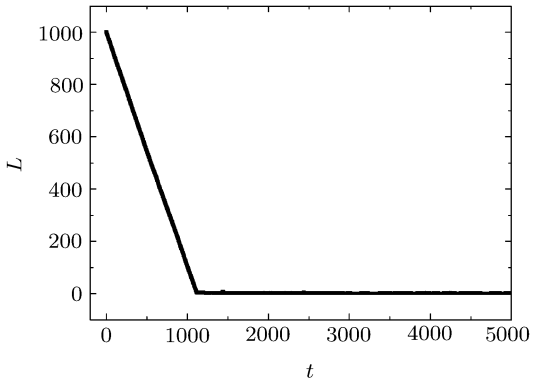


Fig.5. Time-dependence of the maximum length of jams in the system near $\rho = \rho_1$, which starts from a complete jamming initial state.

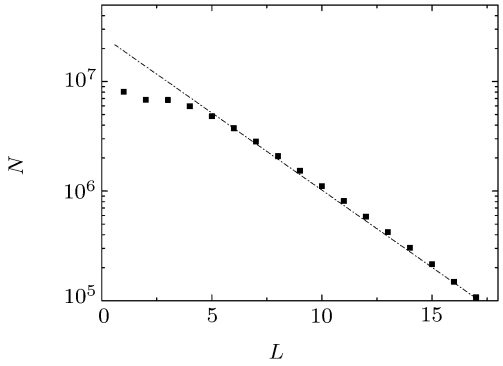


Fig.6. Cluster-size distributions in the system near $\rho = \rho_2$, which starts from a homogeneous initial state. The x-axis is the cluster size L, and the y-axis is the number of vehicles frozen in jams with certain sizes.

Figure 7 simulates a shock in the traffic induced by obstruct for 100 time steps, as the situation when facing a traffic light. The global density is near ρ_1. Vehicles accumulate near the obstruct and dissolve gradually after the traffic resumes, shown as the black parallelogram and the narrow tail in the figure. This process has firstly been studied by the velocity-effect (VE) model before.[21] In their simulation, a shock for 30 time steps induces a black triangle in the spatial-temporal diagram. Compared with the VE model, the HD model is more steady that it can resist much longer shock time when inducing such a process, and has a much shorter relaxation time on the other hand.

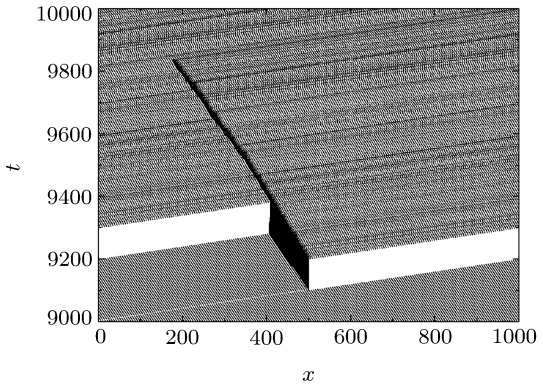


Fig.7. Shock in the traffic induced by obstruct for 100 time steps.

5. Conclusions

In this paper, we propose a new CA model derived from the NS model. In addition, we consider the effect of the headway-distance between two successive cars on the randomization parameter. The mended rules is more fit for real traffic experience. Through simulation, the mended model can have both the benefits of the NS model and the s2s models. It can simulate complex traffic phenomena such as phase transition, phase separation etc. Additionally, it can produce more realistic results compared with existing models, such as hysteresis phenomenon and flow flux adjacent to real measurements. This model has higher traffic efficiency and is more sensitive to the initial states than the NS model. This can be clearly shown on the macroscopic character of the whole system. However, besides its superiority, this model is still very simple, and the variables are coarse. Further work is expected to be developed.

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