Vol 17 No 5, May 2008
1674-1056/2008/17(05)/1840-05

Chinese Physics B

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Design of phase plates for shaping partially coherent beams by simulated annealing*

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(Received 10 May 2007; revised manuscript received 14 September 2007)

Taking the Gaussian Schell-model beam as a typical example of partially coherent beams, this paper applies the simulated annealing (SA) algorithm to the design of phase plates for shaping partially coherent beams. A flow diagram is presented to illustrate the procedure of phase optimization by the SA algorithm. Numerical examples demonstrate the advantages of the SA algorithm in shaping partially coherent beams. An uniform flat-topped beam profile with maximum reconstruction error RE $< 1.74\%$ is achieved. A further extension of the approach is discussed.

Keywords: partially coherent beam, phase plate, simulated annealing (SA), beam shaping
PACC: 4280k, 4230

1. Introduction

Much effort has been recently exhibited in partially coherent laser beams because of their importance in both theoretical and applicative aspects. For some applications, such as micromachining and laser-fusion etc., the far-field beam shaping with phase plates is required.[1,2] The shaping of partially coherent UV-excimer laser beams with multifaceted micro-mirrors and diffractive phase elements has been studied experimentally,[3−5] and designed numerically based on the geometrical transformation and improved iterative Fourier-transform algorithm (IFTA).[6,7] On the other hand, it is known that the simulated annealing (SA) algorithm first proposed by Kirkpatrick *et al*[8] is based on an analogy with the Monte Carlo method to provide an efficient simulation of phase plates to achieve beam shaping. So far, the SA and genetic algorithms etc. have been successfully used for the design of a variety of phase plates illuminated by fully coherent beams.[9−18] However, the application of the SA algorithm to partially coherent beams has not been found. The purpose of this paper is to apply the SA algorithm to the design of phase plates for shaping partially coherent beams, where for the mathematical simplicity a two-dimensional Gaussian Schell-model (GSM) beam is taken as a typical example of partially coherent beams. Section 2 gives the theoretical formulation, in which an expression for the far-field intensity of GSM beams propagating through a phase plate is derived. In Section 3 a flow diagram of phase optimization for shaping GSM beams based on the SA algorithm is presented, and numerical illustrative results are given. Finally, Section 4 summarizes the main results obtained in this paper and discusses the further extension of our results to the three-dimensional case and to other types of partially coherent laser beams.

2. Theoretical formulation

Consider a two-dimensional quasi-monochromatic GSM beam whose mutual intensity at the plane $z = 0$ in the space–time domain reads as[15]

$$J_0(x_1, x_2, z = 0) = A_0 \exp\left[-\frac{x_1^2 + x_2^2}{w_0^2}\right] \exp\left[-\frac{(x_1 - x_2)^2}{2\sigma_0^2}\right], \quad (1)$$

where w_0 and σ_0 denote the waist width and correlation length, respectively. A_0 is a constant and x_1, x_2 denote transversal coordinates of two points at the plane $z = 0$.

Assume that there is a phase plate with complex-valued transmissivity $t(x)$ at the plane $z = 0$

$$t(x) = \exp(\mathrm{i}\theta(x)). \quad (2)$$

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The phase plate is divided into sub-plates with equal interval. The phase delay of a beam propagating through the lth sub-plate is

$$\theta_l(x_l) = \frac{2\pi}{\lambda}(n-1)h_l, \tag{3}$$

where n is the refractive index and h_l denotes the geometric thickness of the l-th sub-plate, $l = 1, 2, 3, \cdots; \lambda$ is the wavelength. The far-field intensity of GSM beams passing through the phase plate is characterized by the propagation formula of the form[19]

$$I(x,z) = \frac{1}{\lambda z}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} J_0(x_1,x_2,0)t(x_1)t^*(x_2) \times \exp\left[-\frac{\mathrm{i}2\pi x}{\lambda_0 z}(x_2-x_1)\right]\mathrm{d}x_1\mathrm{d}x_2. \tag{4}$$

The effective integral interval is divided into sub-intervals with $-\infty < m\Delta x < +\infty$ (m-even number), beyond which the integrals vanish, thus, Eq.(4) can be rewritten as

$$I(x,z) = \frac{1}{\lambda z}\sum_{l=-m/2}^{m/2}\sum_{h=-m/2}^{m/2}\int_{(l-1)\Delta x}^{l\Delta x}\int_{(h-1)\Delta x}^{h\Delta x} \times J_0(x_1,x_2,0)t(x_1)t^*(x_2) \times \exp\left[-\frac{\mathrm{i}2\pi x}{\lambda_0 z}(x_2-x_1)\right]\mathrm{d}x_1\mathrm{d}x_2. \tag{5}$$

Substituting Eqs.(1), (2) and (3) into Eq.(5), by tedious but straightforward diffraction integrals we get

$$\begin{aligned} I(x,z) = {} & \frac{\pi w_0^2\beta A_0}{2\lambda z}\exp\left[-\frac{2\pi^2 w_0^2\beta^2 x^2}{\lambda^2 z^2}\right]\sum_{l=-m/2}^{m/2}\sum_{h=-m/2}^{m/2}\exp[\mathrm{i}(\theta_l-\theta_k)]\left\{\mathrm{erf}\left[\frac{(h+l-2)\Delta x}{\sqrt{2}w_0}\right]-\mathrm{erf}\left[\frac{(h+l)\Delta x}{\sqrt{2}w_0}\right]\right\} \\ & \times\left\{\mathrm{erf}\left[\beta\frac{(h-l)\lambda z\Delta x+(\beta^{-2}-1)\left((h-l)\lambda z\Delta x-\frac{2\pi\mathrm{i}\sigma_0^2 x}{\lambda z}\right)}{\sqrt{2}w_0}\right]\right. \\ & \left.-\mathrm{erf}\left[\beta\frac{(h-l+1)\lambda z\Delta x+(\beta^{-2}-1)\left((h-l+1)\lambda z\Delta x-\frac{2\pi\mathrm{i}\sigma_0^2 x}{\lambda z}\right)}{\sqrt{2}w_0}\right]\right\}, \end{aligned} \tag{6}$$

where erf (•) stands for the error function and $\beta = \left[1+(w_0/\sigma_0)^2\right]^{-1/2}$ denotes the parameter of spatial coherence.[15] Equation (5) is the expression for the far-field intensity of GSM beams passing through a phase plate with complex-valued transmissivity, and indicates that for a given $\theta(x)$ the far-field intensity can be calculated. To the contrary, for a desired far-field intensity $\theta(x)$ can not be straightforwardly obtained from Eq.(5). It is also seen from Eq.(5) that for given initial and far-field intensity distributions the parameter of spatial coherence β additionally affects the phase distribution of the phase plate in the phase-retrieval problem, which differs from the case of fully coherent beams. For our purpose, the SA algorithm is a better choice to achieve phase retrieval, because it delivers a method for finding the global minimum in a limited time, details are found, for example, in Refs.[8,12–14].

3. Partially coherent beam shaping

When the SA algorithm is used to optimize the phase plate for shaping partially coherent beams, the phase described by Eq.(13) is chosen as the variable, and the cost function is defined as

$$\chi_\mu^2(\theta) = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{I(\theta_i)-I_\mathrm{d}(\theta_i)}{\varepsilon^\Delta}\right)^2, \tag{7}$$

where $I(\theta_i)$ and $I_\mathrm{d}(\theta_i)$ denote the reconstructed and desired intensities on the ith sub-plate respectively, ε^Δ stands for the experimental error,[19] N is the total sampling number and in this paper we take $N = 256$. In the iterative procedure the control parameter is the equivalent temperature T. Assume that the initial value of the phase is $\theta_0 = (\theta_1^0, \theta_2^0\ \cdots,\ \theta_{256}^0)$, thus the

initial cost function is $\chi^2_{\mu,0}(\theta_0)$. A given perturbation $\delta\theta = (\delta\theta_1, \delta\theta_2, \cdots, \delta\theta_{256})$ produces a new cost function $\chi^2_\mu\ (\theta_0+\delta\theta)$, and results in a variation of the cost function:

$$\Delta\chi^2_\mu = \chi^2_\mu(\theta_0 + \delta\theta) - \chi^2_{\mu,0}(\theta_0). \tag{8}$$

In accordance with the Metropolis criterion,[20] the perturbation probability is given by

$$P = \begin{cases} \exp[-\Delta\chi^2_\mu/(qT)] & \text{if } \Delta\chi^2_\mu \geq 0, \\ 1 & \text{if } \Delta\chi^2_\mu < 0, \end{cases} \tag{9}$$

where q is the adjustive factor of probability.[21]

The simulation procedure for shaping GSM beams by the SA algorithm is briefly described as follows.

Step 1. Start from the initial cost function χ^2_μ with initial phase $(\theta^0_1, \theta^0_2, \cdots, \theta^0_{256})$ and initial temperature T_0.

Step 2. Set up a new state with the phase $(\theta^0_1, \theta^0_2, \cdots, \theta^0_{256})$ replaced by the phase $(\theta^0_1+\delta\theta_1, \theta^0_2+\delta\theta_2, \cdots, \theta^0_{256}+\delta\theta_{256})$ for a given perturbation.

Step 3. Calculate the variation $\Delta\chi^2_\mu = \chi^2_\mu - \chi^2_{\mu,0}$. If $\Delta\chi^2_\mu \leq 0$, accept and start with the next step; else compare the Metropolis criterion (8) with a randomly generated number σ in the range [0, 1]. If $p > \sigma$, accept; otherwise do not accept and return to Step 2.

Step 4. Decrease the temperature slowly and repeat steps 2 and 3 with the next temperature until the temperature is sufficiently cool. As a result, χ^2_μ should be converged to the global minimum and the corresponding phase distribution $\theta = (\theta_1, \theta_2\ \cdots, \theta_{256})$ is the optimized phase distribution of the phase plate. Numerical illustrative examples are shown in Figs.1–3, where Figs.1(a) and 1(b) give the desired intensity distributions at the plane $z = 4.5\,\mathrm{m}$ corresponding to a flat-topped intensity distribution and an intensity profile with a central dip. The other calculation parameters are $w_0 = 0.2\,\mathrm{mm}$, $\lambda = 1.06\,\mu\mathrm{m}$, $\beta = 0.6$, $\varepsilon^{\Delta} = 1.0 \times 10^{-6}$ and $q = 8$. The optimized reconstructed results by the SA algorithm are given in Figs.2–3, where (a) and (b) correspond to the intensity and phase distributions, respectively. From Figs.1(b) and 2(b) the optical thickness is available, which can be used for the design of phase plates.

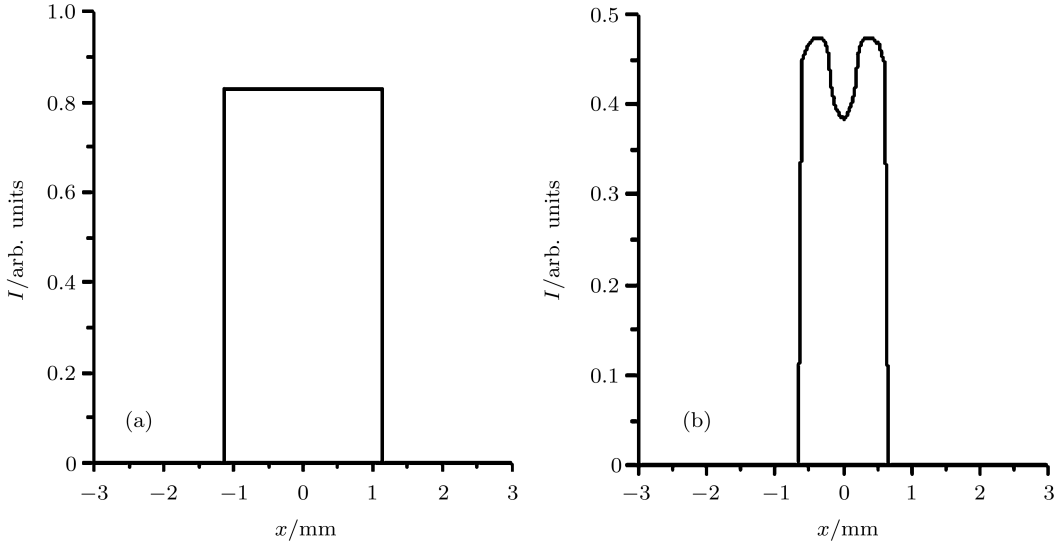


Fig.1. (a) A desired flat-topped intensity profile, (b) A desired intensity profile with a central dip.

The reconstruction error RE is introduced to compare the shaping effect, and is defined as

$$\mathrm{RE}(x_i) = \left|\frac{I(x_i) - I_\mathrm{d}(x_i)}{I_{\mathrm{d,max}}}\right| \times 100\%, \tag{10}$$

where $I(x_i)$ and $I_\mathrm{d}\ (x_i)$ denote the reconstructed and desired intensities at the position x_i, respectively, and $I_{\mathrm{d,max}}$ is the maximum desired intensity. The reconstruction errors corresponding to Figs.2(a) and 3(a) are depicted in Figs.4(a) and 4(b) respectively. As can be seen, the largest value of RE in Fig.2(a) is less than 1.74% which is located in the vicinity of $x = \pm 1.2\,\mathrm{mm}$ on either hand of the beam profile, and an uniform flat-topped profile with $\mathrm{RE} < 0.24\%$ is achieved. For the case of Fig.3(a) the largest value of RE is less than 2.0% on either hand of the beam profile.

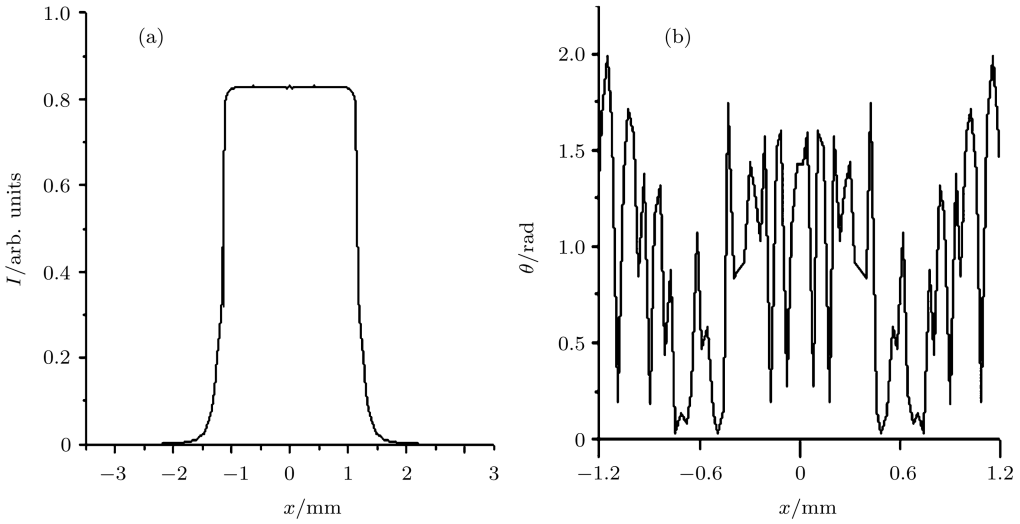


Fig.2. (a) The reconstructed flat-topped intensity profile, (b) The reconstructed phase distribution.

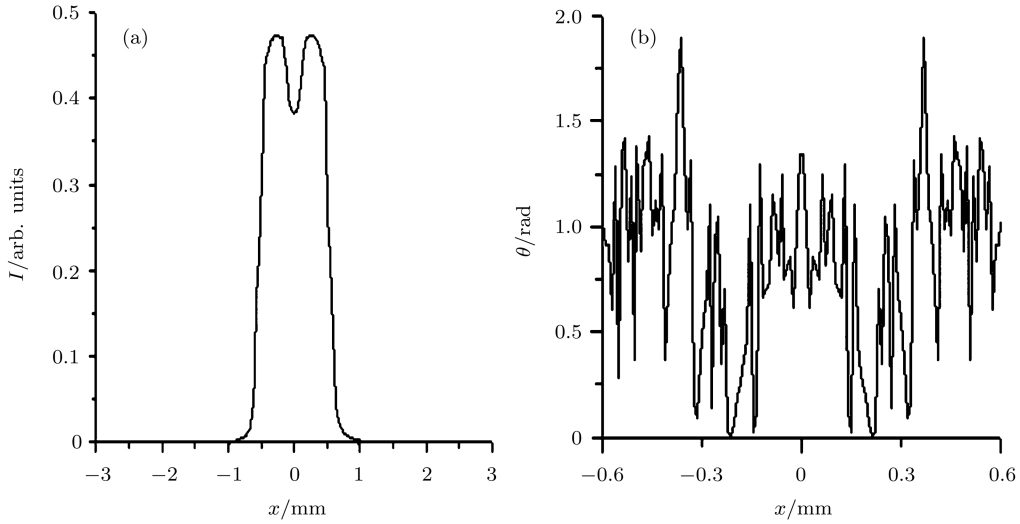


Fig.3. (a) The reconstructed intensity profile with a central dip, (b) The reconstructed phase distribution.

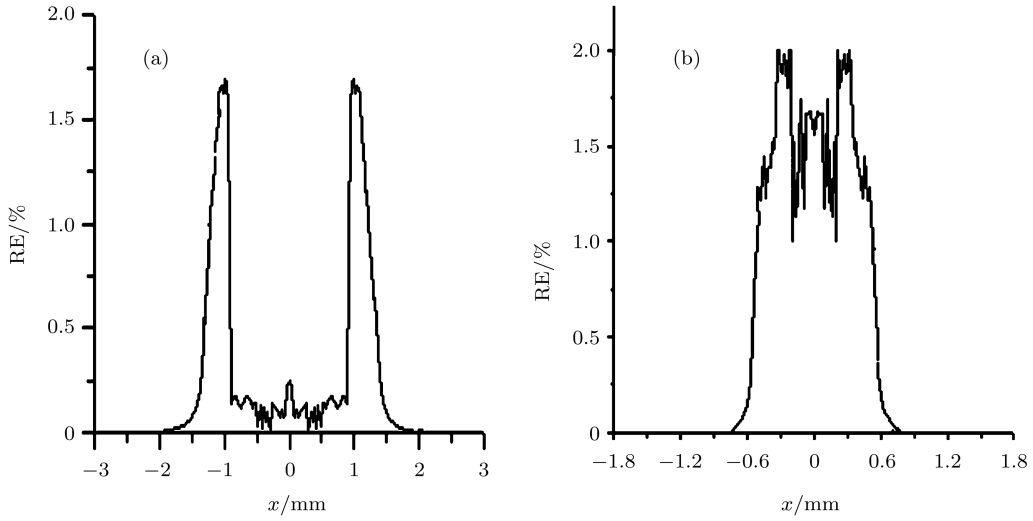


Fig.4. (a) The reconstruction error corresponding to Fig.2(a), (b) The reconstruction error corresponding to Fig.3(a).

4. Conclusion

In this paper the SA algorithm has been used for shaping partially coherent beams, where the effect of partial coherence on the phase distribution of phase plates has been considered, because the parameter of spatial coherence β affects θ. It has been shown that an uniform flat-topped beam profile with maximum reconstruction error RE $< 1.74\%$ has been achieved. The SA algorithm demonstrates the advantages over the geometrical approach and IFTA, because, for example, the IFTA will result in a loss of high frequency information due to the shorter correlation length.[1] For the three-dimensional case the extension is straight-forward, because the variables x and y are separable. Thus, the far-field intensity is expressed as

$$I(x, y, z) = I(x, z)I(y, z), \tag{11}$$

where $I(y, z)$ is obtained simply with x and Δx in Eq.(6) replaced by y and Δy, respectively. A further extension to other types of partially coherent beams is possible, once the initial mutual intensity (or cross-spectral density function in the space-frequency domain) is given. For such a case the closed-form expressions like Eqs.(7) and (11) may not be found, but the SA algorithm is still applicable. The project is under way and related results will be published elsewhere.

Acknowledgment

Li J L is grateful to Prof K. Fu for fruitful discussions relating the subject of this paper.

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