Vol 17 No 5, May 2008
1674-1056/2008/17(05)/1821-05

Chinese Physics B

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Theoretical study on phase conjugation in weakly injected vertical-cavity surface-emitting lasers*

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(Received 27 June 2007; revised manuscript received 14 September 2007)

This paper gives a detailed theoretical investigation on phase conjugation induced by nearly degenerate four-wave mixing in single mode vertical-cavity surface-emitting lasers (VCSELs) with weak optical injection. Considering VCSELs that can work in linearly polarized or elliptically polarized states, it derives the theoretical expression of the conjugated field by small signal analysis based on the vectoral rate equations—the spin-flip model. For linearly polarized state, VCSELs show similar conjugate spectra to edge-emitting semiconductor lasers. For the elliptically polarized state, dichroism and birefringence parameters as well as the spin-flip rate can change the conjugate spectra. Especially, when frequency detuning of the probe and pump waves is between the positive and negative relaxation oscillation frequency, changes are evident. For specific values of parameters, conjugate efficiency between 20 dB to 40 dB are obtained.

Keywords: phase conjugate, VCSELs, polarization, four-wave mixing
PACC: 4260, 4265F, 2975, 4265M

1. Introduction

Phase conjugation through nearly degenerate four-wave mixing (NDFWM) in semiconductor lasers has been extensively studied, owning to its potential applications in future all-optical communications, such as dispersion compensation, wave correction, and wavelength conversion.[1−3] Usually, these studies on conventional edge-emitting semiconductor lasers neglected the polarization issue, since the heterostructure of an edge emitter induces a large anisotropy between TE and TM modes, and only a well defined polarization mode is supported. But for vertical-cavity surface-emitting lasers (VCSELs), the output polarization is complex.[4] Due to weak material and cavity anisotropies, they preferentially emit linearly polarized (LP) light along one of two orthogonal directions (x and y), and they can also emit elliptically polarized (EP) light under particular values of parameters.[5,6] It has been reported experimentally that weakly injected VCSELs can generate phase conjugate waves.[7,8] Recent numerical studies also show that VCSELs subjected to polarization rotated injection can show wave-mixing characteristics in the route to polarization switching.[9] All these motivate a detailed theoretical study on phase conjugation in optical injected VCSELs, taking into account the polarization degrees of freedom.

Recently, the issues of polarization switching, hysteresis, bifurcation, and chaos in optical injected semiconductor lasers have attracted much attention.[10−13] However, theoretical studies on phase conjugation in VCSELs subjected to weak optical injection remain scarce, while being of great theoretical and practical importance.[5−8]

In this paper, we present a detailed theoretical analysis on the phase conjugation of a weakly injected VCSEL. In the analysis, both the LP and EP states of VCSEL are considered. The theoretical expression of the conjugated field is derived by small signal analysis based on the spin-flip model (SFM).[14] To our best knowledge this has not been reported before. The remainder of this paper is organized as follows. In Section 2, the theoretical model is derived. In Section 3, phase conjugation for LP state is discussed. In Section 4, phase conjugation for EP state is discussed. Conclusions are given in Section 5.

*Project supported by the Specialized Research Fund for the Doctoral Program of Higher Education of China (Grant No 20070613058), the Foundation for Key Program of Ministry of Education, China (Grant No 2005105148), and the Doctoral Innovation Fund of Southwest Jiaotong University of China.
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2. Theoretical analysis

To study NDFWM in edge emitting semiconductor lasers, scalar rate equations, coupled wave equations and their modified models have been used.[1−4] While in this paper, our analyses are based on the SFM. These vector rate equations are derived from the Maxwell–Bloch equations, which takes account of the polarization of the laser field by including the spin sublevels of the conduction and valence bands.[13−15] So we can account for the different emission states of VCSELs. Under weak injection, the slowly varying amplitudes of the x-polarized (XP) and y-polarized (YP) modes, $E_x(t)$ and $E_y(t)$, satisfy

$$\begin{aligned}\frac{\mathrm{d}E_{x,y}}{\mathrm{d}t} &= k(1-\mathrm{i}\alpha)(NE_{x,y}-E_{x,y}\\ &\quad \pm \mathrm{i}nE_{y,x})\mu(\gamma_\mathrm{a}+\mathrm{i}\gamma_\mathrm{p})E_{x,y}\\ &\quad +\eta_{x,y}E_\mathrm{inj}\exp(-\mathrm{i}t(\omega_\mathrm{s}+\Omega)), \end{aligned} \tag{1}$$

where k is the field decay rate, α is the linewidth enhancement factor, γ_a is the linear dichroism, and γ_p is the linear birefringence. The last term in Eq.(1) stands for the contribution of optical injection, where η_x and η_y are the injection rate of the XP and YP modes, respectively, E_inj is the injection amplitude, Ω is the frequency detuning between the probe and pump waves, and ω_s is the lasing frequency of the free-running VCSEL, N is the total carrier population, and n is the population difference between the spin-up and spin-down radiation channels. Detailed introductions on rate equations of N and n have been given in Refs.[5, 9, 15], and we do not repeat them here.

It is known that VCSELs have steady-state solutions for LP or EP states.[13−15] We begin our discussions with the EP states, since the LP states in the XP or YP direction can be easily derived from the solutions of the EP states. When the laser is subjected to weak injection, the nonlinear coupling of the probe and pump waves will cause fluctuations of N and n at the beating frequency Ω, and generates a conjugated wave at frequency $\omega_\mathrm{c}=\omega_\mathrm{s}-\Omega$ through NDFWM. The variables in Eq.(1) can be written as

$$\begin{aligned}E_{x,y} &= E_{\mathrm{xs,ys}}\exp(-\mathrm{i}\omega_\mathrm{s}t)+E_{\mathrm{xr,yr}}\exp(-\mathrm{i}t(\omega_\mathrm{s}+\Omega)),\\ &\quad +E_{\mathrm{xc,yc}}\exp(-\mathrm{i}t(\omega_\mathrm{s}-\Omega)), \end{aligned} \tag{2}$$

$$N = N_\mathrm{s}+N_1\exp(-\mathrm{i}\Omega t)+N_1^*\exp(\mathrm{i}\Omega t), \tag{3}$$

$$n = n_\mathrm{s}+n_1\exp(-\mathrm{i}\Omega t)+n_1^*\exp(\mathrm{i}\Omega t), \tag{4}$$

where $E_{xs,ys}$, $E_{xr,yr}$, and $E_{xc,yc}$ are the amplitudes of the steady state, the probe wave, and the conjugated wave, respectively. N_s and N_1 (n_s and n_1) are the steady-state and pulsation of the total carrier population (carrier population difference), respectively. Substituting Eq.(2) into Eq.(1), we can obtain the following equations:

$$E_{xc} = \frac{k(1-\mathrm{i}a)}{\zeta_x+\mathrm{i}\Omega}\left(E_{xs}N_1^*+\mathrm{i}E_{ys}n_1^*+\mathrm{i}E_{yc}n_\mathrm{s}\right), \tag{5}$$

$$E_{yc} = \frac{k(1-\mathrm{i}a)}{\zeta_y+\mathrm{i}\Omega}\left(E_{ys}N_1^*-\mathrm{i}E_{xs}n_1^*-\mathrm{i}E_{xc}n_\mathrm{s}\right), \tag{6}$$

where $\zeta_{x,y}=\pm(\gamma_\mathrm{a}+\mathrm{i}\gamma_\mathrm{p})-k(1-\mathrm{i}\alpha)(N_\mathrm{s}-1)-\mathrm{i}\omega_\mathrm{s}$. Apparently, both the fluctuations of N and n have contributions to the conjugate waves. In addition, the conjugate waves in the two orthogonal directions interact with each other owning to nonzero of n_s.

Similarly, substituting Eqs.(2) and (3) into rate equations of N and n we can obtain

$$N_1 = -\frac{\gamma_\mathrm{e}\left[N_\mathrm{s}(\sigma_x+\sigma_y)+\mathrm{i}n_\mathrm{s}(\sigma_1-\sigma_2^*)+\mathrm{i}n_1\sigma_\mathrm{s}\right]}{\gamma_\mathrm{e}+\gamma_\mathrm{e}P_\mathrm{s}-\mathrm{i}\Omega}, \tag{7}$$

$$n_1 = -\frac{\gamma_\mathrm{e}\left[n_\mathrm{s}(\sigma_x+\sigma_y)+\mathrm{i}N_\mathrm{s}(\sigma_1-\sigma_2^*)+\mathrm{i}N_1\sigma_\mathrm{s}\right]}{\gamma_\mathrm{s}+\gamma_\mathrm{e}P_\mathrm{s}-\mathrm{i}\Omega}, \tag{8}$$

where γ_e is the decay rate of N, and γ_s is the spin-flip rate. $P_\mathrm{s}=|E_{xs}|^2+|E_{ys}|^2$, $\sigma_{x,y}=E_{xs,ys}E_{xc,yc}^*+E_{xs,ys}^*E_{xc,yc}$, $\sigma_{1,2}=E_{ys}E_{xc,xr}^*+E_{xs}^*E_{yr,yc}$, and $\sigma_\mathrm{s}=E_{ys}E_{xs}^*-E_{xs}E_{ys}^*$.

To simplify the calculation, we redefine a set of parameters, as shown in Table 1. Then we can obtain the expression of the conjugated field by using Eqs.(5)–(8)

$$\begin{aligned}\frac{E_{xc,yc}^*}{E_\mathrm{inj}} &= b_{31,41}\frac{d_{23}d_{12}-d_{22}d_{13}}{d_{11}d_{22}-d_{12}d_{21}}\\ &\quad +b_{32,42}\frac{d_{13}d_{21}-d_{11}d_{23}}{d_{11}d_{22}-d_{12}d_{21}}. \end{aligned} \tag{9}$$

It is worth mentioning that Eq.(9) is for elliptical polarization. For linear polarization, $n=0$ and $E_x=0$ (or $E_y=0$), we can obtain the simplified results:
for the XP mode:

$$\frac{E_{xc}^*}{E_\mathrm{inj}} = \frac{a_{51}^*a_{21}a_{34}E_{xs}^*}{1-a_{21}(a_{51}^*E_{xs}+a_{31}E_{xs}^*)}, \tag{10}$$

or for the YP mode

$$\frac{E_{yc}^*}{E_\mathrm{inj}} = \frac{a_{61}^*a_{21}a_{44}E_{ys}^*}{1-a_{21}(a_{61}^*E_{xs}+a_{41}E_{ys}^*)}. \tag{11}$$

Table 1. List of the redefined parameters.

$$a_{11,12,13} = -\frac{\gamma_e}{\gamma_s + \gamma_e P_s - i\Omega}\,[n_s, iN_s, i\sigma_s]\,; \qquad a_{21,22,23} = -\frac{\gamma_e}{\gamma_e + \gamma_e P_s - i\Omega}\,[N_s, in_s, i\sigma_s]\,;$$

$$a_{31,32,33,34} = \frac{k(1-i\alpha)}{\zeta_x - i\Omega}\left[E_{xs}, iE_{ys}, in_s, \frac{\eta_x}{k(1-i\alpha)}\right]; \qquad a_{41,42,43,44} = \frac{k(1-i\alpha)}{\zeta_y - i\Omega}\left[E_{ys}, -iE_{xs}, -in_s, \frac{\eta_y}{k(1-i\alpha)}\right];$$

$$a_{51,52,53} = \frac{k(1-i\alpha)}{\zeta_x + i\Omega}\,[E_{xs}, iE_{ys}, in_s]\,; \qquad a_{61,62,63} = \frac{k(1-i\alpha)}{\zeta_y + i\Omega}\,[E_{ys}, -iE_{xs}, -in_s]\,;$$

$$b_{jm} = \frac{a_{(2+j)m} + a_{(2+j)3}a_{(5-j)m}}{1 - a_{33}a_{43}}; \quad b_{j3} = \frac{a_{(2+j)4} + a_{(2+j)3}a_{(5-j)4}}{1 - a_{33}a_{43}}; \quad b_{(2+j)m} = \frac{a^*_{(4+j)m} + a^*_{(4+j)3}a^*_{(7-j)m}}{1 - a^*_{53}a^*_{63}};$$

$$c_{1m} = E_{xs}b_{3m} + E^*_{xs}b_{1m} + E_{ys}b_{4m} + E^*_{ys}b_{2m}; \qquad c_{2m} = E_{ys}b_{3m} + E^*_{xs}b_{2m} - E_{xs}b_{4m} - E^*_{ys}b_{1m};$$

$$c_{13} = E^*_{xs}b_{13} + E^*_{ys}b_{23}, \qquad c_{23} = E^*_{xs}b_{23} - E^*_{ys}b_{13};$$

$$d_{jj} = a_{j1}c_{1j} + a_{j2}c_{2j} + a_{j3}; \quad d_{j(3-j)} = a_{j1}c_{j(3-j)} - 1; \quad d_{j3} = a_{j1}c_{13} + a_{j2}c_{23};$$
$(j = 1, 2;\ m = 1, 2)$

The above analysis suggests that, if the steady-state solutions are given, we can obtain the generation efficiencies of the conjugated field (ratios of powers of the conjugated field to the injected field).

3. Results and discussions

3.1. Phase conjugation for linear polarization

Firstly, the values of k, α, γ_a, γ_p, γ_e, and γ_s are set at 300ns^{-1}, 3, 0.5ns^{-1}, 20rad/ns, 1 ns^{-1}, 100ns^{-1}, respectively, and the normalized current μ is set at 2. Thus the free-running VCSEL emits in the YP mode, and the steady-state solutions are: $N_s = 1 - \gamma_a/k$, $P_s = \mu/N_s - 1$, and $\omega_s = -\alpha\gamma_a - \gamma_p$. When the laser is subjected to weak injection in the YP direction, FWM is supposed to happen.

Figure 1 presents the conjugated efficiency (CE) as a function of frequency detuning Ω. The probe wave is at the frequency $\omega_s + \Omega$, and the conjugate wave is at the frequency $\omega_s - \Omega$. A central peak appears at 0 detuning, and two symmetrical side peaks also appear when detuning equals the positive and negative value of the relaxation oscillation frequency (ROF). We should point out that, since the laser emits in the YP mode, the term ζ_y always equals 0. So in fact, the spectra in Fig.1 is discontinuous at $\Omega = 0$. Besides, $\zeta_y = 0$ also indicates that changes of γ_a or γ_p cannot affect the expression of CE, but the frequency of the conjugate wave will be changed due to change of ω_s. Physically, the phase conjugation we observed is caused by four wave-mixing related to an optical induced carrier modulation process. So CE keeps at a relative high level when detuning is near the solitary lasing frequency ω_s; but CE decreases markedly when detuning exceeds the ROF, since the carrier density cannot respond efficiently to frequencies much higher than the ROF.

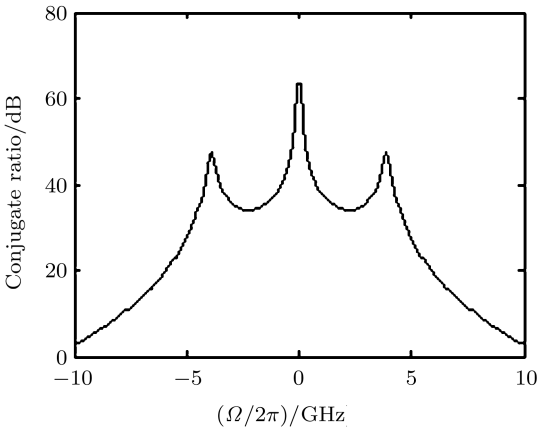


Fig.1. Conjugate efficiency as a function of Ω, with $\eta_y = k$ and $\eta_x = 0$.

The case when the laser emits in the XP mode is similar, and we do not repeat it here. So when VCSELs work in the LP states, their characteristics of phase conjugation are similar to what were observed in edge-emitters.[1−4] The difference is that, for VCSELs, the frequency of the conjugated field is $\omega_s - \Omega$ but not $-\Omega$, and the free running frequency ω_s will change according to the values of dichroism and birefringence.

In Refs.[7, 8], it was reported that VCSELs subjected to weak optical injections can show four wave-mixing effect, and a symmetrical CE spectrum has also been observed. Our theoretical results are in good accordance with these experimental results.

3.2. Phase conjugation for elliptical polarization

To study phase conjugation in this case, we select a set of parameters for which the steady-state solutions corresponding to the EP states exist. Values of γ_a, γ_p and γ_s are given directly in the following discussions, and other parameters keep unchanged. The steady-state solutions N_s, n_s, ω_s, E_{xs}^* and E_{ys}^* for the elliptically polarized states can be obtained through asymptote method, using the relationship functions given in Ref.[5].

Figure 2 gives the conjugated efficiency of the XP mode when the laser works in the EP state. To study the influence of γ_p, we set $\gamma_a = 0$, and choose a small γ_s to make sure that elliptical polarization always exists within the range of values of γ_p that are considered. When γ_p is small, the curves in Fig.2 are similar to those in Fig.1. As γ_p increases, CE is enhanced when Ω is at the negative and positive ROF, and is damped when Ω is near 0. If γ_p still increases further, CE is damped when Ω between the negative and positive ROF. Further simulations find that the curves of CE almost keep unchanged when γ_p takes the same values but with a reverse sign.

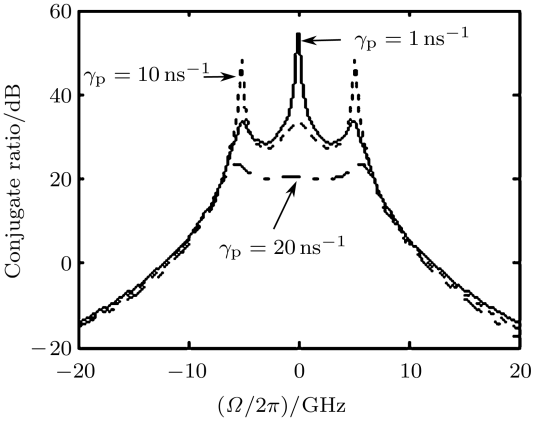


Fig.2. Conjugate efficiency of the XP mode versus Ω, with $\eta_x = \eta_y = k$, $\gamma_a = 0$, $\gamma_s = 10\,\mathrm{ns}^{-1}$.

Figure 3 shows the influence of γ_a. When $\gamma_a = -0.2$, the curves of CE are the most similar to the case of linear polarization. When $\gamma_a = 0$ or -0.4, the value of CE decreases markedly when Ω between the negative and positive ROF. We should point out that the CE of the coexisting YP mode is almost the same as that of the XP mode, so in Figs.2 and 3 only the XP mode is discussed.

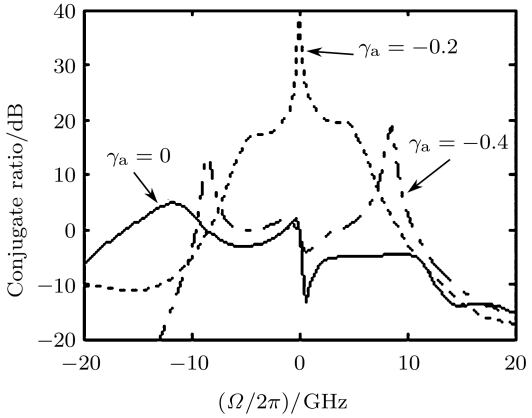


Fig.3. Conjugate efficiency of the XP mode versus Ω, with $\eta_x = \eta_y = k$, $\gamma_p = 50\mathrm{rad/ns}$, $\gamma_s = 10\ \mathrm{ns}^{-1}$.

Compared with the case of linear polarization, dichroism and birefringence in the case of elliptical polarization can not only change the frequencies of the conjugate waves but also decrease the spectra of CE, especially, when Ω is between the positive and negative ROF. In Ref.[15], experimental results show that the modulation instability spectra in photonic crystal fibres is decreased when the probe light is injected through the 45° direction compared with the 0° or 90° direction. To some extent this can justify the decrease of the CE spectra when injection is elliptically polarized. The physical mechanism of distortions of the CE spectra mainly due to the shift of the central frequency ω_s. For linear polarization, ω_s is the solitary solution of the XP or YP mode; while for elliptical polarization, the two LP modes lock to a common frequency ω_s, which depend strongly and complexly on the values of γ_a and γ_p. When ω_s is different for the EP state and for the LP state, distortion of the CE spectra will happen.

Figure 4 shows the values of ω_s for elliptical polarization when γ_p, γ_a and γ_s are changed. From Fig.4(a) we can see that ω_s varies parabolically with γ_p. The curve is symmetric with $\gamma_p = 0$, which well explains why the curves of CE are almost unchanged when γ_p takes a same value but with reverse sign. In Fig.4(b), ω_s varies linearly with γ_a, and only a little change of γ_a can induce a very different ω_s. In Fig.4(c), ω_s varies almost linearly with γ_s, but the range of ω_s is relatively small in comparison with Fig.4(b). Further simulations show that the conjugate efficiency is less

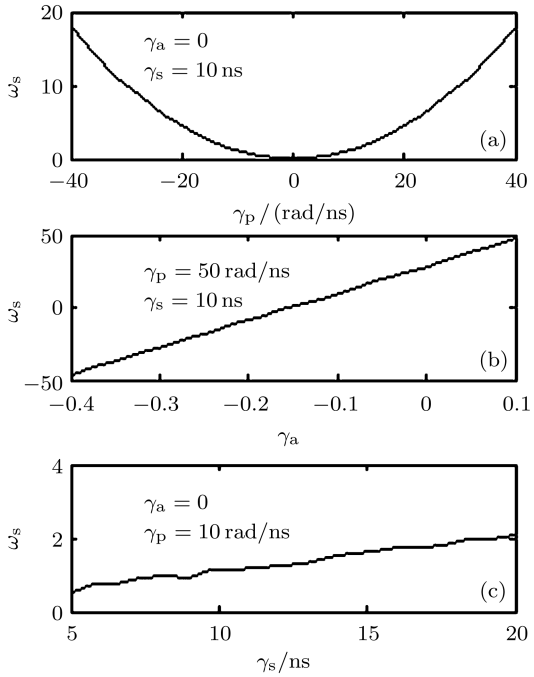


Fig.4. Value of ω_s for the elliptically polarized states. In (a), (b), and (c) γ_p, γ_a, and γ_s are the variables, respectively.

sensitive to the value of γ_s. In addition, in Fig.2 (Fig.3) the case when $\gamma_p = 5$rad/ns ($\gamma_a = -0.2$) is the most similar to that of the linear polarization. At these values ω_s is around 0, so it seems that when $\omega_s \to 0$ the distortions of the spectra of CE is smaller than those in other cases.

4. Conclusion

In conclusion, VCSELs which emit in the LP and the EP states can show different conjugate spectra due to the cavity dichroism and birefringence. For VCSELs working in the LP states, the conjugate spectrum is similar to that in edge-emitting semiconductor lasers, except that the frequency of the conjugate wave depends on the free-running frequency that relates with the dichroism and birefringence. For VCSELs working in the elliptically polarized states, the two LP modes lock to a common frequency, and phase conjugation happens simultaneously in the two modes. The dichroism and birefringence can influence the conjugation efficiency especially when frequency detuning is between the negative and positive ROF. Besides, when free-running frequency of the elliptical state is near 0, large conjugate efficiency can be obtained. This study is not only useful for understanding the nonlinear dynamics in VCSELs, but also of great practical interest.

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