Vol 17 No 5, May 2008
1674-1056/2008/17(05)/1791-07

Chinese Physics B

© 2008 Chin. Phys. Soc.
and IOP Publishing Ltd

Wigner function for the generalized excited pair coherent state*

Meng Xiang-Guo(孟祥国)[a)†], Wang Ji-Suo(王继锁)[a)],
Liang Bao-Long(梁宝龙)[a)], and Li Hong-Qi(李洪奇)[b)]

[a)] *Department of Physics, Liaocheng University, Liaocheng* 252059, *China*
[b)] *Department of Physics, Heze University, Heze* 274015, *China*

(Received 30 August 2007; revised manuscript received 8 November 2007)

This paper introduces the generalized excited pair coherent state (GEPCS). Using the entangled state $|\eta\rangle$ representation of Wigner operator, it obtains the Wigner function for the GEPCS. In the ρ–γ phase space, the variations of the Wigner function distributions with the parameters q, α, k and l are discussed. The tomogram of the GEPCS is calculated with the help of the Radon transform between the Wigner operator and the projection operator of the entangled state $|\eta_1, \eta_2, \tau_1, \tau_2\rangle$. The entangled states $|\eta\rangle$ and $|\eta_1, \eta_2, \tau_1, \tau_2\rangle$ provide two good representative space for studying the Wigner functions and tomograms of various two-mode correlated quantum states.

Keywords: generalized excited pair coherent state, Wigner function, tomogram, entangled state representation

PACC: 4250, 0365

1. Introduction

It is well known that the Wigner function[1,2] of a state plays an important role in modern quantum optics. It gives the most analogous description of quantum mechanics in one dimension to classical statistical mechanics of Hamilton systems. The negative values of the Wigner functions clearly show that these states are nonclassical, so it is interesting to measure the negative values of the Wigner functions for these states. On the other hand, since two marginal distributions of the Wigner function lead to measuring probability density in coordinate space and momentum space, the Wigner function is the most popularly used in various branches of physics.

Recently, various photon states have been generated by the micromaser[3] and the Wigner functions of some cavity fields states in the whole phase space can be measured by a simple scheme based on interaction between cavity fields and atoms.[4,5] More recent, in theory we have obtained Wigner functions and tomogram functions[6,7] for some single-mode quantum states by using the coherent state representation of Wigner operator and the technique of integration within an ordered product (IWOP) of operators, as well as the newly introduced intermediate coordinate-momentum representation in quantum optics. As the further generalization of the work in Refs.[6,7], in this paper we shall introduce a kind of standard two-mode correlated state, i.e., generalized excited pair coherent state (GEPCS), and then evaluate its Wigner function. It is of academic interest to study Wigner function for this state in theory because the state can be generated by nondegenerate two-photon interaction between atom and cavity field.[8] In the Fock space the pair coherent state, which involves strong entanglement in Refs.[9, 10], is defined as

$$|q,\alpha\rangle = C_q \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{(n+q)!n!}} |n+q,n\rangle\,,$$
$$\alpha = |\alpha| \exp(\mathrm{i}\theta), \tag{1}$$

where C_q is the normalization constant,

$$C_q = [(\mathrm{i}|\alpha|)^{-q} J_q(\mathrm{i}2|\alpha|)]^{-1/2}, \tag{2}$$

J_q is the ordinary q-order Bessel function. In fact, the state $|q,\alpha\rangle$ is the common eigenvector of both the pair-annihilation operator (ab) and the difference in the number operator $Q = a^+a - b^+b$ for the two modes (here $[Q, ab] = 0$), i.e.

$$ab\,|q,\alpha\rangle = \alpha\,|q,\alpha\rangle\,, \quad Q\,|q,\alpha\rangle = q\,|q,\alpha\rangle\,. \tag{3}$$

*Project supported by the National Natural Science Foundation of China (Grant No 10574060) and the Natural Science Foundation of Shandong Province of China (Grant No Y2004A09).
†Corresponding author. E-mail: xgmeng@lcu.edu.cn

The GEPCS is constructed by repeated application of the photon creation operators a^+ and b^+ on the state $|q,\alpha\rangle$,

$$\begin{aligned}&|q,\alpha,k,l\rangle\\ =\ & D_n a^{+k}b^{+l}\,|q,\alpha\rangle\\ =\ & D_n\sum_{n=0}^{\infty}\frac{\sqrt{(n+q+k)!(n+l)!}\alpha^n}{(n+q)!n!}\\ & \times|n+q+k,n+l\rangle\,,\end{aligned}\tag{4}$$

the constant D_n is

$$D_n=\left\{\sum_{n=0}^{\infty}\frac{(n+q+k)!(n+l)!}{[(n+q)!n!]^2}|\alpha|^{2n}\right\}^{-1/2}.\tag{5}$$

In particular, when $m=l$, the GEPCS reduces to the excited pair coherent state in Refs.[8, 9]; when $m=l=0$, the GEPCS becomes the pair coherent state in Eq.(1). So the state in Eq.(4) is named as the GEPCS. We want to derive the Wigner function characterizing the state $|q,\alpha,k,l\rangle$, and examine how this Wigner function varies with the parameters q, α, k and l in the ρ–γ phase space, respectively. We also want to derive the tomogram of the state $|q,\alpha,k,l\rangle$. For this purpose, we shall use the Wigner operator in the entangled state $|\eta\rangle$ representation and the two-variable Hermite polynomials, as well as the Radon transform between the Wigner operator and the projection operator of the entangled state $|\eta_1,\eta_2,\tau_1,\tau_2\rangle$ to simplify the calculations.

2. A brief review of the features of entangled state $|\eta\rangle$

In Ref.[11] Fan has shown that the entangled state $|\eta\rangle$

$$\begin{aligned}&|\eta\rangle=\exp\left[\frac{1}{2}|\eta|^2+\eta a_1^+-\eta^* a_2^+ +a_1^+a_2^+\right]|00\rangle\,,\\ &\eta=\eta_1+\mathrm{i}\eta_2,\end{aligned}\tag{6}$$

simultaneously obeys the eigenvector equations

$$(a_1-a_2^+)\,|\eta\rangle=\eta\,|\eta\rangle\,,\quad (a_1^+-a_2)\,|\eta\rangle=\eta^*\,|\eta\rangle\,,\tag{7}$$

where $[a_1-a_2^+,a_1^+-a_2]=0$, and if using the relations

$$X_i=\frac{a_i+a_i^+}{\sqrt{2}},\quad P_i=\frac{a_i-a_i^+}{\sqrt{2}i},\quad i=1,2,\tag{8}$$

we have

$$\begin{aligned}&(X_1-X_2)\,|\eta\rangle=\sqrt{2}\eta_1\,|\eta\rangle\,,\\ &(P_1+P_2)\,|\eta\rangle=\sqrt{2}\eta_2\,|\eta\rangle\,.\end{aligned}\tag{9}$$

This means that the state $|\eta\rangle$ is the common eigenstate of two particles' relative position (X_1-X_2) and the total momentum (P_1+P_2) in two-mode Fock space.

By noticing the normal ordering form of the two-mode vacuum state projector

$$|00\rangle\langle 00|=:\mathrm{e}^{-a_1^+a_1-a_2^+a_2}:\tag{10}$$

and the IWOP technique, we can immediately prove that the completeness relation

$$\int\frac{\mathrm{d}^2\eta}{\pi}\,|\eta\rangle\langle\eta|=\int\frac{\mathrm{d}^2\eta}{\pi}:\mathrm{e}^{-|\eta|^2+(a_1^+-a_2)\eta+(a_2^+-a_1)\eta^*+a_1^+a_2^++a_1a_2-a_1^+a_1-a_2^+a_2}:=1,\tag{11}$$

and from Eq.(7) we can see the orthonormal property

$$\langle\eta\,|\eta'\rangle=\pi\delta(\eta-\eta')\delta(\eta^*-\eta'^*)\equiv\pi\delta^{(2)}(\eta-\eta').\tag{12}$$

3. The Wigner function for the GEPCS

In order to obtain the Wigner function for the GEPCS, first we recall the generating function formula of the two-variable Hermite polynomials $H_{m,n}(\eta,\eta^*)$[12]

$$\sum_{m,n=0}^{\infty}\frac{t^m t'^n}{m!n!}H_{m,n}(\eta,\eta^*)=\exp(-tt'+t\eta+t'\eta^*),\tag{13}$$

where

$$\begin{aligned}&H_{m,n}(\eta,\eta^*)\\ =\ &\sum_{l=0}^{\min(m,n)}\frac{(-1)^l n!m!}{l!(m-l)!(n-l)!}\eta^{m-l}\eta^{*n-l}\\ =\ &\frac{\partial^{m+n}}{\partial t^m\partial t'^n}\exp(-tt'+t\eta+t'\eta^*)|_{t=t'=0}\,,\end{aligned}\tag{14}$$

which yields

$$H^*_{m,n}(\eta,\eta^*)=H_{n,m}(\eta,\eta^*), \tag{15}$$

then in the Fock space we can expand the entangled state $|\eta\rangle$ as

$$|\eta\rangle=\mathrm{e}^{-|\eta|^2/2}\sum_{i,j=0}^{\infty}\frac{(-1)^j}{\sqrt{i!j!}}H_{i,j}(\eta,\eta^*)\,|i,j\rangle\,, \tag{16}$$

where $|i,j\rangle=\dfrac{a_1^{+i}a_2^{+j}}{\sqrt{i!j!}}|00\rangle$ is the two-mode number states. Combining Eqs.(4) and (16) we derive the overlap,

$$\begin{aligned}&\langle\eta\,|q,\alpha,k,l\rangle\\ =\;&D_n\mathrm{e}^{-|\eta|^2/2}\sum_{n=0}^{\infty}\frac{(-1)^{n+l}}{n!(n+q)!}\\ &\times H_{n+l,n+q+k}(\eta,\eta^*)\alpha^n.\end{aligned} \tag{17}$$

Further, using Eq.(15) we obtain

$$\begin{aligned}&\langle\eta\,|q,\alpha,k,l\rangle\\ =\;&D_n\mathrm{e}^{\mathrm{i}(l-k-q)\varphi}\mathrm{e}^{-|\eta|^2/2}\sum_{n=0}^{\infty}\frac{(-1)^{n+l}}{n!(n+q)!}\\ &\times H_{n+l,n+q+k}(|\eta|,|\eta^*|)\alpha^n.\end{aligned} \tag{18}$$

In Ref.[11] under the entangled state representation $|\eta\rangle$ the two-mode Wigner operator $\Delta_{1,2}(\rho,\gamma)$ is expressed as

$$\begin{aligned}&\Delta_{1,2}(\rho,\gamma)\\ =\;&\int\frac{\mathrm{d}^2\eta}{\pi^3}\,|\gamma-\eta\rangle\,\langle\gamma+\eta|\exp(\eta\rho^*-\eta^*\rho).\end{aligned} \tag{19}$$

Using Eqs.(4) and (18), we can obtain the Wigner function of the state $|q,\alpha,k,l\rangle$

$$\begin{aligned}W(\rho,\gamma)=\;&\langle q,\alpha,k,l|\,\Delta_{1,2}(\rho,\gamma)\,|q,\alpha,k,l\rangle\\ =\;&D_mD_n\int\frac{\mathrm{d}^2\eta}{\pi^3}\,\langle q,\alpha,k,l\,|\gamma-\eta\rangle\,\langle\gamma+\eta|\,q,\alpha,k,l\rangle\exp(\eta\rho^*-\eta^*\rho)\\ =\;&D_mD_n\sum_{m,n=0}^{\infty}\frac{(-1)^{m+n}\alpha^{*m}\alpha^n}{m!n!(n+q)!(m+q)!}\\ &\times\int\frac{\mathrm{d}^2\eta}{\pi^3}H_{n+q,n}((\gamma-\eta)^*,(\gamma-\eta))H_{m,m+q}((\gamma+\eta)^*,(\gamma+\eta))\exp(-|\gamma|^2-|\eta|^2+\eta\rho^*-\eta^*\rho)\\ =\;&D_mD_n\sum_{m,n=0}^{\infty}\frac{(-1)^{m+n}\alpha^{*m}\alpha^n}{m!n!(n+q)!(m+q)!}\frac{\partial^{2m+q+k+l}}{\partial t^{m+q+k}\partial t'^{m+l}}\frac{\partial^{2n+q+k+l}}{\partial s^{n+l}\partial s'^{n+q+k}}\\ &\times\int\frac{\mathrm{d}^2\eta}{\pi^3}\exp[-|\gamma|^2-|\eta|^2+\eta\rho^*-\eta^*\rho-tt'+(\gamma-\eta)t+(\gamma-\eta)^*t'\\ &-ss'+(\gamma+\eta)s+(\gamma+\eta)^*s']|_{t=t'=s=s'=0}\,.\end{aligned} \tag{20}$$

Using the following integration formula[11]

$$\begin{aligned}&\int\frac{\mathrm{d}^2z}{\pi}\exp\left(\zeta\,|z|^2+\xi z+\eta z^*+fz^2+gz^{*2}\right)\\ =\;&\frac{1}{\sqrt{\zeta^2-4fg}}\exp\left[\frac{-\zeta\xi\eta+\xi^2g+\eta^2f}{\zeta^2-4fg}\right]\end{aligned} \tag{21}$$

with

$$\mathrm{Re}(\zeta\pm f\pm g)<0,\quad \mathrm{Re}\left(\frac{\zeta^2-4fg}{\zeta\pm f\pm g}\right)<0,$$

we have

$$\begin{aligned}&W(\rho,\gamma)\\ =\;&D_mD_n\sum_{m,n=0}^{\infty}\frac{(-1)^{m+n}\alpha^{*m}\alpha^n}{m!n!(n+q)!(m+q)!}\\ &\times\frac{\partial^{2m+q+k+l}}{\partial t^{m+q+k}\partial t'^{m+l}}\frac{\partial^{2n+q+k+l}}{\partial s^{n+l}\partial s'^{n+q+k}}\\ &\times\exp[(\gamma+\rho-s')t+(\gamma^*-\rho^*-s)t'\\ &+(\gamma-\rho)s+(\gamma+\rho)^*s'-|\gamma|^2\\ &-|\rho|^2]|_{t=t'=s=s'=0}.\end{aligned} \tag{22}$$

From Eq.(22) we find that the Wigner function $W(\rho,\gamma)$ of the GEPCS can be written as a sum of the mixture part $W^{\mathrm{M}}(\rho,\gamma)$ and the quantum interference part $W^{\mathrm{I}}(\rho,\gamma)$:

$$W(\rho,\gamma)=W^{\mathrm{M}}(\rho,\gamma)+W^{\mathrm{I}}(\rho,\gamma), \tag{23}$$

where the mixture part $W^{\mathrm{M}}(\rho,\gamma)$ has the following

form

$$
\begin{aligned}
&W^{\mathrm{M}}(\rho,\gamma)\\
&=\frac{D_n^2}{\pi^2}\sum_{n=0}^{\infty}\sum_{i=0}^{n+l}\sum_{j=0}^{n+q+k}\binom{n+l}{i}\binom{n+q+k}{j}\\
&\times\frac{i!j!|\alpha|^{2n}}{[n!(n+q)!]^2}(-1)^{l+k}\times|\gamma-\rho|^{2(n+l-i)}|\gamma\\
&+\rho|^{2(n+q+k-j)}\exp(-|\gamma|^2-|\rho|^2),
\end{aligned}
\tag{24}
$$

and the interference part $W^{\mathrm{I}}(\rho,\gamma)$ is

$$
\begin{aligned}
&W^{\mathrm{I}}(\rho,\gamma)\\
&=\frac{D_mD_n}{\pi^2}\left(\sum_{m>n}^{\infty}\sum_{i=0}^{m+l}\sum_{j=0}^{m+q+k}+\sum_{n>m}^{\infty}\sum_{i=0}^{n+l}\sum_{j=0}^{n+q+k}\right)\\
&\times\frac{(-1)^{m+n+i+j}\alpha^{*m}\alpha^n i!j!}{n!(n+q)!m!(m+q)!}\binom{n+l}{i}\binom{m+l}{i}\\
&\times\binom{n+q+k}{j}\binom{m+q+k}{j}(\gamma^*-\rho^*)^{m+l-i}\\
&\times(\gamma-\rho)^{n+k-i}(\gamma+\rho)^{q+m+k-j}\\
&\times(\gamma^*+\rho^*)^{q+n+k-j}\exp(-|\gamma|^2-|\rho|^2).
\end{aligned}
\tag{25}
$$

The quantum interference part $W^{\mathrm{I}}(\rho,\gamma)$ gives rise to the phase sensitive nonclassical effects, such as quadrature squeezing effect, which are associated with the variances of the quadrature fluctuations.

Now we would like to discuss changes in the Wigner function as we vary the parameters q, k, l and α. When $q=2, k=l=0$ and $\alpha=0.1$, the shape of the Wigner function is similar to that of the pair coherent state [see Fig.1(a)]. The Wigner function has negative parts, therefore nonclassical effects can be established. From Figs.1(b) and 1(c) it is clearly that, for $k=3$ and $l=1$, the Wigner function $W(\rho,\gamma)$ has two upward peaks when q is an odd number, however there exist two downward peaks when q is an even number at the same position. The feature indicates that the GEPCSs for q being odd numbers and the GEPCSs for q being even numbers are orthogonal. The results are in general valid for all values of q for the GEPCSs. On the other hand, with varing q we see that the negativity of $W(\rho,\gamma)$ has relations with the value of q. This means that the GEPCS exhibits different nonclassical statistical properties when q takes different values. The nonclassicality is more pronounced when q is an even number for the GEPCSs. Furthermore, the comparison of Fig.1(b) with Fig.2 emphasizes that the negativity of $W(\rho,\gamma)$ is associated with the parameters l, k and α. To be concrete, for fixed values of q, l and α, with decreasing k the nonclassicality is more pronounced. However, for fixed values of q, k and α (or l), the nonclassicality of the GEPCS is more pronounced when l (or α) increases. From Figs.1 and 2 we can also see that the Wigner functions $W(\rho,\gamma)$ have different distributions for different parameters q, α, k and l, namely the GEPCSs exhibit remarkably different quantum interference effects. In conslusion, the behaviour of the Wigner function $W(\rho,\gamma)$ is in good agreement with the quantum features of the GEPCS.

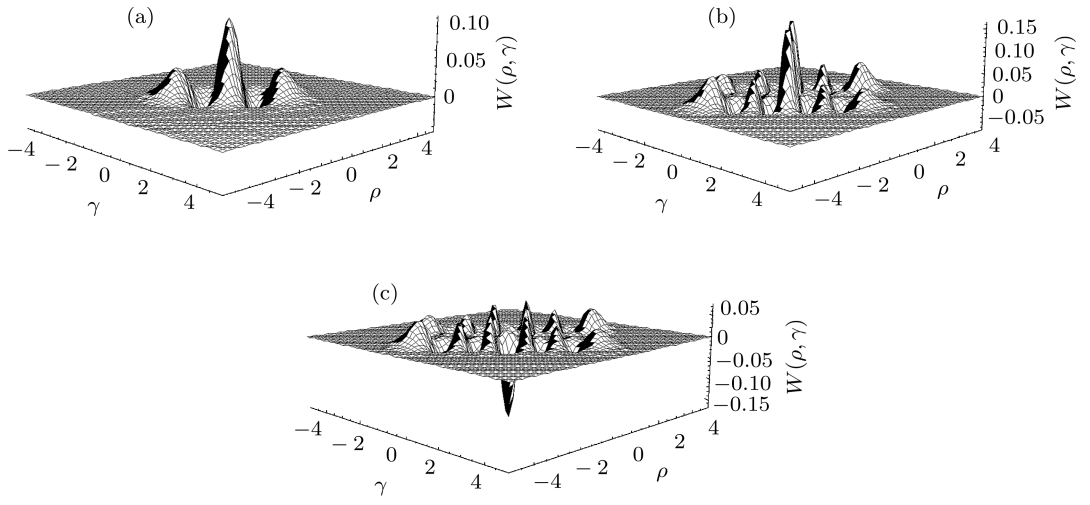


Fig.1. The Wigner functions for the states $|q,\alpha,k,l\rangle$ with ρ and γ for $\alpha=0.1$ and (a) $q=2, k=l=0$; (b) $q=1, k=3$ and $l=1$; (c) $q=2, k=3$ and $l=1$.

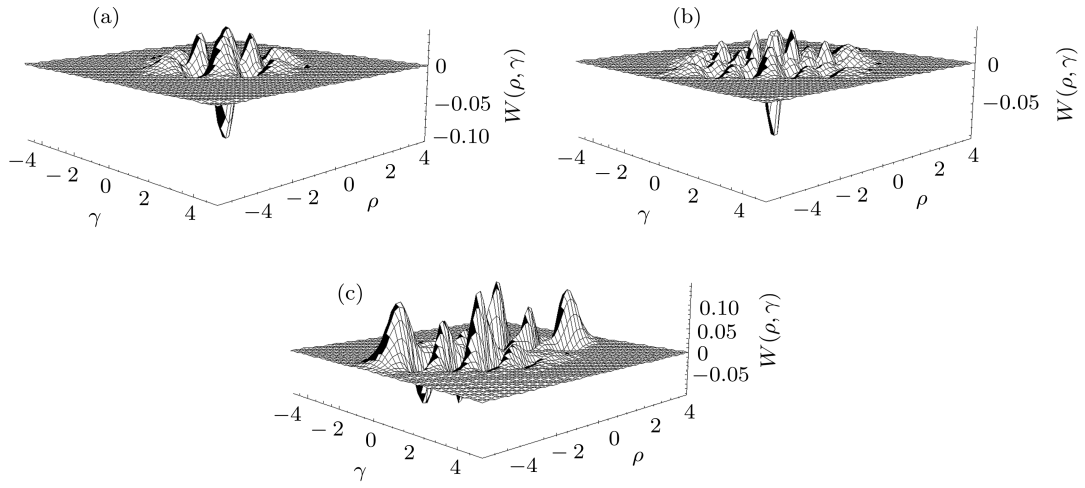


Fig.2. The Wigner functions for the states $|q,\alpha,k,l\rangle$ with ρ and γ for (a) $q=k=l=1$ and $\alpha=0.1$; (b) $q=1,k=l=3$ and $\alpha=0.1$; (c) $q=l=1,k=3$ and $\alpha=0.5$.

Further, performing the integration of $\Delta_{1,2}(\rho,\gamma)$ over $\mathrm{d}^2\rho$ we obtain a projection operator[11]

$$\int \mathrm{d}^2\rho\Delta_{1,2}(\rho,\gamma)=\frac{1}{\pi}\left|\eta\right\rangle\left\langle\eta\right|_{\eta=\gamma}, \tag{26}$$

thus we obtain a marginal distribution of the Wigner function for the state $|q,\alpha,k,l\rangle$ in the γ variable, i.e.

$$\begin{aligned}&\int \mathrm{d}^2\rho W(\rho,\gamma)\\ &=\frac{1}{\pi}\left|\langle\eta|\ q,\alpha,k,l\rangle\right|^2_{\eta=\gamma}\\ &=\frac{\mathrm{e}^{-|\gamma|^2}D_n^2}{\pi}\left|\sum_{n=0}^{\infty}\frac{(-1)^{n+l}\alpha^n}{n!(n+q)!}H_{n+l,n+q+k}(\gamma,\gamma^*)\right|^2.\end{aligned} \tag{27}$$

Similarly, in the entangled state $|\xi\rangle$ representation, the two-mode Wigner operator $\Delta_{1,2}(\rho,\gamma)$ is[11]

$$\begin{aligned}&\Delta_{1,2}(\rho,\gamma)\\ &=\int\frac{\mathrm{d}^2\xi}{\pi^3}\left|\rho-\xi\right\rangle\left\langle\rho+\xi\right|\exp(\xi^*\gamma-\xi\gamma^*),\end{aligned} \tag{28}$$

where $|\xi\rangle$ is the common eigenstate of two particles' centre-of-mass coordinate (X_1+X_2) and the relative momentum (P_1-P_2) in two-mode Fock space. The entangled state $|\xi\rangle$ is given by

$$\begin{aligned}&\left|\xi\right\rangle=\exp\left[-\frac{1}{2}\left|\xi\right|^2+\xi a_1^+ +\xi^* a_2^+ -a_1^+ a_2^+\right]\left|00\right\rangle,\\ &\xi=\xi_1+\mathrm{i}\xi_2,\end{aligned} \tag{29}$$

which is capable of making up the entangled state representation. Performing the integration of $\Delta_{1,2}(\rho,\gamma)$ over $\mathrm{d}^2\gamma$ we obtain another projection operator, i.e.,

$$\int \mathrm{d}^2\gamma\Delta_{1,2}(\rho,\gamma)=\frac{1}{\pi}\left|\xi\right\rangle\left\langle\xi\right|_{\xi=\rho}, \tag{30}$$

therefore, using Eqs.(5) and (30) we have

$$\begin{aligned}&\int \mathrm{d}^2\gamma W(\rho,\gamma)\\ &=\frac{1}{\pi}\left|\langle\xi|\ q,\alpha,k,l\rangle\right|^2_{\xi=\rho}\\ &=\frac{\mathrm{e}^{-|\rho|^2}D_n^2}{\pi}\left|\sum_{n=0}^{\infty}\frac{(-1)^{n+l}\alpha^n}{n!(n+q)!}H_{n+l,n+q+k}(\rho,\rho^*)\right|^2,\end{aligned} \tag{31}$$

which is another marginal distribution of the Wigner function for the state $|q,\alpha,k,l\rangle$ in the ρ variable. Equation (27) (or Eq.(31)) is proportional to the probability for finding the two particles, which have total momentum $\sqrt{2}\gamma_2$ (or relative momentum $\sqrt{2}\rho_2$) and simultaneously relative position $\sqrt{2}\gamma_1$ (or centre-of-mass position $\sqrt{2}\rho_1$), under the GEPCS. Therefore, for an entangled particle system, the physical meaning of the Wigner function $W(\rho,\gamma)$ should lie in that its marginal distributions give the probability of finding the particles in an entangled way in the ρ–γ phase space.

4. Tomogram of the state $|q,\alpha,k,l\rangle$

The use of tomograms in quantum mechanics and quantum optics provides the possibility of describing a quantum state with a positive probability distribution. A direct description of quantum states by means of quantum tomograms for the system observables is

interesting from both the theoretical and experimental points of view. Therefore, in recent years tomogram approach has brought much interest of some physicists. In this section we continue to derive the tomogram of $|q,\alpha,k,l\rangle$, which is defined as

$$\begin{aligned} & T(\eta_1,\eta_2,\tau_1,\tau_2) \\ =\ & \pi\iint \mathrm{d}^2\gamma\mathrm{d}^2\rho\delta(\eta_1-\mu_1\gamma_1-\nu_1\rho_2) \\ & \times\delta(\eta_2-\mu_2\rho_1-\nu_2\gamma_2)W(\rho,\gamma), \end{aligned} \tag{32}$$

where τ_1,τ_2,ρ and γ are complex numbers, $\tau_j = |\tau_j|\mathrm{e}^{\mathrm{i}\theta} = \mu_j+\mathrm{i}\nu_j (j=1,2)$, $\rho=\rho_1+\mathrm{i}\rho_2$ and $\gamma=\gamma_1+\mathrm{i}\gamma_2$. However, it will be very difficult to evaluate the integration if we directly substitute Eq.(23) into Eq.(32), fortunately we use the following Radon transform between the Wigner operator and the projection operator of $|\eta_1,\eta_2,\tau_1,\tau_2\rangle$[13]

$$\begin{aligned} & |\eta_1,\eta_2,\tau_1,\tau_2\rangle\langle\eta_1,\eta_2,\tau_1,\tau_2| \\ =\ & \pi\iint \mathrm{d}^2\gamma\mathrm{d}^2\rho\delta(\eta_1-\mu_1\gamma_1-\nu_1\rho_2) \\ & \times\delta(\eta_2-\mu_2\rho_1-\nu_2\gamma_2)\Delta_{1,2}(\rho,\gamma). \end{aligned} \tag{33}$$

Thus the tomogram of the state $|q,\alpha,k,l\rangle$ is

$$\begin{aligned} & T(\eta_1,\eta_2,\tau_1,\tau_2) \\ =\ & \pi\iint \mathrm{d}^2\gamma\mathrm{d}^2\rho\delta(\eta_1-\mu_1\gamma_1-\nu_1\rho_2) \\ & \times\delta(\eta_2-\mu_2\rho_1-\nu_2\gamma_2) \\ & \times\langle q,\alpha,k,l|\,\Delta_{1,2}(\rho,\gamma)\,|q,\alpha,k,l\rangle \\ =\ & |\langle\eta_1,\eta_2,\tau_1,\tau_2|\,q,\alpha,k,l\rangle|^2, \end{aligned} \tag{34}$$

which shows that for tomographic approach there exists the entangled state $|\eta_1,\eta_2,\tau_1,\tau_2\rangle$, and the Radon transforms of the Wigner operator are just the entangled state density matrices $|\eta_1,\eta_2,\tau_1,\tau_2\rangle\langle\eta_1,\eta_2,\tau_1,\tau_2|$. As a result, the tomogram of quantum states can be considered as the module-square of the states' wave function in this entangled state representation. This is a new way to derive tomogram of quantum states. Using the following integral formula[11]

$$\begin{aligned} & \int\frac{\mathrm{d}^2z}{\pi}z^n\exp(\zeta|z|^2+\xi z+\eta z^*+gz^{*2}) \\ =\ & -\left(\frac{1}{\zeta}\right)^{2n+1}\exp\left[\frac{1}{\zeta^2}(g\xi^2-\zeta\xi\eta)\right] \\ & \times\sum_{k=0}^{[n/2]}\frac{n!}{k!(n-2k)!}(2\xi g-\zeta\eta)^{n-2k}(g\zeta^2)^k \end{aligned} \tag{35}$$

and the entangled state[11]

$$\begin{aligned} |\eta_1,\eta_2,\tau_1,\tau_2\rangle =\ & \frac{1}{\sqrt{|\tau_1\tau_2|}}\exp\left[-\frac{\eta_1^2}{2|\tau_1|^2}-\frac{\eta_2^2}{2|\tau_2|^2}+\left(\frac{\eta_1}{\tau_1^*}+\frac{\eta_2}{\tau_2^*}\right)a^+ +\left(-\frac{\eta_1}{\tau_1^*}+\frac{\eta_2}{\tau_2^*}\right)b^+\right. \\ & \left.+\frac{1}{2}\left(\mathrm{e}^{\mathrm{i}2\theta_1}-\mathrm{e}^{\mathrm{i}2\theta_2}\right)a^+b^+-\frac{1}{4}\left(\mathrm{e}^{\mathrm{i}2\theta_1}+\mathrm{e}^{\mathrm{i}2\theta_2}\right)\left(a^{+2}+b^{+2}\right)\right]|00\rangle, \end{aligned} \tag{36}$$

we have

$$\begin{aligned} & \langle q,\alpha,k,l|\,\eta_1,\eta_2,\tau_1,\tau_2\rangle \\ =\ & \sum_{n=0}^{\infty}\sum_{s=0}^{[(n+q+k)/2]}\sum_{t=0}^{n+q+k-2s}\sum_{m=0}^{[(n+l)/2]}\sqrt{\frac{(n+q+k)!(n+l)!}{n!(n+q)!}} \\ & \times\frac{(-1)^{s+m}D_n\alpha^{*n}}{2^{t+2s+2m}m!s!t!(n+q+k-2s-t)!(n+l-2m-t)!} \\ & \times\left(\frac{\eta_1}{\tau_1^*}+\frac{\eta_2}{\tau_2^*}\right)^{n+q+k-2s-t}\left(-\frac{\eta_1}{\tau_1^*}+\frac{\eta_2}{\tau_2^*}\right)^{n+l-2m-t}(\mathrm{e}^{\mathrm{i}2\theta_1}-\mathrm{e}^{\mathrm{i}2\theta_2})^t(\mathrm{e}^{\mathrm{i}2\theta_1}+\mathrm{e}^{\mathrm{i}2\theta_2})^{s+m} \\ & \times\frac{1}{\sqrt{|\tau_1\tau_2|}}\exp\left(-\frac{\eta_1^2}{2|\tau_1|^2}-\frac{\eta_2^2}{2|\tau_2|^2}\right). \end{aligned} \tag{37}$$

Using Eq.(34) we can obtain the tomogram of the state $|q,\alpha,k,l\rangle$

$$T(\eta,\tau_1,\tau_2) = \left|\sum_{n=0}^{\infty}\sum_{s=0}^{[(n+q+k)/2]}\sum_{t=0}^{n+q+k-2s}\sum_{m=0}^{[(n+l)/2]}\sqrt{\frac{(n+q+k)!(n+l)!}{n!(n+q)!}}\right.$$

$$\times\frac{(-1)^{s+m}D_n\alpha^{*n}}{2^{t+2s+2m}m!s!t!(n+q+k-2s-t)!(n+l-2m-t)!}\times\left(\frac{\eta_1}{\tau_1^*}+\frac{\eta_2}{\tau_2^*}\right)^{n+q+k-2s-t}$$
$$\times\left(-\frac{\eta_1}{\tau_1^*}+\frac{\eta_2}{\tau_2^*}\right)^{n+l-2m-t}\left(\mathrm{e}^{\mathrm{i}2\theta_1}-\mathrm{e}^{\mathrm{i}2\theta_2}\right)^t\left(\mathrm{e}^{\mathrm{i}2\theta_1}+\mathrm{e}^{\mathrm{i}2\theta_2}\right)^{s+m}$$
$$\times\frac{1}{\sqrt{|\tau_1\tau_2|}}\exp\left(-\frac{\eta_1^2}{2|\tau_1|^2}-\frac{\eta_2^2}{2|\tau_2|^2}\right)\Bigg|^2. \tag{38}$$

Hence, experimentally one can measure the module-square of the wave function $|q,\alpha,k,l\rangle$ in the entangled state $|\eta_1,\eta_2,\tau_1,\tau_2\rangle$ representation, then the tomogram of the GEPCS is obtained and the GEPCS may be measured indirectly.

In summary, we have used the entangled state representation of Wigner operator to derive the Wigner function of the GEPCS. In the ρ–γ phase space, we discuss the variations of the Wigner function distribution with the parameters q, α, k and l. It is found that, in the ρ–γ phase space, the variations of the Wigner functions with the parameters q, α, k and l reveal well interference properties of the GEPCSs. On the other hand, the results of the tomogram function for the GEPCS obtained by using the projection operator of the entangled state $|\eta_1,\eta_2,\tau_1,\tau_2\rangle$ may be useful for experiments as references.

References

[1] Wigner E 1932 *Phys. Rev.* **40** 749

[2] Vogel K and Risken H 1989 *Phys. Rev.* A **40** 2847

[3] Vracoe B T H, Brattke S, Weidinger M and Walther H 2000 *Nature* **403** 743
Brattke S, Vracoe B T H and Walther H 2001 *Phys. Rev. Lett.* **86** 3534

[4] Zhang Z M 2003 *Chin. Phys. Lett.* **20** 227
Zhang Z M 2004 *Chin. Phys. Lett.* **21** 5

[5] Zheng S B 2004 *Chin. Phys.* **13** 187

[6] Meng X G, Wang J S and Liang B L 2007 *Acta Phys. Sin.* **56** 2160 (in Chinese)

[7] Meng X G, Wang J S and Li Y L 2007 *Chin. Phys.* **16** 2415

[8] Lu H and Guo G C 1999 *Acta Phys. Sin.* (Overseas Edition) **8** 577
Enaki N and Ciornea V 2004 *Physica* A **340** 436

[9] Agawal G S 1986 *Phys. Rev. Lett.* **57** 827

[10] Agawal G S 1988 *J. Opt. Soc. Am.* B **5** 1940

[11] Fan H Y 2001 *Entangled State Representations in Quantum Mechanics and Their Applications* (Shanghai: Shanghai Jiaotong University Press) (in Chinese)

[12] Fan H Y and Chen J H 2002 *Phys. Lett.* A **303** 311

[13] Fan H Y and Cheng H L 2001 *Commun. Theor. Phys.* **36** 651