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Amplitude-squared squeezing of superposition of coherent states*

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This paper discusses the amplitude-squared squeezing for the superposition of two coherent states with their phase differences being separately $\pi/2$, $3\pi/2$, and π, as well as for the superposition state of two pseudoclassical states. According to the analysis, it is found that the superposition state of two coherent states with their phase differences $\pi/2$ and $3\pi/2$, and the superposition state of two pseudoclassical states both do exhibit the amplitude-squared squeezing. Also, some specific states are found to exhibit even stronger squeezing effects when relative phase of the superposition is equal to the average photon number. Amplitude-squared squeezing is dependent on the difference in phase between two coherent states.

Keywords: nonclassical states, superposition of two coherent states, amplitude-squared squeezing
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1. Introduction

Much attention has been paid to the study of the amplitude-squared squeezing which occurs in the fundamental mode in the second harmonic generation, degenerate parametric amplification, two-photon absorption, two-photon resonance fluorescence, as well as in higher-order processes.[1−4] Amplitude-squared squeezing states can be used to reduce the noise in the output of certain nonlinear optical devices,[3] and this type of squeezing can be converted into normal squeezing. It has also been discussed that the amplitude-squared squeezing states of motion can be generated by using appropriate stimulated Raman couplings between vibrational sidebands.[5]

Quantum mechanical superposition of two coherent states of light results in a new state which displays various nonclassical features.[6] The nonclassical properties of quantum superposition is of considerable importance because of their applications in quantum information theory such as communication,[7,8] quantum teleportation,[9,10] dense coding,[11,12] and the nonclassicality is the necessary input for the entangled states.[13] In a very recent paper[14] the amplitude-squared squeezing of the generalized odd–even coherent states of the anharmonic oscillator in a finite-dimensional Hilbert space has been elaborated. The nonclassical properties of the superposition of two coherent states with their phase differences being separately $\pi/2$, $3\pi/2$, and π, photon-added pair coherent states, as well as the superposition state of two pseudoclassical states have been discussed.[15−21] Hence we, in the present paper, restrict ourselves solely to the discussion on their amplitude-squared squeezing properties.

The present paper is organized in the following sequence: In Sections 2 and 3, we discuss the amplitude-squared squeezing for the superposition state of two coherent states with their phase differences being separately $\pi/2$ and $3\pi/2$, respectively. While in Section 4 the same property is studied for even–odd coherent states and Yurke–Stoler states. Section 5 deals with the amplitude-squared squeezing for the superposition state of two pseudoclassical states. The final section gives a short discussion on the results.

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2. Two coherent states with a phase difference of $\pi/2$

The operators which represent the real and imaginary parts of the squared complex amplitude of the field can be defined as[3]

$$X = \frac{1}{2}\left(a^2 + a^{\dagger 2}\right), \quad \text{and} \quad Y = \frac{1}{2\mathrm{i}}\left(a^2 - a^{\dagger 2}\right). \quad (1)$$

It follows from the above equation that X and Y obey the commutation relation

$$[X,\ Y] = \frac{\mathrm{i}}{2}\left[a^2, a^{\dagger 2}\right] = \mathrm{i}(2N+1), \quad (2)$$

where $N = a^\dagger a$. In general, for any operator $X(Y)$, the squeezing condition is given by

$$S_{X(Y)} = (\Delta X(Y))^2 - \frac{1}{4}\left|\left\langle\left[a^2, a^{\dagger 2}\right]\right\rangle\right| < 0. \quad (3)$$

Now we examine the amplitude-squared squeezing for the superposition of two coherent states with a phase difference of $\pi/2$

$$|\psi_1\rangle = \frac{N_1}{\sqrt{2}}\left(|\alpha\rangle + \mathrm{e}^{\mathrm{i}\phi}\,|\mathrm{i}\alpha\rangle\right), \quad (4)$$

where the normalization constant $N_1 = \Big[1 + \mathrm{e}^{-|\alpha|^2} \times \cos\left(\phi + |\alpha|^2\right)\Big]^{-1/2}$. By making use of expression (3) we find the squeezing conditions for X and Y as follows:

$$S_X = N_1^2\left[\left({x_1}^2 - {x_2}^2\right)^2 - 4\ {x_1}^2{x_2}^2\ \mathrm{e}^{-|\alpha|^2}\cos\left(\phi + |\alpha|^2\right)\right] - \left[N_1^2\ 2\ x_1x_2\ \mathrm{e}^{-|\alpha|^2}\sin\left(\phi + |\alpha|^2\right)\right]^2, \quad (5)$$

$$S_Y = N_1^2\left[4\ {x_1}^2{x_2}^2 - \left({x_1}^2 - {x_2}^2\right)^2\ \mathrm{e}^{-|\alpha|^2}\cos\left(\phi + |\alpha|^2\right)\right] - \left[N_1^2\left({x_1}^2 - {x_2}^2\right)\ \mathrm{e}^{-|\alpha|^2}\sin\left(\phi + |\alpha|^2\right)\right]^2, \quad (6)$$

where x_1 and x_2 are real and imaginary parts of the complex parameter α. It is found from the above equations that squeezing conditions, in which the negative values indicate the amplitude-squared squeezing effects in a quantum state, may be discussed as a function of x_1 and x_2. Although squeezing conditions (5) and (6) are difficult to analyse accurately except for certain values of x_1, x_2 and ϕ, we can approximate the squeezing conditions by letting the first term be less than 0 in expressions (5) and (6). Owing to the negativity of the second term, the amplitude-squared squeezing always takes place if

$$\cos(\phi + |\alpha|^2) \geq \frac{\left({x_1}^2 - {x_2}^2\right)^2 \mathrm{e}^{|\alpha|^2}}{4{x_1}^2{x_2}^2} \quad (7)$$

for the operator X, and if

$$\cos\left(\phi + |\alpha|^2\right) \geq \frac{4\ {x_1}^2{x_2}^2\mathrm{e}^{|\alpha|^2}}{\left({x_1}^2 - {x_2}^2\right)^2} \quad (8)$$

for operator Y.

Evidently from inequalities (7) and (8) we can see that they can be satisfied only in some specific regions because the inequality $|\cos(\phi + |\alpha|^2)| \leq 1$ must be satisfied too. Because the exponential $\exp(|\alpha|^2)$ can increase rapidly with $|\alpha|^2$, the squeezing for the operator X must occur near the regions with $x_1 = \pm x_2$. In other regions there should be no amplitude-square squeezing effect. Such an analysis can be clearly proved from Fig.1, which is depicted directly by squeezing condition expression (5) as a function of real and imaginary parts of the complex parameter α for several values of ϕ. The range and the position of the squeezing regions change with relative phase ϕ. No squeezing occurs in the regions far from the lines $x_1 = \pm x_2$ for operator X. Similarly, the squeezing effect can be analysed for operator Y. Moreover, the squeezing regions are symmetrical with respect to lines $x_1 = x_2$ and $x_1 = -x_2$ for operator X and with respect to lines $x_1 = 0$ and $x_2 = 0$ for operator Y, which can be understood easily because S_X and S_Y depend on x_1^2 and x_2^2.

Now we focus our attention on the amplitude-squared squeezing for the specific states $|\psi_1\rangle^+$ and $|\psi_1\rangle^-$, which are parametrized solely with respect to amplitude α, i.e. for $\phi = -|\alpha|^2$ and $\phi = -|\alpha|^2 + \pi$ respectively. In this case we have

$$S_X^{\pm} = \frac{1}{2}\Big[{x_1}^4 + {x_2}^4 - 6{x_1}^2{x_2}^2 + N_{\pm}^2 N_{\mp}^{-2}({x_1}^2 + {x_2}^2)^2\Big], \quad (9a)$$

$$S_Y^{\pm} = -\frac{1}{2}\Big[{x_1}^4 + {x_2}^4 - 6{x_1}^2{x_2}^2 - N_{\pm}^2 N_{\mp}^{-2}({x_1}^2 + {x_2}^2)^2\Big], \quad (9b)$$

where $N_{\pm} = \left[1 \pm \mathrm{e}^{-|\alpha|^2}\right]^{-1/2}$. The squeezing conditions for the state $|\psi_1\rangle^+$ for both operators X and Y

are presented in Fig.2. It is evident from the figure that the squeezing regions for the specific state are expanded and larger than the state $|\psi_1\rangle$. It can also be easily verified by substituting $x_1 = \pm x_2$ into expression (9a) and $x_1 = 0 = x_2$ into expression (9b) that the amplitude-squared squeezing occurs only for the state $|\psi_1\rangle^+$ but not for the state $|\psi_1\rangle^-$. It is interesting to note that the effects of amplitude-squared squeezing are similar to those discussed by Zeng *et al*[15] for quadrature squeezing. Their main difference is that the regions in the case of amplitude-squared squeezing are located along both the lines $x_1 = x_2$ and $x_1 = -x_2$ for operator X and along the lines $x_1 = 0$ and $x_2 = 0$ for operator Y, and they are symmetrical as well. Here we can conclude that the specific state $|\psi_1\rangle^+$ shows more amplitude-squared squeezings in both the operators X and Y.

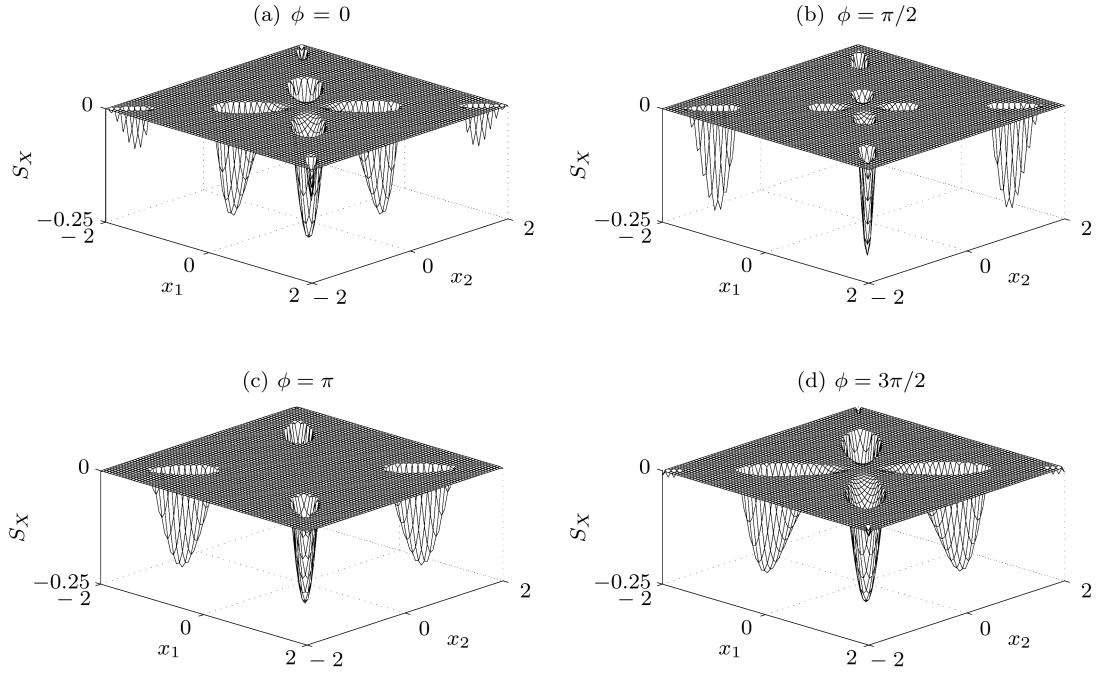


Fig.1. Amplitude-squared squeezing S_X in operator X for quantum superposition state $|\psi_1\rangle$ of expression (4), given by expression (5), showing its dependence on the real and imaginary parts of the complex parameter α for different values of the relative phase ϕ. The amplitude-squared squeezing increases with the value of the relative phase increasing.

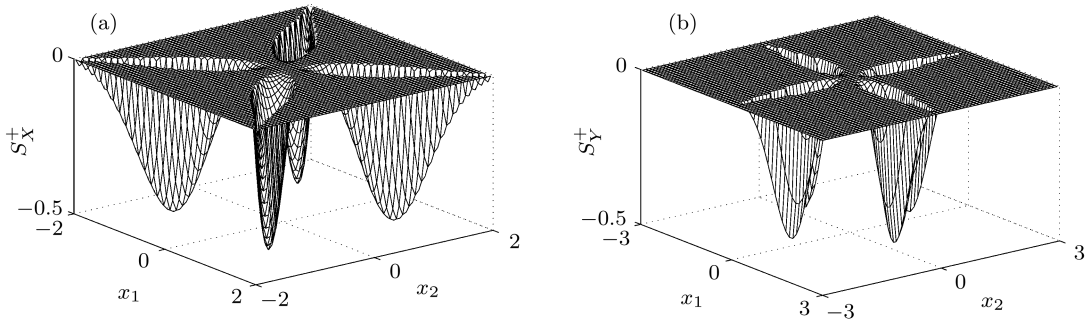


Fig.2. Amplitude-squared squeezing for the specific state when the relative phase of the superposition ϕ is equal to the average photon number $|\alpha|^2$ in its dependence on the real and imaginary parts of the complex parameter α in operator X (a) and in operator Y (b). The amplitude-squared squeezing increases much more in this case.

3. Two coherent states with a phase difference of $3\pi/2$

In this section we consider the superposition of two coherent states with a phase difference of $3\pi/2$ and examine the effects of amplitude-squared squeezing. Superposition of two coherent states with a phase difference of $3\pi/2$ is given as[16]

$$|\psi_2\rangle = \frac{N_2}{\sqrt{2}}\left(|\alpha\rangle + \mathrm{e}^{\mathrm{i}\phi}\,|-\mathrm{i}\alpha\rangle\right), \tag{10}$$

where N_2 is the normalization constant. From expression (3) we find that

$$\begin{aligned} &S_X \\ &= N_2^2\left[\left({x_1}^2 - {x_2}^2\right)^2 - 4{x_1}^2{x_2}^2\mathrm{e}^{-|\alpha|^2}\cos\left(\phi - |\alpha|^2\right)\right] \\ &\quad - \left[N_2^2 2x_1x_2\mathrm{e}^{-|\alpha|^2}\sin\left(\phi - |\alpha|^2\right)\right]^2. \end{aligned} \tag{11}$$

We analyse the squeezing condition expression (11) analytically as done before. The state $|\psi_2\rangle$ will exhibit the amplitude-squared squeezing if

$$\cos\left(\phi - |\alpha|^2\right) \geq \frac{\left({x_1}^2 - {x_2}^2\right)^2\ \mathrm{e}^{|\alpha|^2}}{4\ {x_1}^2{x_2}^2}. \tag{12}$$

There exist many values of $|\alpha|^2$ $\left(|\alpha|^2 = x_1^2 + x_2^2\right)$ and ϕ such that the above inequality in expression (12) is satisfied and thus the state $|\psi_2\rangle$ exhibits the amplitude-squared squeezing. By comparing inequality (12) with inequality (7), one can conclude that the results of the amplitude-squared squeezing for the state $|\psi_2\rangle$ are very similar to those for the state $|\psi_1\rangle$ as discussed in the previous section. This implies that the squeezing regions in phase space in S_X are located along lines $x_1 = \pm x_2$. The squeezing condition for operator X is plotted in Fig.3 as a function of x_1 only for several values of ϕ. The negative value of squeezing condition S_X implies the amplitude-squared squeezing. We skip over the case of squeezing as a function of x_2 in that its result is the same as that in the case of squeezing as a function of x_1 despite the symmetry between x_1 and x_2 in expression (11). We conclude this section by considering the amplitude-squared squeezing for the specific states $|\psi_2\rangle^+$ $\left(\phi = |\alpha|^2\right)$ and $|\psi_2\rangle^-$ $\left(\phi = |\alpha|^2 - \pi\right)$ as

$$\begin{aligned} S_X^{\pm} = \frac{1}{2}\Big[&{x_1}^4 + {x_2}^4 - 6{x_1}^2{x_2}^2 \\ &+ N_{\pm}^2\ N_{\mp}^{-2}\left({x_1}^2 + {x_2}^2\right)^2\Big]. \end{aligned} \tag{13}$$

The squeezing condition for the specific state $|\psi_2\rangle^+$ is plotted in Fig.4 from which it can be obviously seen that the amplitude-squared squeezing may take place at various values of x_1. It means that the specific state $|\psi_2\rangle^+$ does exhibit even stronger effects of the amplitude-squared squeezing as well as other nonclassical features.[16] To verify the results presented in Fig.4 we substitute $x_1 = x_2$ and $x_1 = -x_2$ into expression (13) respectively, and after minor algebra we arrive at

$$S_X^{\pm} = \mp\ 4\ N_{\pm}^2\ {x_1}^4\ \mathrm{e}^{-2x_1^2}, \tag{14}$$

which are the analytical expressions for the amplitude-squared squeezing of the specific states in operator X. On the other hand, positive sign in expression (14) shows that there will be no amplitude-squared squeezing for the specific state $|\psi_2\rangle^-$. In a similar way the squeezing effects for the operator Y can be calculated.

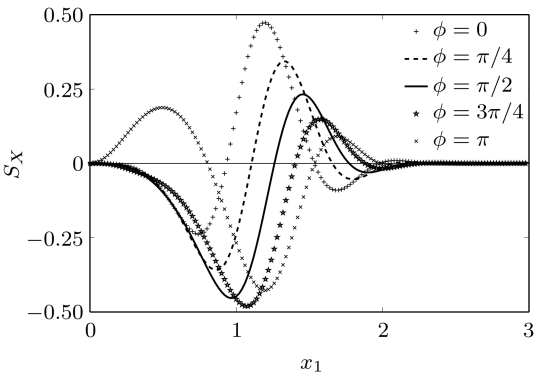


Fig.3. The amplitude-squared squeezing for the state $|\psi_2\rangle$ of expression (10) by making use of $x_1 = x_2$ and $x_1 = -x_2$ in S_X of expression (11) separately. Shown here are only the curves for x_1 and no curves for x_2 due to the symmetry between x_1 and x_2.

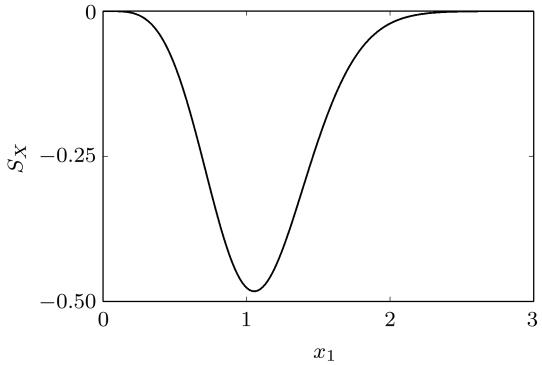


Fig.4. The amplitude-squared squeezing for the specific state $|\psi_2\rangle^+$, that is, when $\phi = |\alpha|^2$ in operator X, corresponding to expression (13).

4. Two coherent states with a phase difference of π

The superposition of two coherent states with a phase difference of π can be defined as

$$|\psi_3\rangle = N_3\left(|\alpha\rangle + \mathrm{e}^{\mathrm{i}\phi}\,|-\alpha\rangle\right). \tag{15}$$

For $\phi = 0$, expression (15) describes the even coherent states while for $\phi = \pi$, it describes the odd coherent states and for $\phi = \pi/2$, we have the Yurke–Stoler states.[19] The squeezing condition for the superposition of two coherent states with a phase difference of π is given by

$$\begin{aligned} S_X &= \frac{1}{4}\left[\alpha^4 + \alpha^{*4} + 2\,|\alpha|^4\right] - \frac{1}{4}\left[\alpha^2 + \alpha^{*2}\right]^2 \\ &= 0, \end{aligned} \tag{16}$$

and similarly the squeezing condition can be obtained for operator Y. From expression (16) it follows that the superposition state $|\psi_3\rangle$ does not exhibit amplitude-squared squeezing at all. Hence, one can conclude that the even–odd coherent states and Yurke–Stoler states do not exhibit amplitude-squared squeezing either because the expressions of the squeezing condition for these states are the same as ones that are given by expression (16). We conclude, therefore, that the superposition states in the forms of expressions (4) and (10) are superior to the even-odd coherent states and Yurke–Stoler states.

In general, the squeezing condition for the superposition of any two coherent states along a circle, if coherent state $|\alpha\rangle$ is taken to be the reference, that is, the state in the form

$$|\psi\rangle = N\left(|\alpha\rangle + \mathrm{e}^{\mathrm{i}\phi}\left|\mathrm{e}^{\mathrm{i}\theta}\alpha\right\rangle\right) \tag{17}$$

can be expressed as

$$S_X = S_1 + S_2 + S_3 - (S_4 + S_5)^2, \tag{18a}$$

$$S_Y = S_1 - S_2 - S_3 + (S_4 - S_5)^2, \tag{18b}$$

where

$$N = \left[2 + 2\,\mathrm{e}^{-|\alpha|^2+|\alpha|^2\cos\theta}\cos\left(\phi + |\alpha|^2\sin\theta\right)\right]^{-1/2}$$

and

$$S_1 = N^2\,|\alpha|^4\left[1 + \mathrm{e}^{-|\alpha|^2+|\alpha|^2\cos\theta}\cos\left(2\theta + \phi + |\alpha|^2\sin\theta\right)\right], \tag{19a}$$

$$\begin{aligned} S_2 = (S_3)^* = \frac{N^2\alpha^4}{4}&\left[\left[1 + \mathrm{e}^{4\mathrm{i}\theta}\right]\left[1 + \mathrm{e}^{-|\alpha|^2+|\alpha|^2\cos\theta}\cos\left(\phi + |\alpha|^2\sin\theta\right)\right]\right. \\ &\left.-\mathrm{i}\left[1 - \mathrm{e}^{4\mathrm{i}\theta}\right]\ \mathrm{e}^{-|\alpha|^2+|\alpha|^2\cos\theta}\sin\left(\phi + |\alpha|^2\sin\theta\right)\right], \end{aligned} \tag{19b}$$

$$\begin{aligned} S_4 = (S_5)^* = \frac{N^2\alpha^2}{2}&\left[\left[1 + \mathrm{e}^{2\mathrm{i}\theta}\right]\left[1 + \mathrm{e}^{-|\alpha|^2+|\alpha|^2\cos\theta}\cos\left(\phi + |\alpha|^2\sin\theta\right)\right]\right. \\ &\left.-\mathrm{i}\left[1 - \mathrm{e}^{2\mathrm{i}\theta}\right]\ \mathrm{e}^{-|\alpha|^2+|\alpha|^2\cos\theta}\sin\left(\phi + |\alpha|^2\sin\theta\right)\right]. \end{aligned} \tag{19c}$$

By changing the value of phase difference θ one can find effects of the amplitude-squared squeezing for any of the states that we have discussed in the previous sections for several values of x_1, x_2 and ϕ. For example, by using $\theta = \pi$ we find that there exists no amplitude-squared squeezing which is the case of superposition state $|\psi_3\rangle$.

5. Two pseudoclassical states

The quantum-mechanical superposition of two coherent states (pseudo-classical states) having the same mean photon numbers but different phases can be defined as[18]

$$|\psi_4\rangle = \frac{N_4}{\sqrt{2}}\left(\left|\alpha\mathrm{e}^{\mathrm{i}\theta/2}\right\rangle + \left|\alpha\mathrm{e}^{-\mathrm{i}\theta/2}\right\rangle\right) \tag{20}$$

with the normalization constant $N_4 = \left[1 + \mathrm{e}^{-|\alpha|^2+|\alpha|^2\cos\theta}\cos\left(|\alpha|^2\sin\theta\right)\right]^{-1/2}$. The squeezing condition S_X for the state $|\psi_4\rangle$ takes the form

$$S_X = \frac{N_4^2}{4}\left[(2{x_1}^4 + 2{x_2}^4 - 12{x_1}^2{x_2}^2)\cos(2\theta) + 2({x_1}^2 + {x_2}^2)^2\right] + N_4^2\left[({x_1}^2 - {x_2}^2)^2\ \mathrm{e}^{-|\alpha|^2+|\alpha|^2\cos\theta}\cos\left(2\theta + |\alpha|^2\sin\theta\right)\right] - N_4^4\left[\ ({x_1}^2 - {x_2}^2)\ \left(\cos\theta + \mathrm{e}^{-|\alpha|^2+|\alpha|^2\cos\theta}\cos(\theta + |\alpha|^2\sin\theta)\right)\right]^2. \quad (21)$$

From the above equation it follows that no amplitude-squared squeezing is found when $\theta = 0$ and $\theta = 2\pi$. For $\theta = \pi$, which is the case of even coherent state, state $|\psi_4\rangle$ does not exhibit the amplitude-squared squeezing. Moreover, by substituting $x_2 = 0$ into expression (21) we can see that the maximum degree of squeezing is obtained for small values of phase difference (see Fig.5). One can also find that state $|\psi_4\rangle$ does exhibit the strong amplitude-squared squeezing when θ is taken to be equal to the average photon number.

The generalized superposition state of two pseudoclassical states can be written as

$$|\Psi\rangle = \frac{N}{\sqrt{2}}\left(\left|\alpha\mathrm{e}^{\mathrm{i}\theta/2}\right\rangle + \mathrm{e}^{\mathrm{i}\phi}\left|\alpha\mathrm{e}^{-\mathrm{i}\theta/2}\right\rangle\right). \quad (22)$$

The squeezing condition for this state in X for $x_2 = 0$ is in the form

$$S_X = \frac{N^2}{2}\left[{x_1}^4\cos(2\theta) + {x_1}^4 + 2\ {x_1}^4\ \mathrm{e}^{-{x_1}^2+{x_1}^2\cos\theta}\cos\left(2\theta + {x_1}^2\sin\theta - \phi\right)\right] - N^4\left[\ {x_1}^2\ \left(\cos\theta + \mathrm{e}^{-{x_1}^2+{x_1}^2\cos\theta}\cos(\theta + {x_1}^2\sin\theta - \phi)\right)\right]^2. \quad (23)$$

It is noted that for $\theta = \pi = \phi$ the state $|\Psi\rangle$ behaves like an odd coherent state, and for $\theta = \pi$ and $\phi = \pi/2$ it attains a state very similar to Yurke–Stoler state with the phase angle rotating through $\pi/2$. In this context the state does not exhibit the amplitude-squared squeezing as we have discussed before (Section 4). Similarly, one can find the squeezing condition for Y.

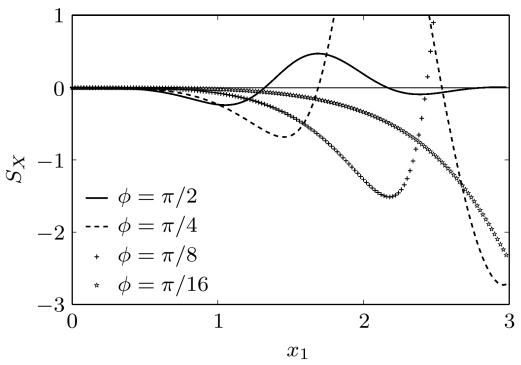


Fig.5. The amplitude-squared squeezing for the superposition state of two pseudoclassical states $|\psi_4\rangle$ in operator X. It is evident that as the phase difference decreases, the amplitude-squared squeezing increases.

6. Conclusion

In the present paper the amplitude-squared squeezing has been investigated for the quantum mechanical superposition of two coherent states with their phase differences of, separately, $\pi/2$, $3\pi/2$, and π, as well as the superposition state of two pseudoclassical states. It has been shown that all the states do exhibit the amplitude-squared squeezing behaviours except the superposition state of two coherent states with a phase difference of π. In particular, we have obtained the following results: (1) even stronger effects of the amplitude-squared squeezing are found in some specific states where the relative phase of the superposition is equal to the average photon number; (2) when the value of relative phase increases, the amplitude-squared squeezing also increases; (3) amplitude-squared squeezing does not occur in the case of famous Schrödinger cat states, namely, the even and the odd coherent states and the Yurke–Stoler states. We hope that such states, in particular, specific states which exhibit even stronger nonclassical features, will find their applications in some future experiments because of the vital role of amplitude-squared squeezed states in nonlinear optics.

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