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Non-geometrical effects on Gaussian beams transmitting through a thin dielectric slab*

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It is shown that a Gaussian light beam transmitting through a planar thin dielectric slab in the air undergoes four different effects, i.e. lateral Goos–Hänchen-like (GHL) displacement, angular deflection, width modification and longitudinal focal shift as compared with the results predicted by geometrical optics. According to the Taylor expansion of the exponent of transmission coefficient when expressed as an exponential form, the lateral GHL displacement and the angular deflection are the first-order effects and can be negative or positive. The width modification and the longitudinal focal shift are the second-order effects and can also be positive or negative. Owing to the waist-width dependent term, the non-geometrical effects of transmitted beam are not identical with the non-specular effects of reflected beam. The conditions for the validity of those effects are suggested and numerical simulations are also given.

Keywords: non-geometrical effects, finite-sized light beam, beam transmission
PACC: 4225B, 4225F, 4282

1. Introduction

A large number of potential applications in linear and nonlinear optics have triggered an extensive research on novel effects of light beams propagation in micro-structure, whose dimension is about on the scale of the wavelength.[1−6] Besides the well-known Goos–Hänchen effect,[7,8] many papers are devoted to the non-specular effects such as angular deflection, focal shift and waist-width modification of beam reflection.[9−11] These effects are inherent to a finite size of the incident beam, and they are particularly pronounced in structures based on partial reflection,[10−13] frustrated total-internal-reflection (FTIR),[14,15] and attenuated[16] total-internal-reflection (ATIR) phenomena.

Although the novel effects of beam reflection have been widely studied, the characteristics of the beam transmission have not attracted much attention probably because they may be obtained in an way similar to that in the case of reflection. However, the beam transmission in micro-structures can also exhibit novel effects and in many circumstances they are different from those for the beam reflection.[17] It was found by Hsue and Tamir[12] that the transmitted light beam through a layered structure underwent a lateral displacement from the position by geometrical optics. Recently, we have reported that the transmission displacement can be zero, backward[13,18] as well as forward.[12] Because the lateral displacement of transmitted beam in this case has nothing to do with the evanescent wave as in the case of FTIR or TIR, we term it Goos–Hänchen-like (GHL) displacement, which results from the boundary effect of the structure on bounded light beams.[19]

The aim of this paper is to investigate the transmission properties of a Gaussian beam transmitting through a thin dielectric slab surrounded by the air. We will show that apart from the GHL displacement, the transmitted beam undergoes other effects that are also different from the prediction by geometrical optics. Much the same as the non-specular effects for the reflected beam, those effects include angular deflection, width modification, and longitudinal focal shift and are thus referred, together with the GHL displacement, to as non-geometrical effects. Particu-

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larly, we find that the GHL displacement, the angular deflection and the width modification of the transmitted beam acquire a waist-width dependent term considering the second-order modification. Owing to this term, those non-geometrical effects are no longer identical with the non-specular effects of the reflected beam.

For validity, it should be pointed out that the layered structure considered here is a 'thin' slab with a thickness much smaller than the beam width. According to the principle of geometrical optics (Snell's law of refraction), when a Gaussian beam of light impinges from the left onto a plane parallel slab of optically denser medium with a refractive index n in the air, the emerging beam from the slab will suffer a lateral displacement,[20] $Y_{\rm g} = a\tan\theta_0'$, along the surface from the position at which the beam is incident on the left surface, where a is the thickness of the slab, θ_0' is the refraction angle of the beam inside the slab which is determined by Snell's law, $\sin\theta_0 = n\sin\theta_0'$, where θ_0 is the incidence angle of the beam. This displacement is always positive for slabs of positively refractive normal media. On the other hand, from the point of view of wave mechanics, the transmitted wave consists of successively transmitted (approximately Gaussian) constituents that arise from multiple reflections of the beam inside the slab as is indicated in Fig.1. For a Gaussian beam with a waist width $2w_0$ and a slab satisfying $2a\tan\theta_0'\cos\theta_0 \gg 2w_0$, the successive constituents are distinguishably separated from one another with descending amplitudes. If $a\tan\theta_0'\cos\theta_0 \approx w_0$, these constituents tend to coalesce, thus producing a net transmitted beam whose shape may generally be quite different from a Gaussian one. If

$$a\tan\theta_0'\cos\theta_0 \ll w_0, \tag{1}$$

the transmitted beam is nearly Gaussian. What we are concerned about in this paper is the last case. The restriction, expression (1), can also be derived when looking into the non-geometrical effects in context, as will be discussed later in Section 6.

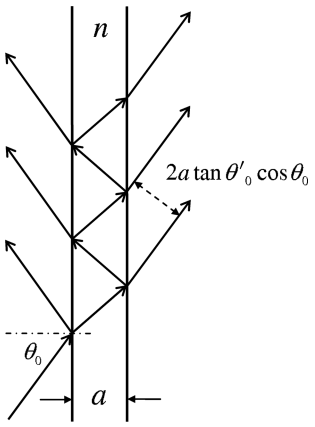


Fig.1. Successive constituents of the transmitted beam due to multiple reflections inside the slab.

2. Properties of transmitted beam predicted by geometrical optics

Consider a Gaussian light beam with a vacuum wavelength λ, which is incident at an angle of θ_0 upon a nonmagnetic dielectric slab in the air as shown in Fig.2. Denote by a, ε, and n the thickness, dielectric constant, and refractive index of the slab extending from 0 to a, respectively. For simplicity, the incident beam is assumed to be two-dimensional, that is, its field function is assumed to be invariant along the z-axis, $\partial/\partial z = 0$.

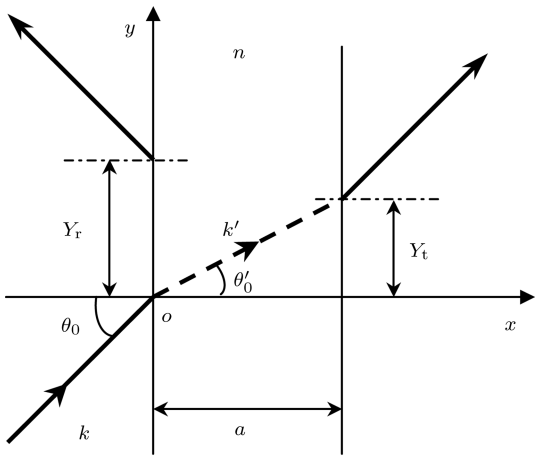


Fig.2. Schematic diagram of a light beam propagating through an optically denser dielectric slab in the air, where $Y_{\rm t}$ denotes the GHL displacement of transmitted beam, and $Y_{\rm r}$ the GHL displacement of reflected beam.

For a TE-polarized (or TM-polarized) Gaussian incident beam whose waist is located at $x = 0$, its electric (or magnetic) field can be expressed as $\Psi_{\rm in}(x) = \dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{-k}^{k} A(k_y)\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x})\mathrm{d}k_y$, where time dependence $\exp(-\mathrm{i}\omega t)$ is implied and suppressed, $\boldsymbol{k} =$

$(k_x, k_y) = (k\cos\theta, k\sin\theta)$, with $k = 2\pi/\lambda$, θ denoting the incidence angle of a plane wave component, and

$$A(k_y) = w_y \exp\left[-\frac{w_y^2}{2}(k_y - k_{y0})^2\right] \tag{2}$$

is the angular spectral distribution[21] that is sharply peaked around $k_{y0} = k\sin\theta_0$, with $w_y = w_0/\cos\theta_0$, and w_0 being half the waist width of the beam. The waist width of the incident beam is assumed to be large enough ($kw_0 \gg 1$), i.e. the divergence angle, $\Delta\theta = 1/(kw_0)$, is small enough. Also, the incidence angle of the beam θ_0 is assumed to be far enough away from $\pi/2$ so that the following condition is satisfied:

$$w_y(k - k_{y0}) > 1, \tag{3}$$

thus the field function of the incident beam can be rewritten as

$$\Psi_{\rm in}(\boldsymbol{x}) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} A(k_y)\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x})\mathrm{d}k_y. \tag{4}$$

Substituting expression (2) into expression (4), we have a field function of the incident Gaussian beam,

$$\Psi_{\rm in}(\boldsymbol{x}) = w_0\left(\frac{k}{\mathrm{i}q}\right)^{1/2}\exp\left[\frac{\mathrm{i}k\cos^2\theta_0}{2q}(y - x\tan\theta_0)^2\right] \times\exp[\mathrm{i}(k_{x0}x + k_{y0}y)],$$

where $k_{x0} = k\cos\theta_0$, and

$$q = \frac{x}{\cos\theta_0} - \mathrm{i}kw_0^2$$

is the complex parameter. For one plane wave component,

$$\psi_{\rm in}(\boldsymbol{x}, k_y) = A(k_y)\exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}), \tag{5}$$

the refracted field by the first surface is $t_1A(k_y)\exp[\mathrm{i}(k'_x x + k_y y)]$, where t_1 is the Fresnel transmission coefficient of the surface, $k'_x = k'\cos\theta'$, where $k' = nk$, and θ' is determined by Snell's law, $\sin\theta = n\sin\theta'$. When this field passes through the slab and is refracted by the second surface, the emerging field from the slab becomes $t_1t_2A(k_y)\exp\{\mathrm{i}[k_x(x-a) + k_y y + k'_x a]\}$, with t_2 being the Fresnel transmission coefficient of the second surface. Then the field of the transmitted beam predicted by geometrical optics would be

$$\Psi_{\rm t}^{\rm g}(\boldsymbol{x}) = \frac{t_1t_2}{\sqrt{2\pi}}\int_{-\infty}^{\infty} A(k_y)\exp\{\mathrm{i}[k_x(x-a) + k_y y + k'_x a]\}\mathrm{d}k_y,$$

where t_1 and t_2 are assumed to be constants with respect to k_y by the nature of geometrical optics and can be worked out from the integral. Under the following paraxial conditions:

$$k_x \approx k_{x0} - (k_y - k_{y0})\tan\theta_0 - \frac{(k_y - k_{y0})^2}{2k\cos^3\theta_0} \tag{6}$$

and

$$k'_x \approx k'_{x0} - (k_y - k_{y0})\tan\theta'_0 - \frac{(k_y - k_{y0})^2}{2k\cos^3\theta'_0},$$

we have

$$\Psi_{\rm t}^{\rm g}(\boldsymbol{x}) = t_1t_2w_0\exp\{\mathrm{i}[k_{x0}(x-a) + k_{y0}y]\} \left(\frac{k}{\mathrm{i}q_{\rm t}^{\rm g}}\right)^{1/2}\exp\left\{\frac{\mathrm{i}k\cos^2\theta_0}{2q_{\rm t}^{\rm g}}[y - a\tan\theta'_0 - (x-a)\tan\theta_0]^2\right\}, \tag{7}$$

where

$$q_{\rm t}^{\rm g} = \frac{x-a}{\cos\theta_0} + \frac{ka\cos^2\theta_0}{k'\cos^3\theta'_0} - \mathrm{i}kw_0^2, \tag{8}$$

$k'_{x0} = k'\cos\theta'_0$, and θ'_0 is determined by $\sin\theta_0 = n\sin\theta'_0$. This would be what we expect from the prediction by geometrical optics for the field of transmitted beam. Obviously, this beam has the same Gaussian shape as the incident beam. It propagates with a wave vector (k_{x0}, k_{y0}), undergoes a lateral displacement of $Y_{\rm g} = a\tan\theta'_0$ from the point ($x = a$, $y = 0$) along the second surface, and has a waist of width $2w_0$. Expression (8) shows that the waist of this beam is located at $X_{\rm g}^{\rm w} = a\left(1 - \dfrac{k\cos^3\theta_0}{k'\cos^3\theta'_0}\right)$ to the left of the second surface.

The wave function of actually transmitted field which is corresponding to plane wave component (5) is

$$\psi_{\rm t}(\boldsymbol{x}, k_y) = T(k_y)A(k_y)\exp\{\mathrm{i}[k_x(x-a) + k_y y]\}, \tag{9}$$

where $T(k_y) = |T|\mathrm{e}^{\mathrm{i}\varphi_{\rm t}}$ is the amplitude transmission coefficient. From Maxwell's equations and boundary conditions, it is not difficult to obtain, for TE polarization,

$$\varphi_{\rm t}(k_y) = \mathrm{int}\left(\frac{k'_x a}{\pi} + \frac{1}{2}\right)\pi + \tan^{-1}\left[\frac{1}{2}\left(\frac{k'_x}{k_x} + \frac{k_x}{k'_x}\right)\tan(k'_x a)\right] \tag{10}$$

and

$$|T(k_y)|^2 = \frac{4k_x^2k_x'^2/k_0^4}{4k_x^2k_x'^2/k_0^4 + \sin^2(k'_x a)}, \tag{11}$$

where $k_0^2 = k_x'^2 - k_x^2 = k'^2 - k^2$ and int$(\cdot)$ represents the integer part of involved number. Similar results can also be obtained for TM polarization. Since the wave inside the slab is a propagating wave, the transmission coefficient $T(k_y)$ results from multiple reflections of the wave inside the slab. With the transmitted component (9), we are ready to calculate the field function of transmitted beam,

$$\Psi_{\rm t}(\boldsymbol{x}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} T(k_y) A(k_y) \times \exp\{{\rm i}[k_x(x-a) + k_y y]\} {\rm d}k_y. \quad (12)$$

3. First-order approximation: GHL displacement and angular deflection

3.1. Wave function of transmitted beam

Now we look at the properties of the transmitted beam in the first-order approximation of the amplitude transmission coefficient. For a slab satisfying expression (1), it is reasonable to express $T(k_y)$ as an exponential form. Expand the exponent in Taylor series at k_{y0}, and retain up to the first-order term, then we will obtain

$$T(k_y) = \exp[\ln T(k_y)] \approx T_0 \exp\left[\frac{1}{T_0}\frac{{\rm d}T}{{\rm d}k_{y0}}(k_y - k_{y0})\right], \quad (13)$$

where $T_0 = T(k_{y0})$, and ${\rm d}/{\rm d}k_{y0}$ denotes the derivative with respect to k_y evaluated at $k_y = k_{y0}$. Introduce two real parameters $L_{\rm t}'$ and $L_{\rm t}''$ defined as

$$L_{\rm t}' + {\rm i}L_{\rm t}'' = \frac{\rm i}{T_0}\frac{{\rm d}T}{{\rm d}k_{y0}},$$

then, in terms of the phase and magnitude of T, we will have

$$L_{\rm t}' = -\frac{{\rm d}\varphi_{\rm t}}{{\rm d}k_{y0}},$$

and

$$L_{\rm t}'' = \frac{\rm d}{{\rm d}k_{y0}} \ln|T|.$$

Substituting expression (13) into expression (12) and using paraxial approximation (6) yield

$$\Psi_{\rm t}'(\boldsymbol{x}) \approx T_0 w_0 \left(\frac{k}{{\rm i}q_{\rm t}'}\right)^{1/2} \exp\left\{\frac{{\rm i}k\cos^2\theta_0}{2q_{\rm t}'}\left[y - L_{\rm t}' - \left(\tan\theta_0 + \frac{\eta'}{kw_0\cos^3\theta_0}\right)(x-a)\right]^2\right\} \times \exp\left\{{\rm i}\left(k_{x0} - \frac{\eta'}{w_0}\tan\theta_0\right)(x-a) + {\rm i}\left(k_{y0} + \frac{\eta'}{w_0}\right)y\right\}, \quad (14)$$

where

$$q_{\rm t}' = \frac{x-a}{\cos\theta_0} - {\rm i}kw_0^2 \quad (15)$$

and

$$\eta' = \frac{L_{\rm t}''\cos^2\theta_0}{w_0}.$$

3.2. GHL displacement

The first exponential factor on the right-hand side of expression (14), together with the form of complex parameter (15), shows that the transmitted beam has the same Gaussian shape as the beam of expression (7) predicted by geometrical optics. But it is displaced laterally to a different place as obtained below. The locus of its peak is expressed as

$$y = L_{\rm t}' + \left(\tan\theta_0 + \frac{\eta'}{kw_0\cos^3\theta_0}\right)(x-a), \quad (16)$$

which indicates that the peak of the beam at the second surface ($x = a$) is displaced from the point ($x = a$, $y = 0$) by

$$Y_{\rm t}' = L_{\rm t}', \quad (17)$$

which is the same as the one given by the stationary-phase approach.[13,22] Such a displacement is referred to as GHL displacement, in that it is different from the prediction, $Y_{\rm g} = a\tan\theta_0'$, by geometrical optics and has nothing to do with the evanescent wave as in the case of Goos–Hänchen displacement. All these amount to a conclusion that the GHL displacement of transmitted beam does not result from the defor-

mation of the beam profile, which is contrary to the conclusion by Hsue and Tamir.[12]

For the case of TE-polarization, we have, by use of expression (10),

$$Y_{\rm t}' = \frac{|T_0|^2 k_0^4}{2k_{x0}^3 k_{x0}'} \left[\frac{k_{x0}^2(k_{x0}^2 + k_{x0}'^2)}{k_0^4} - \frac{\sin(2k_{x0}'a)}{2k_{x0}'a} \right] Y_{\rm g}. \tag{18}$$

We know that $\dfrac{\sin(2k_{x0}'a)}{2k_{x0}'a} < 1$. On the other hand, we have

$$\frac{k_{x0}^2(k_{x0}^2 + k_{x0}'^2)}{k_0^4} < 1, \tag{19}$$

as long as the angle of incidence is large enough. This means that $Y_{\rm t}'$ may be negative. Inequality (19) results in the following necessary condition[13] for $Y_{\rm t}'$ to be negative for the case of TE-polarization:

$$\cos\theta_0 < \left(\frac{n^2-1}{2}\right)^{1/2}.$$

This condition is physically suitable and negative GHL displacements of transmitted beam have been observed in microwave experiments.[18]

3.3. Deflection of propagation direction

The second exponential factor on the right-hand side of expression (14) is a propagating factor, of which the wave vector is $\left(k_{x0} - \dfrac{\eta'}{w_0}\tan\theta_0, k_{y0} + \dfrac{\eta'}{w_0}\right)$ and obviously different from (k_{x0}, k_{y0}), the prediction of geometrical optics. Denoting by $\theta_0 + \delta_{\rm t}'$ the propagation angle of the actually transmitted beam with respect to the normal of the slab, we have an angular deflection,

$$\delta_{\rm t}' = \frac{\eta'}{kw_0\cos\theta_0} = \frac{L_{\rm t}''\cos\theta_0}{kw_0^2}. \tag{20}$$

This is the same as the one obtained from the locus of the peak of transmitted beam,

$$\begin{aligned} y &= L_{\rm t}' + \left(\tan\theta_0 + \frac{\delta_{\rm t}'}{\cos^2\theta_0}\right)(x-a) \\ &\approx L_{\rm t}' + (x-a)\tan(\theta_0 + \delta_{\rm t}'). \end{aligned}$$

According to expression (20), the angular deflection of the transmitted beam, obtained from the prediction of geometrical optics, is closely dependent on the width of the incident beam. The larger the width is, the smaller the angular deflection will be.

For the case of TE polarization, we have, by use of expression (11),

$$L_{\rm t}'' = \frac{|T_0|^2 k_0^4 k_{y0} a^2}{4k_{x0}^2 k_{x0}'^2} \left[\frac{\sin(2k_{x0}'a)}{2k_{x0}'a} - \left(1 + \frac{k_{x0}'^2}{k_{x0}^2}\right)\left(\frac{\sin(k_{x0}'a)}{k_{x0}'a}\right)^2 \right],$$

which can be rewritten as

$$L_{\rm t}'' = -\frac{|T_0|^2 k_0^4 k_{y0}}{4k_{x0}^2 k_{x0}'^4} G_0 \sin(k_{x0}'a)\sin(k_{x0}'a - \alpha), \tag{21}$$

where

$$G_0 = [(1 + k_{x0}'^2/k_{x0}^2)^2 + k_{x0}'^2 a^2]^{1/2},$$
$$\sin\alpha = \frac{k_{x0}'a}{G_0},$$

and

$$\cos\alpha = \frac{1 + k_{x0}'^2/k_{x0}^2}{G_0}.$$

Expression (21) for $L_{\rm t}''$ indicates that the angular deflection of the transmitted beam can be positive or negative. It also unfolds that when $k_{x0}'a = m\pi$ or $k_{x0}'a = m\pi + \alpha$ $(m = 0, 1, 2, \cdots)$, the angular deflection vanishes. That is to say, the conditions for the angular deflection to vanish are independent of the width of the incident beam, though its magnitude depends closely on the width.

4. Second-order approximation: width modification and longitudinal focal shift

4.1. Wave function of transmitted beam

When the exponent of the transmission coefficient is approximated to the second-order term, we have

$$T(k_y) \approx T_0 \exp\left[\frac{1}{T_0}\frac{{\rm d}T}{{\rm d}k_{y0}}(k_y - k_{y0}) + \frac{1}{2}\frac{\rm d}{{\rm d}k_{y0}}\left(\frac{1}{T}\frac{{\rm d}T}{{\rm d}k_y}\right)(k_y - k_{y0})^2\right]. \tag{22}$$

Introduce two new real parameters $F_{\rm t}'$ and $F_{\rm t}''$ defined as

$$F_{\rm t}' + {\rm i}F_{\rm t}'' = -{\rm i}\frac{\rm d}{{\rm d}k_{y0}}\left(\frac{1}{T}\frac{{\rm d}T}{{\rm d}k_y}\right),$$

then, with the phase and magnitude of T, we will have

$$F_{\rm t}' = \frac{{\rm d}^2\varphi_{\rm t}}{{\rm d}k_{y0}^2},$$

and

$$F_{\mathrm{t}}'' = -\frac{\mathrm{d}^2}{\mathrm{d}k_{y0}^2} \ln |T|.$$

Substituting expression (22) into expression (12) and using paraxial condition (6), and omitting some unimportant factors, we finally obtain the following expression:

$$\Psi_{\mathrm{t}}(\boldsymbol{x}) \approx T_0 w_0 \left(\frac{k}{\mathrm{i}q_{\mathrm{t}}}\right)^{1/2} \exp\left\{\mathrm{i}\left(k_{x0} - \frac{\eta_{\mathrm{t}}}{w_{\mathrm{t}0}} \tan\theta_0\right)(x-a) + \mathrm{i}\left(k_{y0} + \frac{\eta_{\mathrm{t}}}{w_{\mathrm{t}0}}\right) y\right\}$$
$$\times \exp\left\{\frac{\mathrm{i}k\cos^2\theta_0}{2q_{\mathrm{t}}}\left[y - L_{\mathrm{t}}' + \frac{\eta_{\mathrm{t}} F_{\mathrm{t}}'}{w_{\mathrm{t}0}} - \left(\tan\theta_0 + \frac{\eta_{\mathrm{t}}}{k w_{\mathrm{t}0}\cos^3\theta_0}\right)(x-a)\right]^2\right\}, \tag{23}$$

where

$$q_{\mathrm{t}} = \frac{x-a}{\cos\theta_0} - kF_{\mathrm{t}}'\cos^2\theta_0 - \mathrm{i}kw_{\mathrm{t}0}^2, \tag{24}$$

$$w_{\mathrm{t}0} = (w_0^2 + F_{\mathrm{t}}''\cos^2\theta_0)^{1/2},$$

and

$$\eta_{\mathrm{t}} = \frac{L_{\mathrm{t}}''\cos^2\theta_0}{w_{\mathrm{t}0}}.$$

The second exponential factor on the right-hand side of expression (23) reveals the Gaussian profile of the transmitted beam, but showing different properties as compared with expression (7), the prediction of geometrical optics.

4.2. Modification of beam width

Quantity $w_{\mathrm{t}0}$ in complex parameter (24) determines the half width of the transmitted beam, which is different from the half width, w_0, of the beam of expression (7) predicted by geometrical optics. We may introduce a modification term, μ_{t}, for the width in such a way $w_{\mathrm{t}0}^2 = w_0^2(1+\mu_{\mathrm{t}})$ that

$$\mu_{\mathrm{t}} = \frac{F_{\mathrm{t}}''\cos^2\theta_0}{w_0^2},$$

where F_{t}'' is given by, for the case of TE polarization,

$$F_{\mathrm{t}}'' = -\left(\frac{1}{k_{y0}^2} + 2\frac{|T_0|^2}{k_{x0}^2} + 2\frac{1+|T_0|^2}{k_{x0}'^2} + \frac{|T_0|^2 k_0^4 a^2}{2k_{x0}^2 k_{x0}'^2}\frac{\sin(2k_{x0}'a)}{2k_{x0}'a}\right) k_{y0} L_{\mathrm{t}}''$$
$$+\frac{|T_0|^2 k_0^4 k_{y0}^2 a^2}{4k_{x0}^4 k_{x0}'^2}\left[\frac{k_{x0}^2}{k_{x0}'^2}\cos(2k_{x0}'a) + 2\frac{k_0^2}{k_{x0}^2}\left(\frac{\sin(k_{x0}'a)}{k_{x0}'a}\right)^2 - \left(2 + \frac{k_{x0}^2}{k_{x0}'^2}\right)\frac{\sin(2k_{x0}'a)}{2k_{x0}'a}\right]. \tag{25}$$

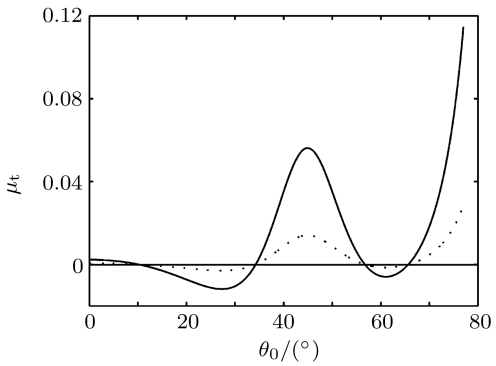


Fig.3. Dependence of μ_{t} on the angle of incidence for the case of TE-polarization, where $\lambda = 32.8\,\mathrm{mm}$, and $a = 2\lambda$, $n = 1.637$, with the solid curve being for $w_0 = 2.4\lambda$, and the dotted curve for $w_0 = 4.8\lambda$.

It is clear that this modification term depends on the width of the incident beam. The waist of the transmitted beam can be reduced as well as expanded, depending on whether F_{t}'' is negative or positive, as shown in Fig.3 for TE-polarization, where $\lambda = 32.8\,\mathrm{mm}$, $a = 2\lambda$, and $n = 1.637$, with the solid curve being for $w_0 = 2.4\lambda$, and the dotted curve for $w_0 = 4.8\lambda$.

4.3. Longitudinal focal shift

Complex parameter (24) shows that the waist of the transmitted beam is located at $X^{\mathrm{w}} = a + kF_{\mathrm{t}}'\cos^3\theta_0$, where F_{t}' for the case of TE polarization

is given by

$$F'_{\rm t} = -\left[\frac{1}{k_{y0}^2} + (1+2|T_0|^2)\left(\frac{1}{k_{x0}^2}+\frac{1}{k_{x0}'^2}\right) + \frac{|T_0|^2 k_0^4 a^2}{2k_{x0}^2 k_{x0}'^2}\frac{\sin(2k'_{x0}a)}{2k'_{x0}a}\right] k_{y0}L'_{\rm t} + \frac{|T_0|^2 k_{y0}^2 a}{2k_{x0}k_{x0}'^2}\left(7+2\frac{k_{x0}'^2}{k_{x0}^2}+\frac{k_{x0}^2}{k_{x0}'^2} - \frac{k_0^4}{k_{x0}^2 k_{x0}'^2}\cos(2k'_{x0}a)\right). \quad (26)$$

Usually we have $F'_{\rm t} \neq -\dfrac{a}{k'\cos^3\theta'_0}$, which means that the actually transmitted beam undergoes a longitudinal focal shift along its propagation direction from the position, $X_{\rm g}^{\rm w} = a\left(1-\dfrac{k\cos^3\theta_0}{k'\cos^3\theta'_0}\right)$, predicted by geometrical optics. The mapping of the focal shift onto the x-axis is $s_x = X^{\rm w} - X_{\rm g}^{\rm w} = k\left(F'_{\rm t}+\dfrac{a}{k'\cos^3\theta'_0}\right)\cos^3\theta_0$. So the longitudinal focal shift itself is

$$s_{\rm t} = \frac{s_x}{\cos\theta_0} = k\left(F'_{\rm t}+\frac{a}{k'\cos^3\theta'_0}\right)\cos^2\theta_0.$$

It is noted that this shift does not depend on the width of the incident beam. Shown in Fig.4 is the dependence of $s_{\rm t}$ on the thickness of the slab for the case of TE-polarization, where $\lambda = 32.8\,$mm, $n = 1.637$, and $\theta_0 = 75°$. It is clear that the focal shift can be either positive or negative.

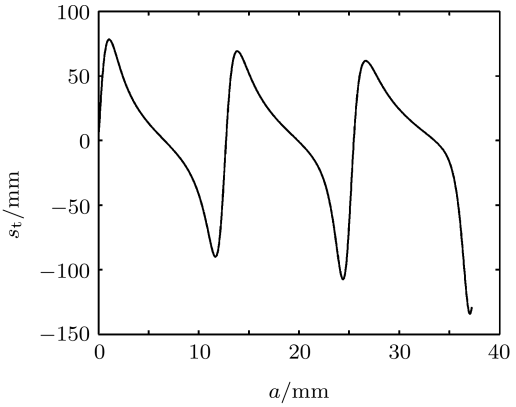


Fig.4. Dependence of the longitudinal focal shift on the thickness of the slab, where the beam is in TE-polarization, $\lambda = 32.8\,$mm, $n = 1.637$, and $\theta_0 = 75°$.

4.4. Correction to the angular deflection

The first exponential factor on the right-hand side of expression (23) shows the propagation direction of the transmitted Gaussian beam by its wave vector $\left(k_{x0}-\dfrac{\eta_{\rm t}}{w_{\rm t0}}\tan\theta_0, k_{y0}+\dfrac{\eta_{\rm t}}{w_{\rm t0}}\right)$, which is different from (k_{x0}, k_{y0}), the prediction by geometrical optics. Denoting by $\theta_0+\delta_{\rm t}$ the angle of the actual transmitted beam with respect to the normal of the slab, we have an angular deflection expressed as

$$\delta_{\rm t} = \frac{\eta_{\rm t}}{kw_{\rm t0}\cos\theta_0} = \frac{L''_{\rm t}\cos\theta_0}{kw_{\rm t0}^2}. \quad (27)$$

This is the same as the one obtained from the locus of the peak of the transmitted beam,

$$y = L'_{\rm t} - \frac{\eta_{\rm t}F'_{\rm t}}{w_{\rm t0}} + \left(\tan\theta_0 + \frac{\delta_{\rm t}}{\cos^2\theta_0}\right)(x-a) \approx L'_{\rm t} - \frac{\eta_{\rm t}F'_{\rm t}}{w_{\rm t0}} + (x-a)\tan(\theta_0+\delta_{\rm t}). \quad (28)$$

Owing to the width modification, the angular deflection here is corrected from the value given by expression (20) in the first-order approximation. Shown in Fig.5 is the dependence of the angular deflection on the incidence angle for TE-polarization, where $\lambda = 32.8\,$mm, $n = 1.637$, $a = 2.8\,$mm, and $w_0 = 2.4\lambda$. For comparison, the case of $w_0 = 4.8\lambda$ is also shown by the dotted curve. This angular deflection has been taken into account in microwave experimental measurements.[18]

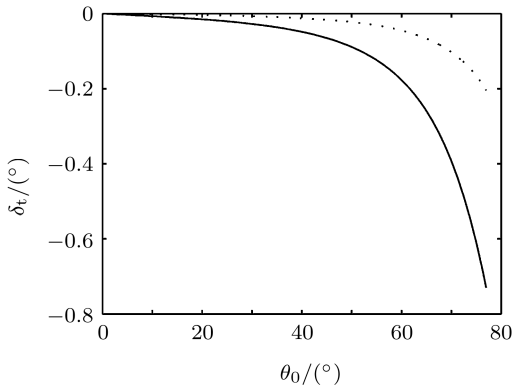


Fig.5. Dependence of $\delta_{\rm t}$ on the incidence angle for the case of TE-polarization, where $\lambda = 32.8\,$mm, $n = 1.637$, and $a = 2.8\,$mm. The solid curve for $w_0 = 2.4\lambda$, and the dotted curve for $w_0 = 4.8\lambda$.

4.5. Correction to the GHL displacement

The locus expression of beam peak is no longer expression (16). Instead, it is represented by expression (28), in which the GHL displacement of the transmitted beam at the second surface ($x = a$) is given as,

$$Y_{\rm t} = L'_{\rm t} - \frac{\eta_{\rm t}F'_{\rm t}}{w_{\rm t0}} = L'_{\rm t} - kF'_{\rm t}\delta_{\rm t}\cos\theta_0. \quad (29)$$

The second term on the right-hand side of the above expression is a second-order correction and depends

on the width of the incident beam. This term implies that the GHL displacement in the second-order approximation is dependent on the width of the incident beam.

5. Effects of transmitted beam are not identical with that of reflected beam

According to the stationary-phase approach, the reflected beam from the slab also undergoes a lateral displacement that is equal to that of the transmitted beam.[13,22] Since the waist-width dependent term of the transmission displacement is closely related to the transmission coefficient $T(k_y)$, it is expected that the reflection displacement also involves a waist-width dependent term that should be related to the reflection coefficient. As a result, the reflection displacement will not be equal to the transmission one.

To show this, we write down the corresponding reflected field in terms of reflection coefficient $R(k_y) = |R(k_y)| \exp(\mathrm{i}\varphi_\mathrm{r})$,

$$\Psi_\mathrm{r}(\boldsymbol{x}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} R(k_y) A(k_y) \times \exp(-\mathrm{i}k_x x + \mathrm{i}k_y y) \mathrm{d}k_y. \tag{30}$$

Expressing $R(k_y)$ in an exponential form, expanding its exponent in Taylor series at k_{y0}, and retaining terms up to $(k_y - k_{y0})^2$, we have

$$R(k_y) \approx R_0 \exp\left[\frac{1}{R_0}\frac{\mathrm{d}R}{\mathrm{d}k_{y0}}(k_y - k_{y0}) + \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}k_{y0}}\left(\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}k_y}\right)(k_y - k_{y0})^2\right], \tag{31}$$

where $R_0 = R(k_{y0})$. Using this expansion and the second-order paraxial approximation (6), carrying out the integration in expression (30) and omitting some unimportant factors, we obtain the field function of reflected beam as follows:

$$\Psi_\mathrm{r}(\boldsymbol{x}) \approx R_0 w_0 \left(\frac{k}{\mathrm{i}q_\mathrm{r}}\right)^{1/2} \exp\left\{\frac{\mathrm{i}k\cos^2\theta_0}{2q_\mathrm{r}}\left[y - L'_\mathrm{r} + \frac{\eta_\mathrm{r} F'_\mathrm{r}}{w_{\mathrm{r}0}} + \left(\tan\theta_0 + \frac{\eta_\mathrm{r}}{k w_{\mathrm{r}0}\cos^3\theta_0}\right)x\right]^2\right\} \times \exp\left\{-\mathrm{i}\left(k_{x0} - \frac{\eta_\mathrm{r}}{w_{\mathrm{r}0}}\tan\theta_0\right)x + \mathrm{i}\left(k_{y0} + \frac{\eta_\mathrm{r}}{w_{\mathrm{r}0}}\right)y\right\}, \tag{32}$$

where

$$q_\mathrm{r} = -\frac{x}{\cos\theta_0} - kF'_\mathrm{r}\cos^2\theta_0 - \mathrm{i}kw_{\mathrm{r}0}^2 \tag{33}$$

is the complex parameter of reflected beam,

$$w_{\mathrm{r}0} = (w_0^2 + F''_\mathrm{r}\cos^2\theta_0)^{1/2} \tag{34}$$

is the half width of the beam, $\eta_\mathrm{r} = \dfrac{L''_\mathrm{r}\cos^2\theta_0}{w_{\mathrm{r}0}}$, and L'_r, L''_r, F'_r, and F''_r are defined as

$$L_\mathrm{r} = L'_\mathrm{r} + \mathrm{i}L''_\mathrm{r} = \frac{\mathrm{i}}{R_0}\frac{\mathrm{d}R}{\mathrm{d}k_{y0}}, \tag{35}$$

$$F_\mathrm{r} = F'_\mathrm{r} + \mathrm{i}F''_\mathrm{r} = -\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}k_{y0}}\left(\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}k_y}\right). \tag{36}$$

It is clear from this expression that the GHL displacement of the reflected beam is $Y_\mathrm{r} = L'_\mathrm{r} - \dfrac{\eta_\mathrm{r}F'_\mathrm{r}}{w_{\mathrm{r}0}} = L'_\mathrm{r} - \dfrac{L''_\mathrm{r}F'_\mathrm{r}\cos^2\theta_0}{w_{\mathrm{r}0}^2}$. The first term on the right-hand side of the above expression is the result of stationary-phase approach which is independent of the waist width of the incident beam. The second term is waist-width dependent, $\Delta Y_\mathrm{r} = -\dfrac{L''_\mathrm{r}F'_\mathrm{r}\cos^2\theta_0}{w_{\mathrm{r}0}^2}$. Since the phase of the reflection coefficient satisfies $\tan\varphi_\mathrm{r} = \tan\varphi_\mathrm{t}$, we have $L'_\mathrm{r} = L'_\mathrm{t}$ and $F'_\mathrm{r} = F'_\mathrm{t}$. The former equation is just what the stationary-phase approach predicts. In addition, from $|R|^2 + |T|^2 = 1$, we obtain

$$L''_\mathrm{r} = -\frac{|T_0|^2}{|R_0|^2}L''_\mathrm{t} \tag{37}$$

and

$$F''_\mathrm{r} = -\frac{|T_0|^2}{|R_0|^2}F''_\mathrm{t} + 2\frac{|T_0|^2}{|R_0|^4}L''^2_\mathrm{t}.$$

Expression (37) means that ΔY_r and ΔY_t are different from each other and have opposite signs. In other words, the reflection displacement is no longer equal to the transmission one when the second-order approximation is taken into account. Shown in Fig.6 is a com-

parison of the reflection displacement with the transmission one for the case of TE-polarization, where the half waist width of the incident beam is chosen to be $w_0 = 2.4\lambda_0$, $\lambda_0 = 32.8\,\mathrm{mm}$, $n = 1.637$, and $\theta_0 = 75°$.

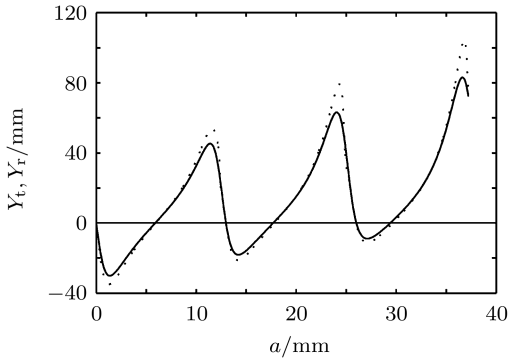


Fig.6. Difference between the reflection and the transmission displacements for the case of TE-polarization, where $w_0 = 2.4\lambda_0$, $\lambda_0 = 32.8\,\mathrm{mm}$, $n = 1.637$, $\theta_0 = 75°$, Y_r is shown by the dotted curve, and Y_t is indicated by the solid curve.

Expression (32) also shows an angular deflection of the reflected beam from the propagation direction predicted by geometric reflection. Letting the reflection angle be $\theta_0 + \delta_r$, we find from expression (32) that

$$\delta_r = \frac{\eta_r}{kw_{r0}\cos\theta_0} = \frac{L_r''\cos\theta_0}{kw_{r0}^2}.$$

Comparing this equation with expression (27) and noting expression (37), we conclude that the angular deflection of the reflected beam is opposite to that of the transmitted beam. Expression (34) shows that the waist width of the reflected beam is also modified from the value given by geometric optics. If we introduce in a similar way a modification term, μ_r, for the waist width of the reflected beam as follows:

$$w_{r0} = w_0(1+\mu_r)^{1/2},$$

we have

$$\mu_r = \frac{F_r''\cos^2\theta_0}{w_0^2}.$$

Since $F_r'' \neq F_t''$, the modification to the reflected beam waist is different from that to the transmitted beam waist. Expression (33) shows that the location of reflected beam waist is shifted by $-kF_r'\cos^2\theta_0$ along its propagation direction from the position predicted by geometric reflection.

6. Discussion

For the analysis given here to be hold true, we have presented the restrictive conditions on the thickness of the slab and the incident beam. Condition (1), i.e. expression (1), means that the slab should be, in general, much thinner than w_0. Condition (3), i.e. expression (3), requires that the divergence angle of the incident beam should be small enough and satisfy the following inequality:

$$\Delta\theta < \frac{1-\sin\theta_0}{\cos\theta_0} = \frac{\cos(\theta_0/2)-\sin(\theta_0/2)}{\cos(\theta_0/2)+\sin(\theta_0/2)} \equiv \sigma(\theta_0) \tag{38}$$

as shown in Fig.7 by the dependence of σ on θ_0.

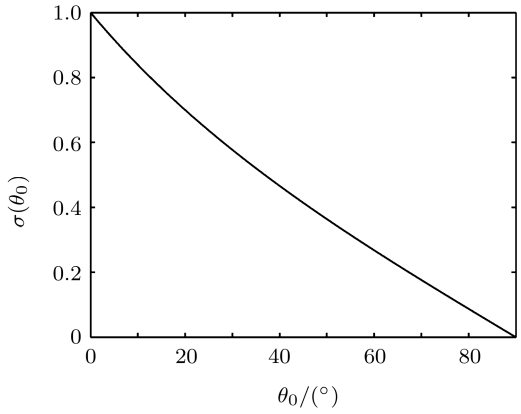


Fig.7. Dependence of σ on incidence angle θ_0.

In particular, the non-geometrical effects obtained above suggest that condition (1) can also be derived from the analytical considerations for the validity of those effects. Obviously, the expressions for the magnitude of the transmission coefficient, $|T(k_{y0})|$, and all the four parameters L_t', L_t'', F_t', and F_t'' contain periodical functions of $k_{x0}'a$ with a period of π as seen from expression (11), (18), (21), (25) and (26). For the case of TE-polarization, those periodical functions produce the oscillations in all the transmission effects that are shown in Fig.5 of Ref.[18] and Figs.3 and 4. Owing to the existence of the divergence angle $\Delta\theta$, the variance of $k_{x0}'a$ caused by $\Delta\theta$ is required to be much smaller than the period, π, of the above-mentioned periodical functions, thus we arrive at the following restriction on the thickness of the slab:[23]

$$a \ll \frac{2\pi w_0(n^2-\sin^2\theta_0)^{1/2}}{\sin(2\theta_0)}, \tag{39}$$

which is equivalent to condition (1).

For experimental parameters $n = 1.637$ and $\theta_0 = 75°$ that were used in Ref.[18], condition (1) requires $a \ll 5.3w_0$, and condition (38), i.e. inequality (38), requires $\Delta\theta < 0.132$. The microwave beam and the thickness of the slabs that were used in Ref.[18] do

satisfy those conditions. The divergence angle of the incident microwave beam is $\Delta\theta = 0.066$ ($w_0 = 2.4\lambda$). All the slabs that were used are much thinner than w_0. Numerical simulations have shown that for a Gaussian incident beam, the transmitted beam remains Gaussian under the experimental conditions, as shown in Fig.8, which displays the normalized profile of transmitted beam at the second surface ($x = a$) for TE polarization and the following parameters: $\lambda = 32.8\,\mathrm{mm}$, $n = 1.637$, $a = 2.8\,\mathrm{mm}$, $\theta_0 = 75°$, and $w_0 = 2.4\lambda$.

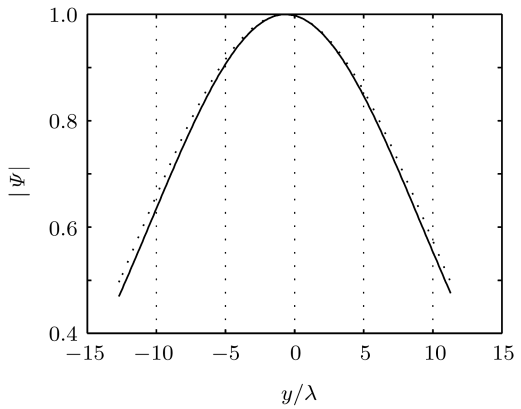


Fig.8. Normalized profile of transmitted beam for TE polarization at the second surface ($x = a$), where $\lambda = 32.8\,\mathrm{mm}$, $n = 1.637$, $a = 2.8\,\mathrm{mm}$, $\theta_0 = 75°$, and $w_0 = 2.4\lambda$. The solid curve represents the Gaussian beam (23), and the dotted curve denotes the result of numerical calculation.

The solid curve represents the Gaussian beam of expression (23), and the dotted curve denotes the result of numerical calculation, which is given by the following integration:

$$\Psi_{\mathrm{t}}^{n}(\boldsymbol{x}) = \frac{1}{\sqrt{2\pi}} \int_{-k}^{k} T(k_y) A(k_y) \times \exp\{\mathrm{i}[k_x(x-a) + k_y y]\} \mathrm{d}k_y.$$

In summary, we have investigated in detail the non-geometrical effects on Gaussian beams transmitting through a dielectric slab. According to expression (22), the lateral GHL displacement and the angular deflection are the first-order effects (though both of them contain the second-order corrections), and the width modification and longitudinal focal shift are the second-order effects. The lateral GHL displacement can be negative or positive and does not result from the deformation of the transmitted beam, which is contrary to the conclusion by Hsue and Tamir.[12] The angular deflection from the propagation direction predicted by geometrical optics depends on the width of incident beam and can be negative or positive. Owing to the width modification, the width of the transmitted beam can be reduced as well as expanded. The longitudinal focal shift of transmitted beam waist from the position predicted by geometrical optics can also be negative or positive and is independent of the width of incident beam. Owing to a waist width dependent term, the GHL displacement, the angular deflection and the width modification of the transmitted beam are not identical with those of the reflected beam. All the four effects hold true as long as conditions (1) and (38) are satisfied. Examples for TE-polarized beams are given. The analytical method used here can be applied to other planar micro-structures, such as the FTIR structure and the multi-layered structures.[24] Different structures and different dielectric materials result in different transmission coefficients and exhibit their own different effects on transmitted beams. Detailed considerations of these non-geometrical effects of such micro-structures will open a realm of applications in optical devices and integrated optics.

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