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Spatial correlation properties and phase singularity annihilation of Gaussian Schell-model beams in the focal region*

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By using the generalized Debye diffraction integral, this paper studies the spatial correlation properties and phase singularity annihilation of apertured Gaussian Schell-model (GSM) beams in the focal region. It is shown that the width of the spectral degree of coherence can be larger, less than or equal to the corresponding width of spectral density, which depends not only on the scalar coherence length of the beams, but also on the truncation parameter. With a gradual increase of the truncation parameter, a pair of phase singularities of the spectral degree of coherence in the focal plane approaches each other, resulting in subwavelength structures. Finally, the annihilation of pairs of phase singularities takes place at a certain value of the truncation parameter. With increasing scalar coherence length, the annihilation occurs at the larger truncation parameter. However, the creation process of phase singularities outside the focal plane is not found for GSM beams.

Keywords: singular optics, spatial correlation, annihilation of phase singularities, truncation parameter
PACC: 4225, 4225K

1. Introduction

Much interest has been exhibited in singular optics, which has been recently extended from primarily dealing with fully coherent, quasi-monochromatic wavefields[1,2] to treating fully coherent, polychromatic wavefields, and partially coherent, quasi-monochromatic wavefields.[3,4] When a partially coherent field is considered, its phase becomes random and is no longer well defined. However, the spectral degree of coherence of the field is found to possess phase singularities called coherence vortices, at which the spectral degree of coherence has zero value. The behaviour of coherence vortices and their connection with the corresponding fully coherent intensity vortices have been studied by several groups.[3−9]

The purpose of the present paper is to study the spatial correlation properties and phase singularity annihilation of apertured partially coherent, quasi-monochromatic fields in the focal region, where the Gaussian Schell-model (GSM) beam is taken as a typical example, and the scalar spatial coherent length and truncation parameter on the spatial correlation properties and phase singularities are analysed by using the generalized Debye diffraction integral.[10] Specifically, the annihilation process of phase singularities in the focal plane is studied. A comparison with the cases of Schell-model beams and fully coherent Gaussian beams is also made.

2. Focused field of GSM beams

As shown in Fig.1, assume that a quasi-monochromatic scalar, partially coherent beam is incident upon an aperture of half width a in an opaque screen and converges toward a point 0 at a distance f from the aperture. In accordance with the generalized Debye diffraction integral the cross-spectral density at a pair of points ($\boldsymbol{r}_1$, $\boldsymbol{r}_2$) in the focal region reads as[10]

$$W(\boldsymbol{r}_1,\boldsymbol{r}_2)=\left(\frac{kf}{2\pi}\right)^2\iint\limits_{\Omega}W^{(0)}(-f\boldsymbol{s}_1,-f\boldsymbol{s}_2)\times\exp\left[\mathrm{i}k(\boldsymbol{s}_2\cdot\boldsymbol{r}_2-\boldsymbol{s}_1\cdot\boldsymbol{r}_1)\right]\mathrm{d}\Omega_1\mathrm{d}\Omega_2,\quad(1)$$

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where k is the wave number related to the wavelength λ by $k{=}2\pi/\lambda$, $W^{(0)}(-f\boldsymbol{s}_1, -f\boldsymbol{s}_2)$ is the cross-spectral density of the incident beam at two points on the aperture. The unit vector $\boldsymbol{s}_i$ is expressed as $\boldsymbol{s}_i = (-\sin\theta_i\cos\phi_i,\ -\sin\theta_i\sin\phi_i,\ \cos\theta_i)$ with θ_i, ϕ_i being the azimuthal and polar angles, respectively, $\boldsymbol{r}_i{=}(x_i,\ y_i,\ z_i)$, and Ω_i is the solid angle subtended by the aperture at the geometrical focus, $i{=}1,\ 2$ (unless otherwise stated).

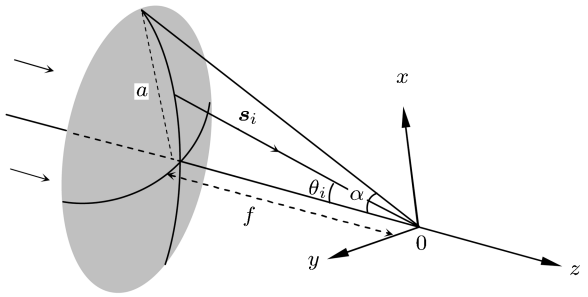


Fig.1. Illustration of the focusing system and notation.

The spectral degree of coherence is defined as[11]

$$\mu(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{W(\boldsymbol{r}_1, \mathbf{r}_2)}{[S(\boldsymbol{r}_1)S(\boldsymbol{r}_2)]^{1/2}}, \tag{2}$$

where $S(\boldsymbol{r}_i)$ specifies the spectral density (intensity) at the position $\boldsymbol{r}_i$ and is given by

$$S(\boldsymbol{r}_i) = W(\boldsymbol{r}_i, \boldsymbol{r}_i). \tag{3}$$

It is noted that the intensity $S(\boldsymbol{r})$ is always a real-valued function, whereas $\mu(\boldsymbol{r}_1,\ \boldsymbol{r}_2)$ is complex-valued in general. The spectral degree of coherence $\mu(\boldsymbol{r}_1,\ \boldsymbol{r}_2)$ is a measure of the correlation of the field at two points $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, and $0{\leq}\ |\mu|\ {\leq}1$, where $|\mu|{=}0,\ 1$ correspond to the fully incoherent and fully coherent cases, respectively. The intermediate values $0 < |\mu| < 1$ characterize partial coherence.[11]

From Eqs.(1)–(3) we have

$$W(-\boldsymbol{r}_1, -\boldsymbol{r}_2) = W(\boldsymbol{r}_2, \boldsymbol{r}_1) = W^*(\boldsymbol{r}_1, \boldsymbol{r}_2), \tag{4}$$

$$\mu(-\boldsymbol{r}_1, -\boldsymbol{r}_2) = \mu(\boldsymbol{r}_2, \mathbf{r}_1) = \mu^*(\boldsymbol{r}_1, \boldsymbol{r}_2), \tag{5}$$

$$S(-\boldsymbol{r}) = S(\boldsymbol{r}). \tag{6}$$

For the sake of simplicity and without loss of generality, we consider the two-dimensional case, thus the generalized Debye diffraction integral reduces to[10]

$$W(\rho_1, \rho_2) = \frac{kf}{2\pi}\int\int_{-\alpha}^{\alpha} W^{(0)}(x_{10}, x_{20}) \times \exp\left[\mathrm{i}k(\boldsymbol{s}_2\cdot\boldsymbol{\rho}_2 - \boldsymbol{s}_1\cdot\boldsymbol{\rho}_1)\right]\mathrm{d}\theta_1\mathrm{d}\theta_2, \tag{7}$$

where $\boldsymbol{s}_i = (-\sin\theta_i,\ \cos\theta_i)$, $\boldsymbol{\rho}_i = (x_i,\ z_i)$, α is the semi-aperture angle of the focusing geometry. For GSM beams considered in this paper we have[11]

$$W^{(0)}(x_{10}, x_{20}) = \exp\left(-\frac{x_{10}^2 + x_{20}^2}{2w_0^2}\right) \times \exp\left[-\frac{(x_{10} - x_{20})^2}{2\sigma^2}\right], \tag{8}$$

where w_0 and σ denote the waist width and spatial coherence length, respectively.

The substitution from Eq.(8) into Eq.(7) yields

$$W(\rho_1, \rho_2) = \frac{kf}{2\pi}\int\int_{-\alpha}^{\alpha} \exp\left[\frac{-\left(\delta^2 + \dfrac{1}{\varepsilon^2}\right)\left(\sin^2\theta_1 + \sin^2\theta_2\right) + \dfrac{2\sin\theta_1\sin\theta_2}{\varepsilon^2}}{2\sin^2\alpha}\right] \times \exp\left[-\mathrm{i}k(z_1\cos\theta_1 - x_1\sin\theta_1) + \mathrm{i}k(z_2\cos\theta_2 - x_2\sin\theta_2)\right]\mathrm{d}\theta_1\mathrm{d}\theta_2, \tag{9}$$

where

$$\delta = \frac{a}{w_0}, \qquad \text{(truncation parameter)} \tag{10}$$

$$\varepsilon = \frac{\sigma}{a}. \qquad \text{(scalar coherence length)} \tag{11}$$

In the derivation of Eq.(9) the relation $a = f\sin\alpha$ is used which is valid in the paraxial approximation.

If pairs of axial points $\boldsymbol{\rho}_i{=}(0,\ z_i)$ are considered, the cross-spectral density simplifies to

$$W(0, z_1; 0, z_2) = \frac{kf}{2\pi}\int\int_{-\alpha}^{\alpha} \exp\left[\frac{-\left(\delta^2 + \dfrac{1}{\varepsilon^2}\right)\left(\sin^2\theta_1 + \sin^2\theta_2\right) + \dfrac{2\sin\theta_1\sin\theta_2}{\varepsilon^2}}{2\sin^2\alpha}\right] \times \left[\cos(kz_1\cos\theta_1 - kz_2\cos\theta_2) - \mathrm{i}\sin(kz_1\cos\theta_1 - kz_2\cos\theta_2)\right]\mathrm{d}\theta_1\mathrm{d}\theta_2, \tag{12}$$

and the axial spectral density is given by

$$S(0,z)=\frac{kf}{2\pi}\int\int_{-\alpha}^{\alpha}\exp\left[\frac{-\left(\delta^2+\frac{1}{\varepsilon^2}\right)\left(\sin^2\theta_1+\sin^2\theta_2\right)+\frac{2\sin\theta_1\sin\theta_2}{\varepsilon^2}}{2\sin^2\alpha}\right]\times\cos\left[kz(\cos\theta_1-\cos\theta_2)\right]\mathrm{d}\theta_1\mathrm{d}\theta_2. \tag{13}$$

For pairs of points in the focal plane $\boldsymbol{\rho}_i=(x_i,\,0)$ the cross-spectral density reduces to

$$W(x_1,0;x_2,0)=\frac{kf}{2\pi}\int\int_{-\alpha}^{\alpha}\exp\left[\frac{-\left(\delta^2+\frac{1}{\varepsilon^2}\right)\left(\sin^2\theta_1+\sin^2\theta_2\right)+\frac{2\sin\theta_1\sin\theta_2}{\varepsilon^2}}{2\sin^2\alpha}\right]\times\cos(kx_1\sin\theta_1-kx_2\sin\theta_2)\mathrm{d}\theta_1\mathrm{d}\theta_2. \tag{14}$$

It follows from Eq.(14) that for such a case $W(x_1,0;x_2,0)$ is real-valued. As a result, the spectral degree of coherence in the focal plane is real-valued.

The spectral density in the focal plane reads as

$$S(x,0)=\frac{kf}{2\pi}\int\int_{-\alpha}^{\alpha}\exp\left[\frac{-\left(\delta^2+\frac{1}{\varepsilon^2}\right)\left(\sin^2\theta_1+\sin^2\theta_2\right)+\frac{2\sin\theta_1\sin\theta_2}{\varepsilon^2}}{2\sin^2\alpha}\right]\times\cos\left[kx(\sin\theta_1-\sin\theta_2)\right]\mathrm{d}\theta_1\mathrm{d}\theta_2. \tag{15}$$

3. Numerical calculation results and discussions

To illustrate the spatial correlation properties and phase singularities of apertured GSM beams in the focal region, numerical calculations were performed by using the formulae derived in the Section 2. In the following calculations f=3000λ, α=15° are kept fixed, hence the Fresnel number $N=a^2/\lambda f=(\sin\alpha)^2 f/\lambda >> 1$ insures the validity of the Debye theory.[10] Figures 2(a)–2(c) give the axial spectral degree of coherence $|\mu(0,\,0;\,0,\,z)|$ (absolute value) at pairs of points (0, 0) and (0, z) and normalized axial spectral density $S(0,z)/S(0,0)$ of apertured GSM beams for different values of the truncation parameter δ, and (a) ε=0.25, (b) ε=0.5, and (c) ε=5. As can be seen, there are no zeros of $|\mu(0,\,0;\,0,\,z)|$ and $S(0,z)/S(0,0)$ for GSM beams in general. If the width is defined as the width at half maximum of curves, for ε=0.25 in Fig.2(a) the width of the spectral degree of coherence w_μ is larger than the width of the spectral density w_s at δ=2, whereas we have $w_\mu=w_s$ at δ=1.24, and $w_\mu<w_s$ at δ=0.5. For ε=0.5 and 5 in Figs.2(b) and 2(c) the width of the spectral density is always less than that of the spectral degree of coherence $w_s<w_\mu$ for the selective values of δ, and the larger the truncation parameter δ, the greater the difference between w_s and w_μ.

The spectral degree of coherence and spectral density of apertured GSM beams in the focal plane are presented in Figs.3(a)–3(c) for different values of the truncation parameter δ, and (a) ε=0.25, (b) ε=0.5, and (c) ε=5, where pairs of points (0, 0), (x, 0) are chosen. It is seen that, depending on both δ and ε, w_s can be larger, less than or equal to w_μ, e.g., for ε=0.25 in Fig.3(a) we have $w_\mu>w_s$ at δ=3.3, $w_\mu=w_s$ at δ=2.83, and $w_\mu<w_s$ at δ=0.5. However, differing from Figs.2(a)–2(c), there are zeros of $\mu(0,\,0;\,x,0)$ in the focal plane for the selective values of δ, and for the large value of ε=5 in Fig.3(c), i.e., in the coherent limit, zeros of the spectral density appear and coincide with those of the corresponding spectral degree of coherence. For example, at δ=0.5 the positions of the first and second singularities of $\mu(0,\,0;\,x,0)$ and $S(x,\,0)$ in the focal plane are x_0=1.97λ and 3.89λ, respectively. However, for the small values of ε=0.25 and 0.5 in Figs.3(a) and 3(b) zeros of the spectral density are absent. A comparison with Schell-model beams,

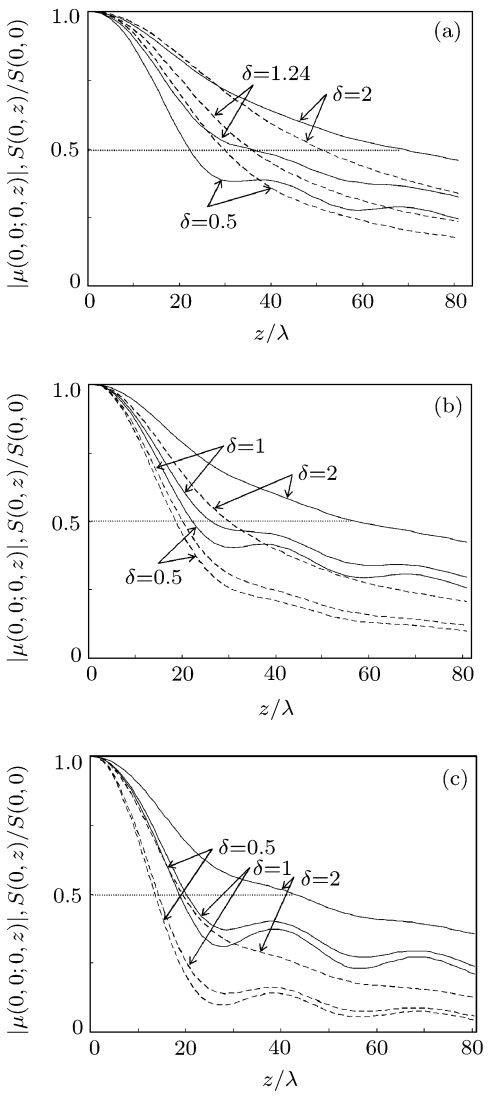


Fig.2. The axial spectral degree of coherence (absolute value) $|\mu(0,\,0;\,0,\,z)|$ and normalized spectral density $S(0,z)/S(0,0)$ versus the normalized distance z/λ for different values of the truncation parameter δ, —: $|\mu(0,\,0;\,0,\,z)|$, – – –: $S(0,z)/S(0,0)$, (a) ε=0.25, (b) ε=0.5, and (c) ε=5.

namely, GSM beams with uniform intensity,[8] shows that when $\varepsilon \geq 0.5$, for Shell-model beams the width of spectral density w_s is always less than that of the spectral degree of coherence w_μ, $w_s < w_\mu$ (see Fig.7 in Ref.[8]), whereas for GSM beams, as shown in Fig.3(b), when ε=0.5, depending on the truncation parameter, we have $w_s < w_\mu$ at δ=2, but $w_s = w_\mu$ at δ=1.1, and $w_s > w_\mu$ at δ=0.5.

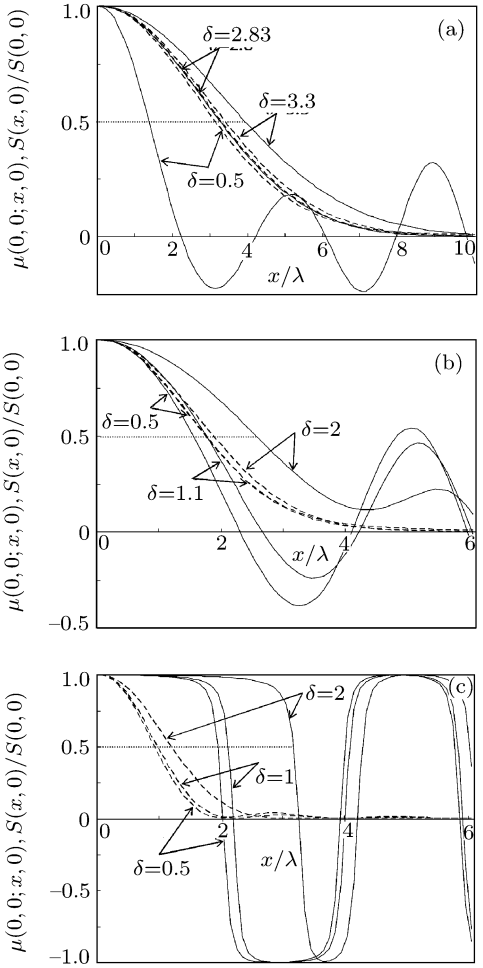


Fig.3. The spectral degree of coherence $\mu(0,\,0;\,x,\,0)$ and normalized spectral density $S(x,0)/S(0,0)$ in the focal plane versus the normalized distance x/λ for different values of the truncation parameter δ, —: $\mu(0,\,0;\,x,0)$, – – –: $S(x,0)/S(0,0)$, (a) ε=0.25, (b) ε=0.5, and (c) ε=5.

The contour lines of the spectral degree of coherence $|\mu(0,z;x,z)|$ in the focal region are depicted in Figs.4(a)–4(c) for different values of (a) δ=0.5, (b) δ=1.56, (c) δ=1.69, and ε=0.25. Figure 4(a) indicates that there are zeros of $\mu(0,0;x,0)$ at the positions of x_0=2.15λ, 4.18λ, 6.03λ, 7.97λ and 9.76λ in the focal plane z=0, where the spectral degree of coherence $\mu(0,0;x_0,0)$ possesses phase singularities, thus the field at the points (0, 0) and (x_0, 0) are completely uncorrelated. With increasing truncation parameter δ to 1.56 in Fig.4(b), phase singularities move and the position of a pair of phase singularities approaches each other, which results in subwavelength structures, i.e., at δ=1.56 the distance between the first and second singularities is 0.9λ. A further increase of δ to 1.69 in Fig.4(c) leads to the annihilation of them. In comparison with fully coherent Gaussian beams, whose airy pattern creation / annihilation in the focal region was detailedly analysed in Refs.[12, 13] where, accompanying the annihilation, the creation of phase

singularities outside the focal plane takes place, however, the creation process of phase singularities outside the focal plane is not found for GSM beams. Additionally, if pairs of points $(-x, 0)$, $(x, 0)$ are chosen, similar numerical calculations show also the annihilation of phase singularities in the focal plane, but no creation of phase singularities outside the focal plane for GSM beams.

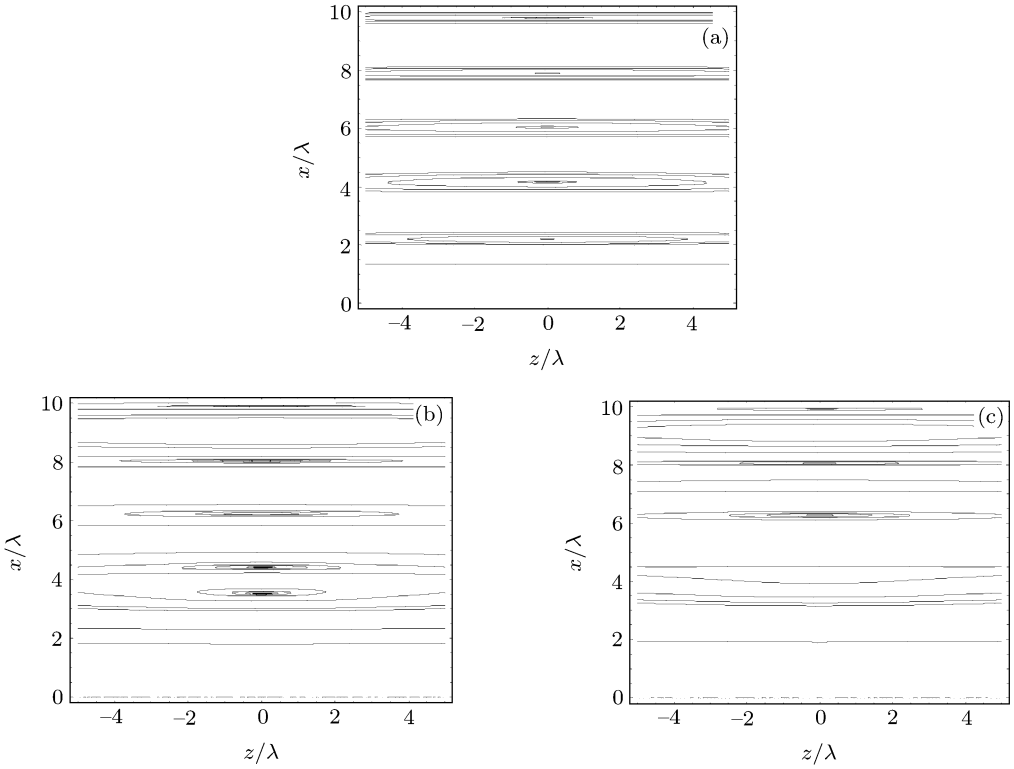


Fig.4. Contour lines of the spectral degree of coherence $|\mu(0, z; x, z)|$ for ε=0.25, (a) δ=0.5, (b) δ=1.56, and (c) δ=1.69.

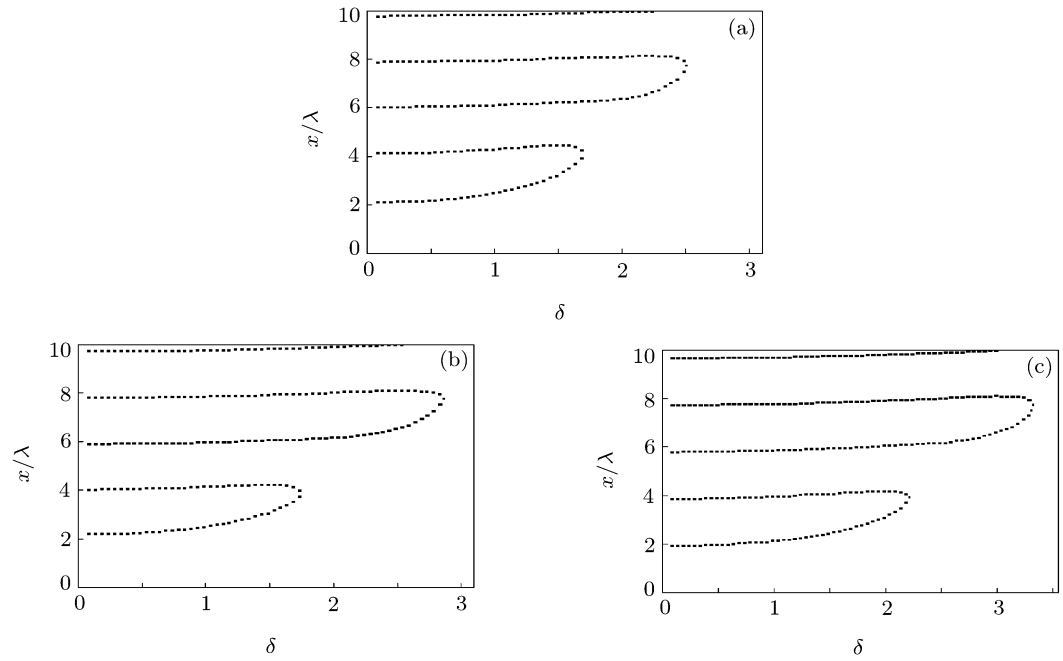


Fig.5. The positions of phase singularities in the focal plane versus the truncation parameter δ, (a) ε=0.25, (b) ε=0.5, and (c) ε=5.

The positions of phase singularities (zeros of $\mu(0,0;x,0)$) along the x axis in the focal plane versus the truncation parameter δ are plotted in Figs.5(a)–5(c) for different values of (a) ε=0.25, (b) ε=0.5 and (c) ε=5, from which it is readily seen that, as the truncation parameter δ increases, a pair of phase singularities approaches gradually, and the distance between them decreases to infinitesimal value. Finally, the annihilation of pairs of singularities takes place at a certain value of the truncation parameter δ. For example, as shown in Fig.5(a), for ε=0.25 a pair of the first and second singularities disappears at δ=1.69. With increasing scalar coherence length ε, the annihilation occurs at the larger truncation parameter δ, e.g., for ε=0.5 and 5 the annihilation of the first and second singularities occurs at δ=1.74 and 2.20 in Figs.5(b) and 5(c), respectively.

4. Conclusions

This paper has make a detailed study of the spatial correlations properties and phase singularity annihilation of apertured partially coherent, quasi-monochromatic fields in the focal region, where the GSM beam has been taken as a typical example. It has been found that the width of the spectral degree of coherence w_μ can be larger, less than or equal to the corresponding width of the spectral density w_s, which is dependent not only on the scalar coherence length ε, but also on the truncation parameter δ. By varying ε and/or δ, w_μ and w_s are controllable which may be of interest for some practical applications. On the axis there are no zeros of the spectral degree of coherence, whereas zeros of the spectral degree of coherence are presented in the focal plane. In the coherent limit, zeros of the spectral degree of coherence coincide with those of the spectral density, namely, coherence vortices become intensity vortices. The truncation parameter δ and scalar coherence length ε both affect the presence and spatial distribution of phase singularities in the focal plane. With gradually increasing truncation parameter δ, a pair of phase singularities approaches each other, resulting in subwavelength structures. A further increase of the truncation parameter δ leads to the annihilation of phase singularities. With increasing scalar coherence length ε, the annihilation process appears at larger truncation parameter. However, unlike the case of fully coherent apertured Gaussian beams, the creation of phase singularities outside the focal plane is not found for GSM beams. Finally, we would like to mention that, although the two-dimensional GSM beams have been treated here, starting from Eq.(1), the extension to the three-dimensional case is straightforward, because the variables x and y of GSM are separable.

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