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# Cascades with coupled map lattices in preferential attachment community networks*

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In this paper, cascading failure is studied by coupled map lattice (CML) methods in preferential attachment community networks. It is found that external perturbation $R$ is increasing with modularity $Q$ growing by simulation. In particular, the large modularity $Q$ can hold off the cascading failure dynamic process in community networks. Furthermore, different attack strategies also greatly affect the cascading failure dynamic process. It is particularly significant to control cascading failure process in real community networks.

**Keywords:** community networks, modularity, coupled map lattices, cascading failure
**PACC:** 0590, 0200

## 1. Introduction

Much current attention has been focused on complex networks which characterize many natural and man-made systems, such as internet networks, airline transport complex networks, urban traffic complex networks,[1−3] power grid complex infrastructures and social interaction complex networks.[4,5] The community structure has been found in many real technological, biological and social complex networks recently.[6−8] The study of community structure is useful to understand the mechanism of complex networks. The general notion of community networks is first pointed out in the physics literature by Girvan and Newman,[9,10] and it refers to the fact that nodes appear to community structure in which edge connection intensity of inner-community is larger than that of inter-community in many complex networks.

The purely topological characterization will miss important properties which are often encountered in many real systems. A step has already been put forward to realistically thinking over physical dynamic features. The dynamic behaviours in complex networks have recently attracted much attention of researchers in different areas.[11−19] Cascading failure is a common dynamic phenomenon in most communication and transportation networks. A typical example is the largest blackout event in the northeastern US power transmission networks in the year of 2003. It also can occur in many physical systems, such as the electrical power grids, the internet networks and the social economic systems. Cascading failure is triggered by breakdown of one node, and the single node breakdown will cause main changes in the balance of loads. Finally, the subsequent overload failures are easily taken place. The coupled map lattices (CML) methods can interpret the spatiotemporal chaos effect through the basic knowledge of dynamic systems theory. The methods have been widely investigated over past decades to interpret rich space–time dynamic behaviours of complex systems. And they are more suitable for computational studies because of discrete nature of time and space. Gade *et al* have researched synchronization with CML methods in small-world or scale-free coupling topologies.[20,21] Wang X F *et al* have successfully interpreted cascading failure with CML methods in globally complex networks, small world networks, and scale-free networks.[22−24] However, there are few works on cascading failure with CML methods by considering community structure in complex networks. In this paper, we will fill this gap by studying the cascades with CML methods in community networks. In Section 2, we introduce the generation of preferential attachment community net-

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works. In Section 3, the CML methods are adopted to interpret cascading failure process in preferential attachment community networks. Simulation results are given in Section 4. Finally, conclusions are drawn in Section 5.

## 2. The generation of preferential attachment community networks

The scale-free networks with degree distribution $P(k)$ give a power-law behaviour, i.e. $P(k) \sim k^{-\gamma}$, where $k$ is node degree and $\gamma$ is scale-free networks exponent. There are two main mechanisms which are responsible for emergence of scale-free pattern. One principal consequence of such distribution is that it promotes the appearance of hub node (one node with high degree). Another concept of understanding dynamic scale-free networks is grown. Inspired by these two above mentioned mechanisms and the previous works by Li and Maini,[25] the generation of preferential attachment community networks is described as follows. (1) Initialization: starting from $h$ communities denoted by $H_1, H_2, H_3, \ldots, H_h$, each community has small initial nodes $\phi_0$. And the $\dfrac{h(h-1)}{2}$ inter-community edges connect each community with other $h-1$ communities. It is assured that there is one edge between inter-communities. The edges of inner-community are randomly linked in each community. (2) Growth: at each time step, a newly added node connects to other $\phi(1 \leq \phi < \phi_0)$ nodes in same community through $\phi$ inner-edges (edges connected to nodes in same community). At the same time, the new node with probability $\alpha$ connects to $\eta(1 < \eta < \phi)$ nodes in other $h-1$ communities through inter-edges (edges connected nodes in two different communities), simultaneously. After $t$ time step, the scheme generates every community with $N$ nodes. Furthermore, total community networks have $hN = h\phi_0 + t$ nodes and $\dfrac{h\phi_0(\phi_0-1)}{2} + \dfrac{h(h-1)}{2} + \phi t + \text{integer}(\alpha\eta t)$ edges. Here parameters $\alpha$ and $\eta$ control the growing ratio of inter-edge and inner-edge. (3) Preferential Attachment: the new node is more likely to connect with the hub node than the random node (one node with low degree) for both inner-community and inter-community. For clarity purposes, the generated community networks with total 90 nodes are shown in Fig.1. Additionally, its degree distributions $P(k)$ of total nodes including inner-community and inter-community have strong stabilities in growing process. The mean-field theory[25] is applied to analyse scaling property of inner-community degree distribution $\left(P(k) = \dfrac{2\phi^2 t}{h\phi_0 + t} k^{-3}\right)$, inter-community degree distribution $\left(P(k) = \dfrac{2\eta^2 t}{h\phi_0 + t} k^{-3}\right)$ and total degree distribution of community networks $\left(P(k) = \dfrac{2(\phi+\eta)^2 t}{h\phi_0 + t} k^{-3}\right)$. Consequently, all of them follow the power-law distribution with exponent 3, i.e. $P(k) \propto k^{-3}$.

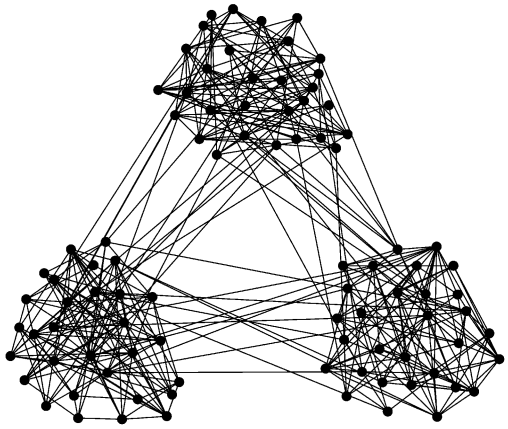


**Fig.1.** The preferential attachment community networks are generated by the proposed scheme under the condition of $N = 90$, $h = 3$, $\phi_0 = 6$, $\phi = 4$, $\eta = 1$ and $\alpha = 0.1$.

In general, a recent measure has been widely accepted in community networks. This measure, introduced by Newman and Girvan,[26] is based on the concept of modularity:

$$Q = \sum_r (g_{rr} - b_r^2). \tag{1}$$

The $g_{rw}$ is fraction of edges in community networks which connect nodes in community $r$ to nodes in community $w$, and the row (or column) sums $b_r = \sum_w g_{rw}$ give the fraction of edges which connect to nodes in community $r$. And $g_{rr}$ is the fraction of edges which connect two nodes in the same community $r$. We adjust community numbers $h$ to change modularity $Q$ of community structure. The larger value of $Q$ is, the more accurate a partition into communities will be. In practice, $0.3 < Q < 0.7$ is considered a good indication of community structure.

# 3. Cascading failure with CML methods in community networks

The CML methods have been widely used to study the dynamic complex systems over past decades. In CML methods, dynamic elements are situated at discrete points in space, time is discrete, and state variable is continuous.[27−29] Each spatial unit is coupled to its neighbours. The selection of neighbours is determined by structure of complex networks. The traditional complex networks are also brighter with spatiotemporal property, owing to spatiotemporal property of the CML methods. For simplicity, the CML methods[22,23] are introduced to describe dynamics of cascades in community networks.

$$\begin{aligned} x_{m_i}(t+1) = \Bigg| & (1-\varepsilon_{\text{inner}}-\varepsilon_{\text{inter}}) f(x_{m_i}(t)) \\ & + \varepsilon_{\text{inner}} \Bigg( \sum_{s=m} \sum_{\substack{j=1 \\ j\neq i}}^{N} a_{m_i s_j} \left( \frac{f(x_{s_j}(t))}{k_{m_i \text{inner}}} \right) \Bigg) \\ & + \varepsilon_{\text{inter}} \Bigg( \sum_{\substack{s=1 \\ s\neq m}}^{h} \sum_{j=1}^{N} a_{m_i s_j} \frac{f(x_{s_j}(t))}{k_{m_i \text{inter}}} \Bigg) \Bigg| \end{aligned} \tag{2}$$

($i = 1, 2, \cdots, N; m = 1, 2, \cdots, h$; $N$=total nodes of each community; $h$=total number of the communities).

In Eq.(2), the $x_{m_i}(t+1)$ is state variable of node $i$ in community $m$ at time step $t+1$. The adjacency matrix $A = (a_{m_i s_j})_{hN \times hN}$ represents edge connection information between node $i$ in community $m$ and node $j$ in community $s$. If there is an edge between two nodes, $a_{m_i s_j}$ is represented by number 1, otherwise, by number 0. In particular, we assume that no two different nodes can have more than one edge between them and no node can have an edge with itself. $A$ is symmetric $0-1$ matrix with diagonal elements zero. The $k_{m_i \text{inner}}$ defines total number of edges which connect node $i$ to other nodes inside community $m$. The $k_{m_i \text{inner}}$defines total number of edges which connect node $i$ community $m$ to other nodes in different communities. The coupled strength of inner-community and inter-community are represented by $\varepsilon_{\text{inner}}, \varepsilon_{\text{inter}} \in (0,1)$, respectively. To guarantee non-negative state of each node, we use absolute value notation in Eq.(3). Here mapping function $f(x)$ defines local dynamics which is chosen to be chaotic logistic map $f(x) = dx(1-x)$, $d \in (0,4]$.[22,23]

Furthermore, the CML methods are applied to describe the dynamic patterns of cascading failure in community networks. In order to show cascading failure dynamic process, an external perturbation $R$ is added to node $c$ in community $m$ at the $l$th time step, we can get

$$\begin{aligned} x_{m_c}(l+1) = \Bigg| & (1-\varepsilon_{\text{inner}}-\varepsilon_{\text{inter}}) f(x_{m_c}(l)) \\ & + \varepsilon_{\text{inner}} \Bigg( \sum_{s=m} \sum_{\substack{j=1 \\ j\neq i}}^{N} a_{m_c s_j} \left( \frac{f(x_{s_j}(l))}{k_{m_i \text{inner}}} \right) \Bigg) \\ & + \varepsilon_{\text{inter}} \Bigg( \sum_{\substack{s=1 \\ s\neq m}}^{h} \sum_{j=1}^{N} a_{m_c s_j} \frac{f(x_{s_j}(l))}{k_{m_i \text{inter}}} \Bigg) \Bigg| + R. \end{aligned} \tag{3}$$

In order to show the dynamic process clearly, the state variable $x_{m_i}(t)$ can be defined as follows. (1) The node $i$ is in a normal state at the $l$th time step, if the condition $0 < x_{m_i}(t) < 1$, $t \leq l$ are satisfied. (2) Otherwise, if $0 < x_{m_i}(t) < 1 (t < l,\ x_{m_i}(l) \geq 1)$, then node $i$ is said to be failed at the $l$th time step and we assume in this case that $x_{m_i}(t) = 0$, $t > l$. (3) If initial state of each node in Eq.(3) is in the interval $(0,1)$ and there is no external perturbation $R$, then total nodes in community networks will be in normal states forever.

Node $c$ in community $m$ will be failed at the $l$th time step and we have $x_{m_c}(t) \equiv 0$ for all time steps $t > l$. In the following simulations, external perturbation $R \geq 1$ is added to node $c$ in community $m$. Cascading failure can be characterized by total number of failed nodes $I(t)$ before the $(t+1)$th time step. $I \equiv \lim\limits_{t\to\infty} I(t)$ measures the size of whole cascading failure in community networks.

# 4. Simulations and results

The community networks are generated by the proposed scheme (see Section 2) under the condition of $\phi_0 = 10, \phi = 9, \eta = 1, \alpha = 0.1, h = 3$ and $hN = 2000$ nodes. The inner-degree distribution of community networks is shown in Fig.2(a), which obeys a power-law distribution $P(k) \sim k^{-\gamma}$ with $\gamma \approx 3$. The inter-degree distribution of community networks is shown in Fig.2(b), which obeys a power-law degree distribution $P(k) \sim k^{-\gamma}$ with $\gamma \approx 2.87$.

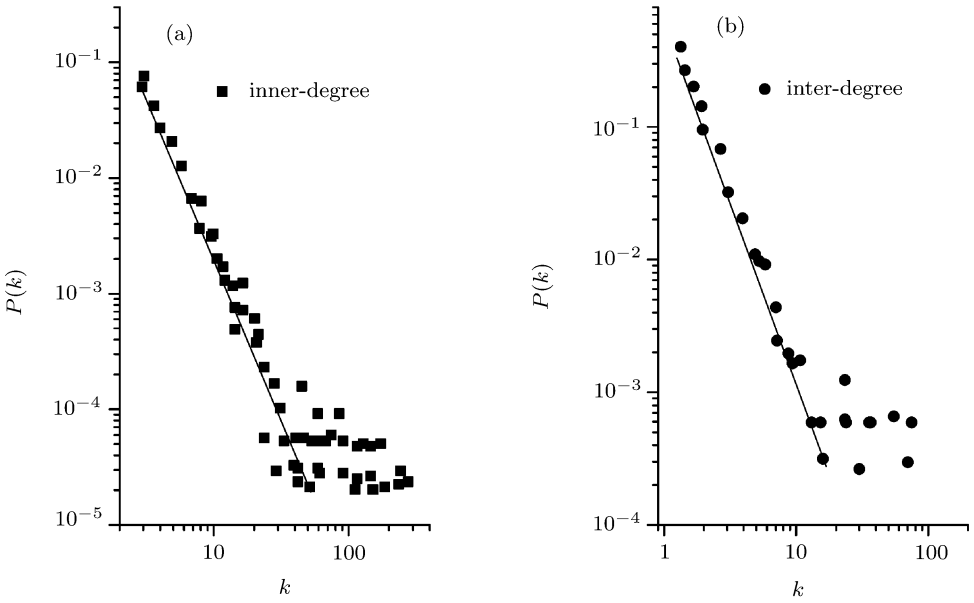



**Fig.2.** The community networks is generated under the condition of $\phi_0 = 10$, $\phi = 9$, $\eta = 1$, $\alpha = 0.1$, $h = 3$ and $hN = 2000$ nodes. (a): inner-community degree distribution (b): inter-community degree distribution. The graphs (a) and (b) correspond to average 20 realizations, respectively.

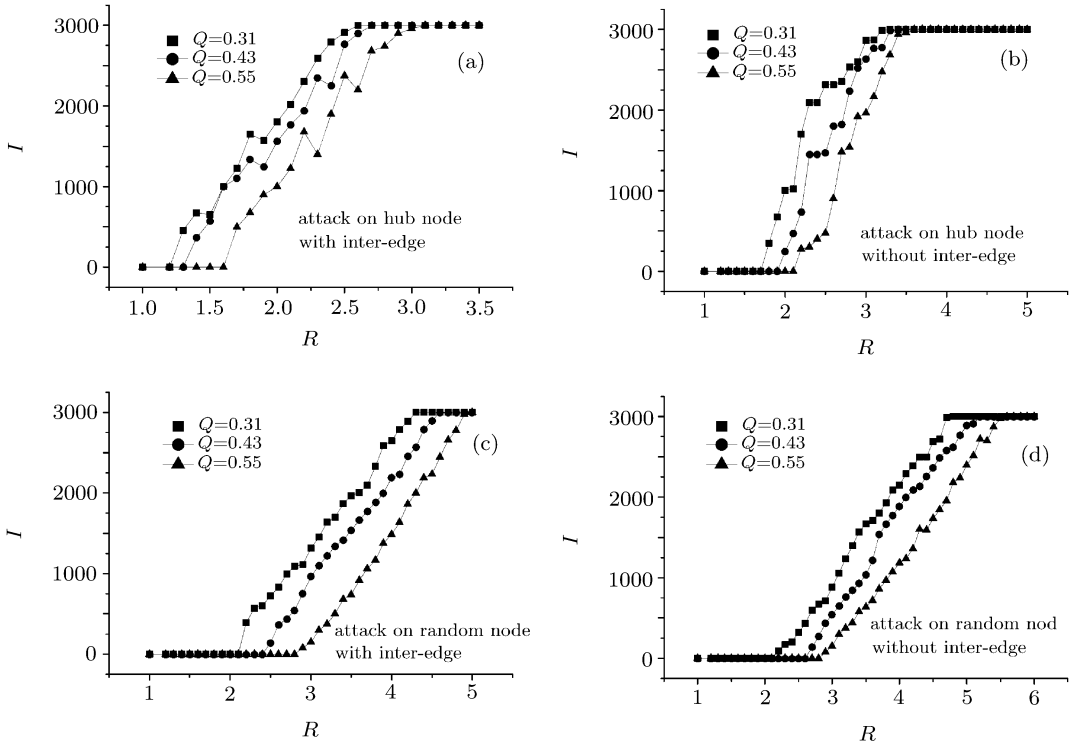



**Fig.3.** Size $I$ of cascading failure is as a function of amplitude for external perturbation $R$ with CML methods in community networks under the condition of $Q = 0.31(\phi = 9, \eta = 1, \alpha = 0.1)$, $Q = 0.43(\phi = 11, \eta = 1, \alpha = 0.076)$, $Q = 0.55(\phi = 15, \eta = 1, \alpha = 0.063)$ respectively. Cascading failure is triggered by adding external perturbation strategies to an randomly chosen node: (a) attack on hub node which connects with inter-edge; (b) attack on hub node which does not connect with inter-edge; (c) attack on random node which connects with inter-edge; (d) attack on random node which does not connect with inter-edge. The simulations are averaged 20 realizations of community networks.

Here we consider cascading failure with CML methods in community networks. In the first place, we define two kinds of edge: inner-edge is the edge whose endpoints are in same community; inter-edge is the edge whose endpoints are in two different communities. According to this situation, we adopt four different attack strategies to trigger cascading failure with CML methods: attack on hub node which connects with inter-edge, attack on random node which connects with inter-edge, attack on hub node which does not connect with inter-edge, attack on random node which does not connect with inter-edge. Figures 3(a), 3(b), 3(c) and 3(d) plot $I$ as a function of amplitude $R$ with different modularity $Q = 0.31$, $Q = 0.42$ and $Q = 0.55$ under four attack strategies, respectively (with fixed coupled strength $\varepsilon_{\text{inner}} = 0.45$, $\varepsilon_{\text{inter}} = 0.35$). According to Fig.3, we can get the conclusion that cascading failure is easily taken place in community networks with smaller modularity $Q$ under four different attack strategies. For the fixed modularity $Q$, external perturbation $R$ under attacking on hub node with inter-edge is smaller than that under attacking on hub node without inter-edge. Especially, attacking on the node with inter-edge can expedite cascading failure process. Figure 4 reveals the close relationship between modularity $Q$ and external perturbation $R$. By simulations, we can see that external perturbation $R$ is increasing, while modularity $Q$ is growing. And the general trend of external perturbation $R$ is towards a stable increase in the scale of $Q \in [0.3, 0.7]$, which is provided with good indication community structure of complex networks. This result is also consistent with some recent works on dynamic behaviours of cascading failure process in scale-free networks with community structure,[30] where it also found that small modularity is much easier to trigger cascade failure than that of the larger one by other methods instead of CML methods.

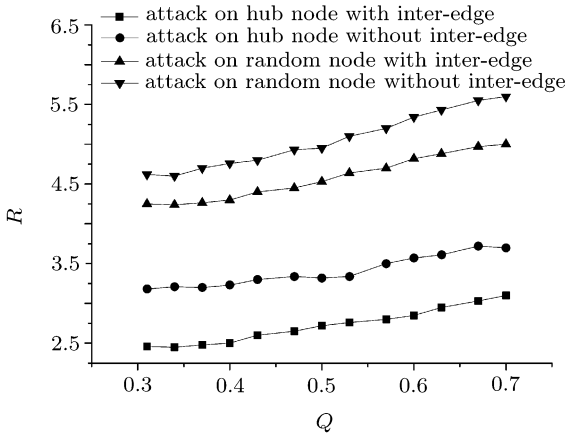



**Fig.4.** External perturbation $R$ with CML methods is as a function of modularity $Q$ in community networks. The graph corresponds to the average 20 realizations of community networks.

## 5. Conclusion

Cascading failure, triggered by an initial perturbation, but spreading to other constituents successively, is one of intriguing problems in physics. In this paper, we aim at studying how the topology structure affects cascading failure with CML methods in community networks. By simulations, we can see that external perturbation $R$ is increasing with the modularity $Q$ growing. In addition, decreasing attacking on hub node and inter-edge can efficiently delay cascading failure process in community networks.

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