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Impulsive control of permanent magnet synchronous motors with parameters uncertainties*

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The permanent magnet synchronous motors (PMSMs) may have chaotic behaviours for the uncertain values of parameters or under certain working conditions, which threatens the secure and stable operation of motor-driven. It is important to study methods of controlling or suppressing chaos in PMSMs. In this paper, robust stabilities of PMSM with parameter uncertainties are investigated. After the uncertain matrices which represent the variable system parameters are formulated through matrix analysis, a novel asymptotical stability criterion is established. Some illustrated examples are also given to show the effectiveness of the obtained results.

Keywords: permanent magnet synchronous motors, impulsive control, parameters uncertainties, robust stability

PACC: 0545

1. Introduction

Permanent magnet synchronous motors (PMSMs) are of great interest, particularly for industrial applications in the low-medium power range, since it has superior features such as compact size, high torque/weight ratio, high torque/inertia ratio and absence of rotor losses.[1] Moreover, in comparison with induction motors, PMSMs have the advantages of higher efficiency, due to the absence of rotor losses and lower no-load current below the rated speed.[2]

The control technique analysis and systematic design for PMSMs are among the most important issues because the performance of the PMSM is very sensitive to external load disturbances and parameter variations in the plant. Thus, a considerable amount of control techniques have been developed to overcome these problems,[1−13] and the related chaotic control methods are widely studied.[14−21] For examples, Baik and Kim proposed two robust nonlinear speed control schemes for a PMSM which guarantee the robustness of the PMSM with parameter variations and speed measurement error using boundary layer integral sliding mode control technique in Ref.[3] and using adaptive and sliding mode control techniques in Ref.[4]. Ahmad M Harb and Ashraf A Zaher[5] used the bifurcation analysis for PMSM and introduced a nonlinear robust model-reference controller which shows how to stabilize the system, while having a satisfactory performance. Wei *et al*[6] deduced a law of controlling chaos in PMSMs by using the exact linearization theory of differential geometry. Karunadasa and Renfrew[7] presented a sliding mode position control system, with an allowance for the non-infinite sampling rates and supply source limitation encountered in practice, and the effects of these on system performance have been fully investigated. A sliding-mode controller with integral-operation switching surface which is theoretically robust with regard to plant uncertainties was designed in Ref.[8]. Wai[9] successfully demonstrated the application of the total sliding-mode and recurrent-fuzzy-neural-network sliding-mode control systems to control the rotor position of a field-oriented PM synchronous servo motor drive. Recently, the H_∞ control strategy has also been widely celebrated for its robustness in counteracting uncertainty perturbations and external disturbances.[2,10,11] Lee *et al*[2] presented a robust PMSM drive system equipped with an adaptive H_∞ controller under the influence

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of uncertainties and external disturbances. By using feed-forward compensation, the robust performance control problem is formulated as a nonlinear H_∞ problem and solved by a quadratic storage function. Attaianese *et al*[10] applied H_∞ control theory to design a state feedback static controller of a separately excited direct current motor in order to achieve position control. The problem of finding an optimal H_∞ controller was considered. Hsien *et al*[11] presented a H_∞ design method for a robust speed controller in a sensorless PMSM drive. The main point of the H_∞ control is to synthesize a feedback law that renders the closed-loop system to satisfy a prescribed H_∞-norm constraint which represents desired stability or tracking requirements.

The control schemes for the PMSMs described above have been widely used in industry. However, the values of parameters of the PMSM model are not constant, but span some interval. The main reason is that the permanent magnetic-flux will vary with the temperature rise of permanent-magnet material and rotary inertia will vary with the attrition of the device, or modelling errors, measurement inaccuracy, and so on. So the control strategy of PMSMs with parameters variation is quite valuable. Impulsive control has the advantages of higher reliability and stronger antijamming ability. If apply analogous voltage to control motor, the control voltage will raise to the limit rapidly when the blunder of wiring topology or some element breakage happens. But this incident will not happen if we apply impulse to control motor as control signal. So the impulsive control of PMSMs with parameters variation will be discussed in this paper. To the best of our knowledge, no reports about the stability results for PMSMs with parameters variation and few for uncertain impulsive dynamical systems have been presented in the open literatures.

The organization of the paper is as follows. In the next section, the problems investigated in this paper are formulated, and some preliminaries are presented. In Sections 3, the asymptotic stability criteria for the equilibrium points of PMSMs with parameters variation are derived. Several numerical examples and simulations including comparison analysis of conservatism are presented in Section 4. Finally, some conclusions are drawn in Section 5.

2. Problem formulation and preliminaries

The machine model of a PMSM can be described in the rotor rotating reference frame as follows:

$$\begin{aligned}
\frac{\mathrm{d}i_d}{\mathrm{d}t} &= \left(-R_1 i_d + \omega L_q i_q\right)/L_d,\\
\frac{\mathrm{d}i_q}{\mathrm{d}t} &= \left(-R_1 i_q - \omega L_d i_d - \omega\psi_r\right)/L_q,\\
\frac{\mathrm{d}\omega}{\mathrm{d}t} &= \left[n_\mathrm{p}\psi_r i_q + n_\mathrm{p}(L_d - L_q) i_d i_q - T_\mathrm{L} - \beta\omega\right]/J, \qquad (1)
\end{aligned}$$

where i_d and i_q are d, q axis stator current, L_d and L_q are d, q axis stator inductance, R_1 is stator resistance, ω is rotor speed, T_L is the external load torque, ψ_r is the maximum flux induced by the rotor magnet, n_p is number of pole-pairs, β represents the damping coefficient, and J is the moment of inertia of the rotor.

Applying the affine transformation and time scale transformation to system (1) with $x = \lambda\tilde{x}$, $t = \tau\tilde{t}$, where $x = [i_d \ i_q \ \omega]^\mathrm{T}$, $\tilde{x} = [\tilde{i}_d \ \tilde{i}_q \ \tilde{\omega}]^\mathrm{T}$, $b = \dfrac{L_q}{L_d}$, $k = \dfrac{\beta}{n_\mathrm{p}\tau\psi_\mathrm{r}}$, $\tau = \dfrac{L_q}{R_1}$,

$$\lambda = \begin{bmatrix} \lambda_d & 0 & 0 \\ 0 & \lambda_q & 0 \\ 0 & 0 & \lambda_\omega \end{bmatrix} = \begin{bmatrix} bk & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1/\tau \end{bmatrix},$$

we can describe the dimensionless mathematical model of PMSM as follows:

$$\begin{aligned}
\frac{\mathrm{d}\tilde{i}_d}{\mathrm{d}t} &= -\tilde{i}_d + \tilde{\omega}\tilde{i}_q,\\
\frac{\mathrm{d}\tilde{i}_q}{\mathrm{d}t} &= -\tilde{i}_q - \tilde{\omega}\tilde{i}_d + \gamma\tilde{\omega},\\
\frac{\mathrm{d}\omega}{\mathrm{d}t} &= \sigma(\tilde{i}_q - \tilde{\omega}) + \varepsilon\tilde{i}_d\tilde{i}_q - \tilde{T}_\mathrm{L}, \qquad (2)
\end{aligned}$$

where $\gamma = -\dfrac{\psi_\mathrm{r}}{kL_q}$, $\sigma = \dfrac{\beta\tau}{J}$, $\varepsilon = \dfrac{n_\mathrm{p} b\tau^2 k^2 (L_d - L_q)}{J}$, $\tilde{T}_\mathrm{L} = \frac{\tau^2}{J} T_\mathrm{L}$.

Considering the PMSM with smooth air gap, then we have $L_d = L_q = L$, so $\varepsilon = \dfrac{n_\mathrm{p} b\tau^2 k^2 (L_d - L_q)}{J} = 0$.

Let $x = [x_1\ x_2\ x_3]^{\mathrm{T}} = [\tilde{i}_d\ \tilde{i}_q\ \tilde{\omega}]^{\mathrm{T}}$, so the chaotic mathematical model of smooth air gap PMSM with parameters variation and impulsive effects can be expressed by

$$\begin{aligned}
&\dot{x}_1 = -x_1 + x_3x_2,\\
&\dot{x}_2 = -x_3x_1 - x_2 + \gamma x_3, \quad t \neq \tau_k, k = 1, 2, \cdots,\\
&\dot{x}_3 = \sigma(x_2 - x_3),\\
&\Delta x|_{t=t_k} \triangleq x\left(\tau_k^+\right) - x\left(\tau_k^-\right) = Bx, \quad k = 1, 2, \cdots, \quad (3)
\end{aligned}$$

where $\gamma \in N[r_1, r_2]$, $\sigma \in N[l_1, l_2]$ due to parameters variation.

Obviously, $x = [\gamma - 1, \sqrt{\gamma - 1}, \sqrt{\gamma - 1}]$, $x = [\gamma - 1, -\sqrt{\gamma - 1}, -\sqrt{\gamma - 1}]$, $x = [0, 0, 0]$ are the equilibrium points of system (3). We will discuss the stability of the origin of the system (3).

Let R^n denote the n-dimensional Euclidean space. Let $R_+ = [0, +\infty)$, $N = \{1, 2, \cdots\}$. The following sets will be used in the sequel:

$$S_\rho = \{x \in R^n : \|x\| \leq \rho\}.$$

Definition 2.1. For $(t, x) \in (\tau_{k-1}, \tau_k] \times R^n$, we define

$$\begin{aligned}
D^+V(t, x) = \lim_{h \to 0^+} \sup \frac{1}{h}[V(t + h, x + hf(t, x))\\
- V(t, x)]. \quad (4)
\end{aligned}$$

Definition 2.2. The uncertain impulsive system is called asymptotically stable, if the trivial solution of the system is asymptotically stable.

3. The impulsive stability of the PMSM with parameters variation

Let $\Phi(x) = \begin{bmatrix} x_3x_2 & -x_1x_3 & 0 \end{bmatrix}^{\mathrm{T}}$, under the initial condition $x\left(t_0^+\right) = x_0 \geq 0$, the system (3) becomes

$$\begin{aligned}
&\dot{x} = Ax + \Phi(x), \qquad t \neq \tau_k, k = 1, 2, \cdots,\\
&\Delta x|_{t=\tau_k} = Bx, \qquad k = 1, 2, \cdots,\\
&x\left(\tau_0^+\right) = x_0 \geq 0, \quad (5)
\end{aligned}$$

where $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & \gamma \\ 0 & \sigma & -\sigma \end{bmatrix}$.

The matrix A is uncertain matrix due to $\gamma \in N[r_1, r_2]$ and $\sigma \in N[l_1, l_2]$.

In order to obtain stability result for system (3), we shall establish some lemmas first.

Lemma 3.1.[15] Let $X \in R^{n \times n}$ be a $n \times n$ positive definite matrix and $Y \in R^{n \times n}$ a symmetric matrix. Then for any $x \in R^n$, $t \in R_+$, the following inequality holds:

$$x^{\mathrm{T}}Yx \leq \lambda_{\max}\left(X^{-1}Y\right)x^{\mathrm{T}}Xx, \quad (6)$$

where $\lambda_{\max}(\cdot)$ is the maximal eigenvalue function.

Lemma 3.2. Let $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & \gamma \\ 0 & \sigma & -\sigma \end{bmatrix}$, where $\gamma \in N[r_1, r_2]$, $\sigma \in N[l_1, l_2]$, and let $P = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & r_1 \\ 0 & l_1 & -l_2 \end{bmatrix}$, $Q = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & r_2 \\ 0 & l_2 & -l_1 \end{bmatrix}$, then A can be written as:

$$A = A_0 + E\Sigma F, \quad (7)$$

where

$$\begin{aligned}
&A_0 = \frac{1}{2}(P + Q), H = (h_{ij})_{3\times 3} = \frac{1}{2}(Q - P),\\
&\Sigma \in \Sigma^* = \{\Sigma \in R^{9\times 9} : \Sigma = \mathrm{diag}(\varepsilon_{11}, \varepsilon_{12}, \cdots, \varepsilon_{13}, \cdots, \varepsilon_{31}, \cdots, \varepsilon_{33}), \ \ |\varepsilon_{ij}| \leq 1, \varepsilon_{32} = \varepsilon_{33}, \ \ i, j = 1, 2, 3\},\\
&E = (\sqrt{h_{11}}e_1, \sqrt{h_{12}}e_1, \sqrt{h_{13}}e_1, \cdots, \sqrt{h_{31}}e_3, \cdots, \sqrt{h_{33}}e_3) \in R^{3\times 9}\\
&F = (\sqrt{h_{11}}e_1, \sqrt{h_{12}}e_2, \sqrt{h_{13}}e_3, \cdots, \sqrt{h_{31}}e_1, \cdots, \sqrt{h_{33}}e_3)^{\mathrm{T}} \in R^{9\times 3},\\
&e_1 = (1, 0, 0)^{\mathrm{T}}, e_2 = (0, 1, 0)^{\mathrm{T}}, e_3 = (0, 0, 1)^{\mathrm{T}}.
\end{aligned}$$

Remarks. Clearly, for any $\Sigma \in \Sigma^*$, we obtain

(1) $\Sigma\Sigma^{\mathrm{T}} = \Sigma^{\mathrm{T}}\Sigma \leq I$(3×3 identity matrix);

(2) $EE^{\mathrm{T}} = \mathrm{diag}\left\{\sum\limits_{j=1}^{3} h_{1j}, \sum\limits_{j=1}^{3} h_{2j}, \sum\limits_{j=1}^{3} h_{3j}\right\}$,

$$F^{\mathrm{T}}F = \mathrm{diag}\left\{\sum_{i=1}^{3} h_{i1}, \sum_{i=1}^{3} h_{i2}, \sum_{i=1}^{3} h_{i3}\right\}; \quad (8)$$

(3) by Lemma 3.2, the system (3) can be rewritten as

$$\begin{aligned}&\dot{x} = A_0x + E\Sigma Fx + \Phi(x), \quad t \neq \tau_k, k = 1, 2, \cdots,\\ &\Delta x|_{t=\tau_k} = Bx, \quad k = 1, 2, \cdots,\\ &x\left(\tau_0^+\right) = x_0 \geq 0, \end{aligned} \quad (9)$$

where A_0, E, Σ, F are defined as in Lemma 3.2.

Lemma 3.3.[15] If $\Sigma \in \Sigma^*$, then for any positive scalar $\lambda > 0$ and for any $\xi \in R^n$, the following inequality holds:

$$2\xi^{\mathrm{T}}\sum\eta \leq \frac{1}{\lambda}\xi^{\mathrm{T}}\xi + \lambda\eta^{\mathrm{T}}\eta, \quad (10)$$

especially, the inequality holds for $\lambda = 1$.

Lemma 3.4.[16] Given any real matrices Σ_1, Σ_2, Σ_3 of appropriate dimensions and a scalar $\varepsilon > 0$ such that $0 < \Sigma_3 = \Sigma_3^{\mathrm{T}}$, then, the following inequality holds:

$$\Sigma_1^{\mathrm{T}}\Sigma_2 + \Sigma_2^{\mathrm{T}}\Sigma_1 \leq \varepsilon\Sigma_1^{\mathrm{T}}\Sigma_3\Sigma_1 + \varepsilon^{-1}\Sigma_2^{\mathrm{T}}\Sigma_3^{-1}\Sigma_2, \quad (11)$$

especially, the inequality holds for $\varepsilon = 1, \Sigma_3 = E_0$(identity matrix).

Theorem 3.1. Assume that β_k and $\lambda(\alpha)$ are the maximum eigenvalue of $\frac{1}{2}(I + B_k)^{\mathrm{T}}(I + B_k)$ and $\frac{1}{2}\left(A_0^{\mathrm{T}} + A_0 + F^{\mathrm{T}}F + EE^{\mathrm{T}}\right)$ respectively, and $0 \leq \varepsilon_k = \tau_k - \tau_{k-1} < \infty (k = 1, 2, \cdots)$ are impulsive spacing. If there exists a constant $\delta > 1$ satisfying

$$\ln(\delta\beta_k) + \lambda(\alpha)\varepsilon_k \leq 0, k = 1, 2, \cdots, \quad (12)$$

then the system (3) is robust and asymptotically stable.

Proof. Let Lyapunov function $V(x) = \frac{1}{2}x^{\mathrm{T}}x$. When $t = \tau_k (k = 1, 2, \cdots)$, by Lemma 2.1, we obtain

$$\begin{aligned}V(\tau_k^+, x) =& \frac{1}{2}\left(x(\tau_k) + B_kx(\tau_k)\right)^{\mathrm{T}}\left(x(\tau_k) + B_kx(\tau_k)\right) = \frac{1}{2}x(\tau_k)^{\mathrm{T}}(I + B_k)^{\mathrm{T}}(I + B_k)x(\tau_k)\\ \leq& \lambda_{\max}\left(\frac{1}{2}(I + B_k)^{\mathrm{T}}(I + B_k)\right)x^{\mathrm{T}}(\tau_k)x(\tau_k) = \beta_kV(\tau_k, x). \end{aligned} \quad (13)$$

When $t \neq \tau_k$, note that $\Phi(x) = [x_3x_2 \quad -x_1x_3 \quad 0]^{\mathrm{T}}$, so $\Phi^{\mathrm{T}}(x)x + x^{\mathrm{T}}\Phi(x) = 0$. By the assumption and Lemmas 3.1, 3.2, 3.3 and 3.4, for all $(t, x) \in (\tau_{k-1}, \tau_k] \times R^n \quad (k = 1, 2, \cdots)$, we obtain

$$\begin{aligned}D^+V(t, x) =& \frac{1}{2}\left(\dot{x}^{\mathrm{T}}x + x^{\mathrm{T}}\dot{x}\right) = \frac{1}{2}\left(A_0x + E\Sigma Fx + \Phi(x)\right)^{\mathrm{T}}x + \frac{1}{2}x^{\mathrm{T}}\left(A_0x + E\Sigma Fx + \Phi(x)\right)\\ =& \frac{1}{2}\left(x^{\mathrm{T}}A_0^{\mathrm{T}} + x^{\mathrm{T}}F^{\mathrm{T}}\Sigma^{\mathrm{T}}E^{\mathrm{T}} + \Phi^{\mathrm{T}}(x)\right)x + \frac{1}{2}x^{\mathrm{T}}\left(A_0x + E\Sigma Fx + \Phi(x)\right)\\ =& \frac{1}{2}x^{\mathrm{T}}\left[A_0^{\mathrm{T}} + A_0\right]x + \frac{1}{2}\left[x^{\mathrm{T}}F^{\mathrm{T}}\Sigma^{\mathrm{T}}E^{\mathrm{T}}x + x^{\mathrm{T}}E\Sigma Fx\right]\\ \leq& \frac{1}{2}x^{\mathrm{T}}\left[A_0^{\mathrm{T}} + A_0\right]x + \frac{1}{2}\left[\frac{1}{2}\left((Fx)^{\mathrm{T}}(Fx) + (E^{\mathrm{T}}x)^{\mathrm{T}}(E^{\mathrm{T}}x)\right) + \frac{1}{2}\left((E^{\mathrm{T}}x)^{\mathrm{T}}(E^{\mathrm{T}}x) + (Fx)^{\mathrm{T}}(Fx)\right)\right]\\ =& \frac{1}{2}x^{\mathrm{T}}\left[A_0^{\mathrm{T}} + A_0\right]x + \frac{1}{2}\left[(Fx)^{\mathrm{T}}(Fx) + (E^{\mathrm{T}}x)^{\mathrm{T}}(E^{\mathrm{T}}x)\right]\\ =& \frac{1}{2}x^{\mathrm{T}}\left[A_0^{\mathrm{T}} + A_0 + F^{\mathrm{T}}F + EE^{\mathrm{T}}\right]x\\ \leq& \lambda_{\max}\left(\frac{1}{2}\left(A_0^{\mathrm{T}} + A_0 + F^{\mathrm{T}}F + EE^{\mathrm{T}}\right)\right)x^{\mathrm{T}}x = \lambda(\alpha)V(t, x). \end{aligned} \quad (14)$$

Let $k = 1$, for all $t \in (\tau_0, \tau_1]$, by (14), we have

$$V(t, x) \leq V(\tau_0, x)\exp\left(\lambda(\alpha)(t - \tau_0)\right).$$

Then

$$V(\tau_1, x) \leq V(\tau_0, x)\exp\left(\lambda(\alpha)(\tau_1 - \tau_0)\right). \quad (15)$$

By Eq.(13), we have

$$V(\tau_1^+, x) \leq \beta_1V(\tau_1, x) \leq \beta_1V(\tau_0, x)\exp\left(\lambda(\alpha)(\tau_1 - \tau_0)\right).$$

For all $t \in (\tau_1, \tau_2]$, we have

$$\begin{aligned}V(t, x) \leq& V(\tau_1^+, x)\exp\left(\lambda(\alpha)(t - \tau_1)\right)\\ \leq& \beta_1V(\tau_0, x)\exp\left(\lambda(\alpha)(t - \tau_0)\right).\end{aligned}$$

Similarly, for all k and $t \in (\tau_k, \tau_{k+1}]$, we have

$$V(t,x) \leq \beta_k \cdots \beta_2 \beta_1 V(\tau_0, x) \exp\left(\lambda(\alpha)(t-\tau_0)\right).$$

From Eq.(12), we have

$$\beta_k \exp(\lambda \varepsilon_k) \leq \frac{1}{\delta}, \quad k = 1, 2, \cdots.$$

Hence, for all $t \in (\tau_k, \tau_{k+1}]\,(k = 1, 2, \cdots)$, we have

$$\begin{aligned} V(t,x) \leq & \beta_k \cdots \beta_2 \beta_1 V(\tau_0, x) \exp\left(\lambda(\alpha)(t-\tau_0)\right) \\ = & V(\tau_0, x)\left[\beta_k \exp\left(\lambda(\alpha)\varepsilon_k\right)\right] \cdots \left[\beta_1 \exp\left(\lambda(\alpha)\varepsilon_1\right)\right] \\ & \times \exp\left(\lambda(\alpha)(t-\tau_k)\right) \\ \leq & V(\tau_0, x) \frac{1}{\delta^k} \exp\left(\lambda(\alpha)(t-\tau_k)\right). \end{aligned} \tag{16}$$

So, if $t \to \infty$, then $k \to \infty$ and $V(t,x) \to 0$. It follows that the conclusion holds.

Corollary 3.1. Assume that $\varepsilon_i \equiv \zeta$ for all $i \in N$. If $\zeta \leq \min_i \left(-\frac{1}{\lambda(\alpha)} \ln\left(\delta \beta_i\right)\right)$, the system (3) is robust and asymptotically stable, where $\lambda(\alpha)$, δ and $\beta_i (i = 1, 2, \cdots)$ are defined as in Theorem 3.1.

Corollary 3.2. Based on the conditions of corollary 3.1 and the applying the uniform impulsive control, i.e. for all k, $B_k = B$, if $\zeta \leq -\frac{1}{\lambda(\alpha)} \ln(\gamma\beta)$, the system (3) is robust and asymptotically stable, where $\beta = \lambda_{\max}\left(\Gamma^{-1}(I+B)^{\mathrm{T}} \Gamma (I+B)\right)$.

Remark. When the system parameters are constant, we can obtain the following results. Assume β_i and $\lambda(\alpha)$ are the maximum eigenvalue of $\frac{1}{2}(I+B_i)^{\mathrm{T}}(I+B_i)$ and $\left[A + A^{\mathrm{T}}\right]$ respectively, and $0 \leq \varepsilon_i = \tau_i - \tau_{i-1} < \infty$ $(i = 1, 2, \cdots)$ are impulsive spacing. If there exists a constant $\delta > 1$ satisfying $\ln(\delta\beta_i) + \lambda(\alpha)\varepsilon_i \leq 0$, $(i = 1, 2, \cdots)$, then the system (3) is robust and asymptotically stable.

4. Examples

Example 1. Impulsive control of PMSMs with fixed parameters values.

When system operating parameters are taken the fixed values: σ= 5.45, γ= 20, the PMSM displays chaos. The typical chaotic attractor is shown in Fig.1.

Let $B = \mathrm{diag}([-0.55, \ -0.75, \ -0.65])$ and $x_0 = [\,0.01 \ 0.01 \ 0.01\,]^{\mathrm{T}}$. The phase diagram of PMSM with impulsive effects is shown Fig.2.

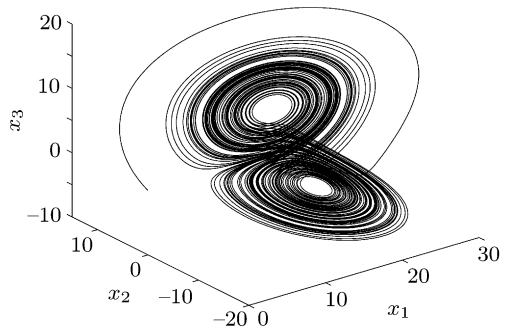


Fig.1. Typical chaotic attractor in PMSM with system fixed parameters: σ= 5.45, γ=20.

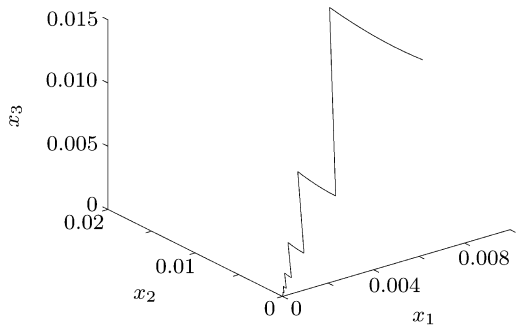


Fig.2. The phase diagram of PMSM with impulsive control.

Example 2. Impulsive control of PMSMs with parameters uncertainties.

In the system (11), we let $\sigma = 5.45 \pm 0.2$, $\gamma = 20 \pm 1$. During our experiment, we set σ in the interval $[\,5.25 \ 5.65\,]$ and set γ in the interval $[\,19 \ 21\,]$ randomly. The PMSM will display chaos without impulsive control. The chaotic attractor is shown in Fig.3.

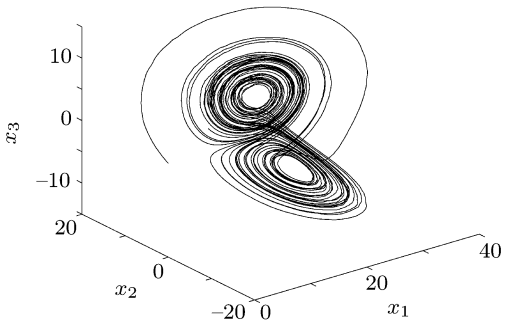


Fig.3. The chaotic attractor in PMSM with system parameters variation.

Set $B_k = \mathrm{diag}([-0.55, -0.75, -0.65])$ for all $k \in N$ and $x_0 = [\,0.01 \ 0.01 \ 0.01\,]^{\mathrm{T}}$. Assume that the impulsive spacing is equal, i.e. $\varepsilon_k \equiv \zeta$. Let Γ be an identity matrix and $\delta = 2$. We can easily obtain $\beta = 0.1012$, $\lambda(\alpha) = 14.0020$. By the Remark, if

$\zeta \leq -\frac{1}{\lambda(\alpha)} \ln(\delta\beta) = 0.1141$. Figures 4–8 illustrate the impulsive control phase diagram of PMSM with system parameters variation for different impulsive spacing ζ.

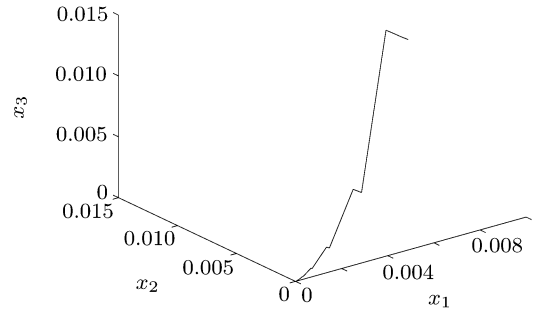


Fig.4. The phase diagram of PMSM with system parameters variation (ζ=0.01).

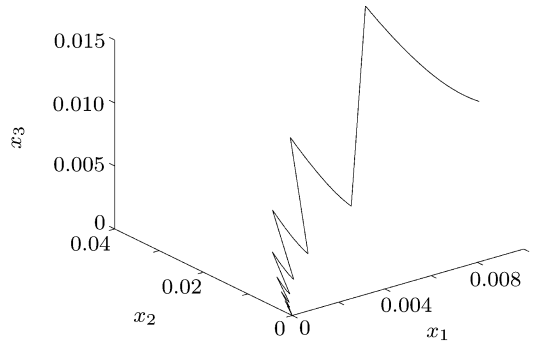


Fig.5. The phase diagram of PMSM with system parameters variation (ζ=0.1).

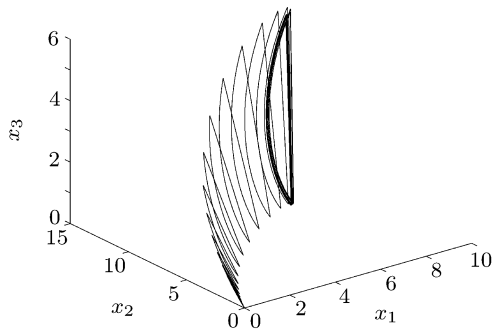


Fig.6. The phase diagram of PMSM with system parameters variation (ζ=0.2).

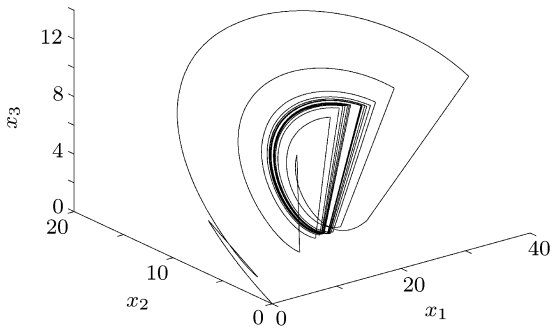


Fig.7. The phase diagram of PMSM with system parameters variation (ζ=0.5)

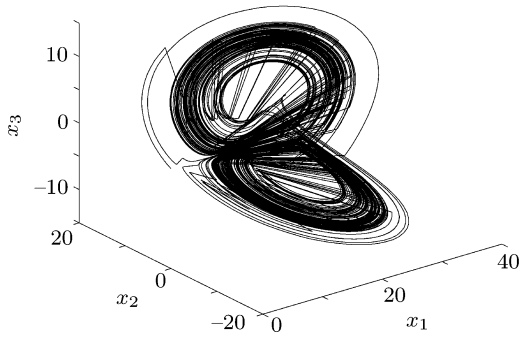


Fig.8. The phase diagram of PMSM with system parameters variation (ζ=0.8)

In summary, according to the above simulation experiments, we think that the proposed impulsive control for PMSM with system parameters variation and the corresponding results can provide the good effectiveness with PMSM control.

5. Conclusions

In this paper, we have investigated impulsive control of PMSMs. The parameters variation in the system is considered. The variable system parameters are formulated through matrix analysis. The sufficient conditions of the robust asymptotic stability are established by employing the method of Lyapunov functions, so as to apply more flexibly. The effectiveness of our methods has been shown by computer simulation. The illustrated example shows that the system is robust and asymptotic stable under the conditions of Theorem 3.1.

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