Vol 17 No 5, May 2008
1674-1056/2008/17(05)/1670-08

Chinese Physics B

© 2008 Chin. Phys. Soc.
and IOP Publishing Ltd

New results on global exponential stability of competitive neural networks with different time scales and time-varying delays*

Cui Bao-Tong(崔宝同), Chen Jun(陈 君)†, and Lou Xu-Yang(楼旭阳)

Research Center of Control Science and Engineering, Jiangnan University, Wuxi 214122, *China*

(Received 6 August 2007; revised manuscript received 20 September 2007)

This paper studies the global exponential stability of competitive neural networks with different time scales and time-varying delays. By using the method of the proper Lyapunov functions and inequality technique, some sufficient conditions are presented for global exponential stability of delay competitive neural networks with different time scales. These conditions obtained have important leading significance in the designs and applications of global exponential stability for competitive neural networks. Finally, an example with its simulation is provided to demonstrate the usefulness of the proposed criteria.

Keywords: competitive neural network, different time scale, global exponential stability, delay
PACC: 0545

1. Introduction

The competitive neural networks with different time scales are extensions of Grossberg's shunting network (Grossberg, 1976) and Amari's model for primitive neuronal competition (Amari, 1983).[1−3] In recent years, a class of competitive neural network with different time scales and delay has attracted attentions of many researchers and used in many fields such as processing, pattern recognition, neural computing, and control.[1−23]

The neural network can model the dynamics of cortical cognitive maps with unsupervised synaptic modifications. Therefore, the competitive neural network model can be characterized by a system of differential equations with two types of variables, the short-term memory (STM) variables describing the fast neural activity and the long-term memory (LTM) variables describing the slow unsupervised synaptic modifications.[3] On the other hand, there are two time scales in these neural networks, one corresponds to the fast changes of the neural network states, another corresponds to the slow changes of the synapses by external stimuli.[3]

We consider the system that is similar to the spatiotemporal system proposed by Amari. The neural network can be mathematically expressed as

$$\begin{aligned}
&\text{STM}: \varepsilon \dot{x}_j(t) \\
&= -a_j x_i(t) + \sum_{i=1}^{N} D_{ji} f(x_j(t)) \\
&\quad + \sum_{i=1}^{N} D^{\tau}_{ji} f(x_j(t-\tau(t))) \\
&\quad + B_j \sum_{i=1}^{\bar{p}} m_{ji} y_i, \\
&\text{LTM}: \dot{m}_j(t) = -m_{ij}(t) + y_j f(x_j(t)), \\
&i = 1, 2, \cdots, \bar{p}, j = 1, 2, \cdots, N, t \in [0, +\infty), \quad (1)
\end{aligned}$$

where $x_j(t)$ is the neuron current activity level, $f(x_j(t))$ is the output of neurons, $m_{ij}(t)$ is the synaptic efficiency, y_j is the constant external stimulus, D_{ji} represents the connection weight between the ith neuron and the jth neuron, $\bar{p}$ denotes the number of the constant external stimulus, D^{τ}_{ji} is the synaptic weight of delayed feedback, a_j is a positive function, representing the time constant of the neuron, $\tau(t) > 0$ characterizes a time-varying differentiable function and $\tau = \sup_{t\geq 0}\{\tau(t)\}, \mu = \sup_{t\geq 0}\{\dot{\tau}(t)\} < 1$, B_j is the strength of the external stimulus, ε is the time scale of STM

*Project supported by National Natural Science Foundation of China (Grant No 60674026), the Jiangsu Provincial Natural Science Foundation of China (Grant No BK2007016) and Program for Innovative Research Team of Jiangnan University of China.
†Corresponding author. E-mail: chenjun8348@163.com

state (Meyer-Baese *et al*, 2003 [4]).

Recently, several efforts have been devoted to the study of general competitive neural networks by using monotone dynamical system theory.[3−6] A competitive neural network model was recently proposed to describe the dynamics of cortical maps by Lu and He,[3] where there are two types of memories: LTM and STM memories. They analysed the global exponential stability of the delayed competitive neural networks with different time scales by using the nonsmooth analysis techniques to prove the existence and uniqueness of the equilibrium, and derived a new sufficient condition ensuring global exponential stability of the networks. At the same time, the delays in artificial neural networks are usually time-varying. In Ref.[3], the exponential synchronization problem for a class of competitive neural networks was investigated. Moreover, some exponential synchronization criteria were devised by Lyapunov functionals, linear matrix inequality (LMI) approach, the Leibniz–Newton formula.

As far as we know, this kind of competitive dynamics is seldom considered. Our objective in this paper is to study further exponential stability of the competitive neural networks with different time scales and time-varying delays. And after setting, where $S_i(t) = \sum_{i=1}^{\bar{p}} m_{ij}(t)y_j = y^{\mathrm{T}}m_i(t)$, $y = (y_1, \cdots, y_{\bar{p}})$, $m_i(t) = (m_{i1}, \cdots, m_{i\bar{p}})$ and summing up the LTM over j, we can rewrite the network Eq.(1) as the state-space form as follows:

$$\begin{aligned}
\varepsilon\dot{x}_j(t) &= -a_j x_i(t) + \sum_{i=1}^{N} D_{ji} f(x_j(t)) \\
&\quad + \sum_{i=1}^{N} D^{\tau}_{ji} f(x_j(t-\tau(t))) + B_j S_i(t), \\
\dot{S}_i(t) &= -s_i(t) + |y|^2 f(x_j(t)), \\
&\quad i, j = 1, 2, \cdots, N, t \in [0, +\infty), \qquad (2)
\end{aligned}$$

where $|y|^2 = y_1^2 + \cdots + y_p^2$ is a constant. Without loss of generality, the input stimulus vector y is assumed to be normalized with unit magnitude $|y|^2 = 1$, and the fast time scale parameter ε is also assumed to be unit, then the network is simplified to

$$\begin{aligned}
\varepsilon\dot{x}_j(t) &= -a_j x_i(t) + \sum_{i=1}^{N} D_{ji} f(x_j(t)) \\
&\quad + \sum_{i=1}^{N} D^{\tau}_{ji} f(x_j(t-\tau(t))) + B_j S_i(t), \\
\dot{S}_i(t) &= -s_i(t) + f(x_j(t)),
\end{aligned}$$

$$i, j = 1, 2, \cdots, N, t \in [0, +\infty). \qquad (3)$$

The initial conditions of the two competitive neural networks Eqs.(2) and (3) are given, respectively, as follows:

$$x_i(t) = \phi_i(t) \in Z([-\tau, 0] : R), i = 1, 2, \cdots, N,$$

and

$$\bar{x}_i(t) = \psi_i(t) \in Z([-\tau, 0] : R), i = 1, 2, \cdots, N,$$

where $Z([-\tau, 0] : R)$ denotes the set of all continuous functions from $[-\tau, 0]$ to R.

The rest of paper is organized as follows. In Section 2, we introduce some notations, lemmas and definitions. Then, in Section 3, main results are presented for the global exponential stability of competitive neural networks with different time scales and delays. In Section 4, an example and its picture of simulation is given to illustrate our results. Finally, we will give some conclusions in Section 5.

2. Some preliminaries

Assume that the functions a_j and f_j satisfies the following conditions:

(H1) Each function $a_j : R \longrightarrow R$ is locally Lipschitz and nondecreasing, and there exists a positive real $\bar{a}_j > 0$, such that $\dot{a_j}(x) \geq \bar{a}_j$, for any $x \in R$ at which $\bar{a}_j$ is differentiable.

(H2) Each function $f_j : R \longrightarrow R$ is monotonic nondecreasing and globally Lipschitz, i.e. there exists a positive real $f_j > 0$, such that $0 \leq (f_j(x) - f_j(y))/(x - y) \leq k_j$, for any $x, y \in R, x \neq y$.

(H2)* $0 < (f_j(x) - f_j(y))/(x - y) \leq k_j$, for any $x, y \in R, x \neq y$.

(H3) $\tau = \sup_{t\geq 0}\{\tau(t)\}, \mu = \sup_{t\geq 0}\{\dot{\tau}(t)\} < 1$, for $j = 1, 2, \cdots, N$, and $t \geq 0$, where τ and μ are constants.

Throughout this paper, we will use the following notations: $\bar{B}^{\mathrm{T}}, \bar{B}^{-1}, \lambda_{\mathrm{m}}(\bar{B}), \lambda_{\mathrm{M}}(\bar{B})$ denote, respectively, the transpose of , the inverse of, the minimum eigenvalue of, the maximum eigenvalue of a square matrix B. The notation $\bar{B} > 0$ ($\bar{B} < 0$) means that $\bar{B}$ is symmetric and positive definite (negative definite). Denote $A = \mathrm{diag}(a_1, a_2, \cdots, a_N)$; $K = \mathrm{diag}(k_1, k_2, \cdots, k_N)$; $D = (D_{ij})_{N\times N}$; $D^{\tau} = (D^{\tau}_{ij})_{N\times N}$, $B = \mathrm{diag}(B_1, B_2, \cdots, B_N)$.

Definition 2.1[3] The equilibrium $x^* = (x_1^*,\ x_2^*,\ \cdots,\ x_m^*)$ of the system (3) under the initial value condition is said to be globally exponentially stable, if there exist constants $M \geq 1, \epsilon > 0$ such that for all $t \geq 0$

$$\sum_{i=1}^{m} |(x_i(t) - x_i^*)| \leq M \left[\sum_{i=1}^{m} \sup_{s\in[-\tau,0]} |(x_i(s) - x_i^*)| \mathrm{e}^{-\epsilon t} \right].$$

Lemma 2.1[13] For any given real matrices Q_1, Q_2, Q_3 with $Q_3 = Q_3^{\mathrm{T}}$, the following inequality holds:

$$-Q_2^{\mathrm{T}} Q_3^{-1} Q_2 + Q_2^{\mathrm{T}} Q_1 + Q_1^{\mathrm{T}} Q_2 \leq Q_1^{\mathrm{T}} Q_3 Q_1.$$

Lemma 2.2[7] There exists an equilibrium point for the competitive neural network with different time scales and delays described by Eq.(2).

3. Main results

The main results of this paper are presented as follows:

Theorem 3.1 Suppose that the assumptions (H1)–(H3) and the condition of Lemma 2.2 hold. The global exponential stability of the unique equilibrium (x^0, S^0) is ensured, i.e. all solutions of the delayed neural network Eq.(3) converge to the unique equilibrium (x^0, S^0) exponentially fast, if there exist a positive definite matrix P and a positive diagonal matrix C, such that the following matrices are satisfied:

$$\begin{aligned} \omega = & -2k_j CA + 2k_j^2 CD + k_j^2 CB \\ & + \eta (D^\tau \mathrm{e}^{\varepsilon\tau})^{\mathrm{T}} P D^\tau \mathrm{e}^{\varepsilon\tau} k_j^2 \\ & + \eta^{-1} \frac{1}{1-\mu} k_j^2 C P^{-1} C \leq 0, \\ AB - 2\bar{A} + \bar{A} k_j + & k_j \alpha C B \leq 0. \end{aligned}$$

Proof According to Theorem 3.1, the competitive neural network Eq.(3) with delay has a unique equilibrium (x^0, S^0). Now we make the variable changes $\phi_i(t) = (x_i(t) - x_i^0)\mathrm{e}^{\varepsilon t}$, $\psi_i(t) = (S_i(t) - S_i^0)\mathrm{e}^{\varepsilon t}$ to shift the origin to (x^0, S^0).

From Eq.(3), we have

$$\begin{aligned} \dot{\phi}_i(t) =\ & -\beta_i \phi(t) + \varepsilon\phi(t) + \sum_{i=1}^{N} D_{ij} g_j(\phi(t)) \\ & + \sum_{i=1}^{N} D_{ij}^\tau \mathrm{e}^{\varepsilon\tau} g_j(\phi(t-\tau(t))) + B_j \psi_i(t), \\ \dot{\psi}_i(t) =\ & (-1+\varepsilon)\psi_i(t) + g_j(\phi(t)), \\ i, j =\ & 1, 2, \cdots, N, t \in [0, +\infty), \end{aligned}$$

where the functions $\beta_i(\phi_i(t)) = [a_j(\phi_i(t)\mathrm{e}^{-\varepsilon t} + x_i^0) - a_j(x_i^0)]\mathrm{e}^{\varepsilon t}$, $g_i(\phi_i(t)) = [f_j(\phi_i(t))\mathrm{e}^{-\varepsilon t} + x_i^0) - f_j(x_i^0)]\mathrm{e}^{\varepsilon t}$, satisfying $\beta_i(0) = 0$, $g_i(0) = 0$, $0 \leq \phi_i(t) g(\phi_i(t)) \leq k_j \phi_i^2(t)$, and according to the Lebourg Theorem in Ref.[8], there exist $d_i \geq \bar{a}_j$, such that $\beta_i \phi(t) = d_i \phi(t), i = 1, 2, \cdots, N$.

Consider the Lyapunov functional as following:

$$\begin{aligned} V(t) =\ & \phi^{\mathrm{T}}(t) A \phi(t) + \psi^{\mathrm{T}}(t) \bar{A} \psi(t) \\ & + 2\alpha \sum_{i=1}^{n} c_i \int_0^{\phi_i(t)} g_i(s) \mathrm{d}s \\ & + \alpha\eta \int_{t-\tau(t)}^{t} g^{\mathrm{T}}(x(s)) (D^\tau \mathrm{e}^{\varepsilon\tau})^{\mathrm{T}} \\ & \times P D^\tau \mathrm{e}^{\varepsilon\tau} g(x(s)) \mathrm{d}s \\ & + \frac{1}{1-\mu} \int_{t-\tau(t)}^{t} g^{\mathrm{T}}(\phi(s)) \\ & \times (\gamma I + (D^\tau \mathrm{e}^{\varepsilon\tau})^{\mathrm{T}} D^\tau \mathrm{e}^{\varepsilon\tau}) g(\phi(s)) \mathrm{d}s, \quad (4) \end{aligned}$$

where $P = P^{\mathrm{T}} > 0$, $c_i > 0$, $i = 1, 2, \cdots, N$, and α, A, $\bar{A}$, η, γ, are positive.

Denote $A = \beta$. Calculating the derivative of $V(t)$ along the solution of system Eq.(3) and adding and subtracting the positive term $\alpha\eta^{-1} \frac{1}{1-\dot{\tau}(t)} g^{\mathrm{T}}(\phi(t)) C P^{-1} C g(\phi(t))$, we obtain

$$\begin{aligned} \dot{V}(t) \leq & -\phi^{\mathrm{T}}(t) A^2 \phi(t) - \phi^{\mathrm{T}}(t) A^2 \phi(t) \\ & + 2\phi^{\mathrm{T}}(t) A D g(\phi(t)) \\ & + 2\phi^{\mathrm{T}}(t) A D^\tau \mathrm{e}^{\varepsilon\tau} g(\phi(t-\tau(t))) \\ & + 2\phi^{\mathrm{T}}(t) A B \psi(t) \\ & - 2\psi^{\mathrm{T}}(t) \bar{A}(1-\varepsilon)\psi(t) + 2\psi^{\mathrm{T}}(t) \bar{A} g(\phi(t)) \\ & - 2\alpha g^{\mathrm{T}}(\phi(t)) C A \phi(t) \\ & + 2\alpha g^{\mathrm{T}}(\phi(t)) C D g(\phi(t)) \\ & + 2\alpha g^{\mathrm{T}}(\phi(t)) C D^\tau \mathrm{e}^{\varepsilon\tau} g((\phi(t-\tau(t))) \\ & + 2\alpha g^{\mathrm{T}}(\phi(t)) C B \psi(t) \\ & + \alpha\eta g^{\mathrm{T}}(\phi(t)) (D^\tau \mathrm{e}^{\varepsilon\tau})^{\mathrm{T}} P D^\tau \mathrm{e}^{\varepsilon\tau} g(\phi(t)) \\ & - \alpha\eta(1-\dot{\tau}(t)) g^{\mathrm{T}}(\phi(t-\tau(t))) (D^\tau \mathrm{e}^{\varepsilon\tau})^{\mathrm{T}} \\ & \times P D^\tau \mathrm{e}^{\varepsilon\tau} g(\phi(t-\tau(t))) + \frac{1}{1-\mu} g^{\mathrm{T}}(\phi(t)) \\ & \times (\gamma I + (D^\tau \mathrm{e}^{\varepsilon\tau})^{\mathrm{T}} D^\tau \mathrm{e}^{\varepsilon\tau}) g(\phi(t)) \\ & - \frac{1-\dot{\tau}(t)}{1-\mu} g^{\mathrm{T}}(\phi(t-\tau(t))) (\gamma I + (D^\tau \mathrm{e}^{\varepsilon\tau})^{\mathrm{T}} \\ & \times D^\tau \mathrm{e}^{\varepsilon\tau}) g(\phi(t-\tau(t))) \end{aligned}$$

$$+\alpha\eta^{-1}\frac{1}{1-\dot\tau(t)}g^{\mathrm T}(\phi(t))CP^{-1}Cg(\phi(t))$$
$$-\alpha\eta^{-1}\frac{1}{1-\dot\tau(t)}g^{\mathrm T}(\phi(t))CP^{-1}Cg(\phi(t)). \quad (5)$$

In the light of Lemma 2.1, we can write the following equalities:

$$-\phi^{\mathrm T}(t)A^2\phi(t)+2\phi^{\mathrm T}(t)ADg(\phi(t))$$
$$\le g^{\mathrm T}\phi(t))D^{\mathrm T}Dg(\phi(t)), \quad (6)$$
$$-\phi^{\mathrm T}(t)A^2\phi(t)+2\phi^{\mathrm T}(t)AD^{\tau}\mathrm e^{\varepsilon\tau}g(\phi(t-\tau(t)))$$
$$\le g^{\mathrm T}(\phi(t-\tau(t)))(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}D^{\tau}\mathrm e^{\varepsilon\tau}g(\phi(t-\tau(t))), \quad (7)$$
$$2\alpha g^{\mathrm T}(\phi(t))CD^{\tau}\mathrm e^{\varepsilon\tau}g(\phi(t-\tau(t)))$$
$$-\alpha\eta^{-1}\frac{1}{1-\dot\tau(t)}g^{\mathrm T}(\phi(t))CP^{-1}Cg(\phi(t))$$
$$\le \alpha\eta(1-\dot\tau(t))g^{\mathrm T}(\phi(t-\tau(t)))(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}$$
$$\times PD^{\tau}\mathrm e^{\varepsilon\tau}g(\phi(t-\tau(t))). \quad (8)$$

From (H2), we have

$$-2\alpha g^{\mathrm T}(\phi(t))CA\phi(t)\le -2\alpha g^{\mathrm T}(\phi(t))CAk_j^{-1}g\phi(t). \quad (9)$$

By using Eqs.(6)–(9), from Eq.(5) we can conclude that

$$\begin{aligned}\dot V(t)\le{}& g^{\mathrm T}\phi(t))D^{\mathrm T}Dg(\phi(t)))+g^{\mathrm T}(\phi(t-\tau(t)))\\ &\times(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}D^{\tau}\mathrm e^{\varepsilon\tau}g(\phi(t-\tau(t)))\\ &+2\phi^{\mathrm T}(t)AB\psi(t)-2\psi^{\mathrm T}(t)\bar A(1-\varepsilon)\psi(t)\\ &+2\psi^{\mathrm T}(t)\bar Ag(\phi(t))\\ &-2\alpha g^{\mathrm T}(\phi(t))CAk_j^{-1}g_j(\phi(t))\\ &+2\alpha g^{\mathrm T}(\phi(t))CDg(\phi(t))+2\alpha g^{\mathrm T}(\phi(t))CB\psi(t)\\ &+\alpha\eta(1-\dot\tau(t))g^{\mathrm T}(\phi(t-\tau(t)))\\ &\times(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}PD^{\tau}\mathrm e^{\varepsilon\tau}g(\phi(t-\tau(t)))\\ &-\alpha\eta(1-\dot\tau(t))g^{\mathrm T}(\phi(t-\tau(t)))\\ &\times(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}PD^{\tau}\mathrm e^{\varepsilon\tau}g(\phi(t-\tau(t)))\\ &+\alpha\eta g^{\mathrm T}(\phi(t))(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}PD^{\tau}\mathrm e^{\varepsilon\tau}g(\phi(t))\\ &+\frac{1}{1-\mu}g^{\mathrm T}(\phi(t))(\gamma I+(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}D^{\tau}\mathrm e^{\varepsilon\tau})g(\phi(t))\\ &-\frac{1-\dot\tau(t)}{1-\mu}g^{\mathrm T}(\phi(t-\tau(t)))\\ &\times(\gamma I+(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}D^{\tau}\mathrm e^{\varepsilon\tau})g(\phi(t-\tau(t)))\\ &+\alpha\eta^{-1}\frac{1}{1-\dot\tau(t)}g^{\mathrm T}(\phi(t))CP^{-1}Cg(\phi(t)). \quad (10)\end{aligned}$$

Since $\dfrac{1-\dot\tau(t)}{1-\mu}\ge 1$, it follows that

$$-\frac{1-\dot\tau(t)}{1-\mu}g^{\mathrm T}(\phi(t-\tau(t)))(\gamma I+(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}PD^{\tau}\mathrm e^{\varepsilon\tau})$$
$$\times g(\phi(t-\tau(t)))+g^{\mathrm T}(\phi(t-\tau(t)))(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}D^{\tau}\mathrm e^{\varepsilon\tau}$$
$$\times g(\phi(t-\tau(t)))\le 0. \quad (11)$$

Hence, it is easy to verify that

$$\begin{aligned}\dot V(t)\le{}& g^{\mathrm T}\phi(t))D^{\mathrm T}Dg(\phi(t)))\\ &+2\phi^{\mathrm T}(t)AB\psi(t)-2\psi^{\mathrm T}(t)\bar A(1-\varepsilon)\psi(t)\\ &+2\psi^{\mathrm T}(t)\bar Ag(\phi(t))\\ &-2\alpha g^{\mathrm T}(\phi(t))CAk_j^{-1}g_j(\phi(t))\\ &+2\alpha g^{\mathrm T}(\phi(t))CDg(\phi(t)))\\ &+2\alpha g^{\mathrm T}(\phi(t))CB\psi(t)\\ &+\alpha\eta g^{\mathrm T}(\phi(t))(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}PD^{\tau}\mathrm e^{\varepsilon\tau}g(\phi(t))\\ &+\frac{1}{1-\mu}g^{\mathrm T}(\phi(t))(\gamma I+(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}D^{\tau}\mathrm e^{\varepsilon\tau})g(\phi(t))\\ &+\alpha\eta^{-1}\frac{1}{1-\dot\tau(t)}g^{\mathrm T}(\phi(t))CP^{-1}Cg(\phi(t)). \quad (12)\end{aligned}$$

Since

$$\begin{aligned}&2\phi^{\mathrm T}(t)AB\psi(t)\\ &\le \phi^{\mathrm T}(t)AB\phi(t)+\psi^{\mathrm T}(t)AB\psi(t), \quad (13)\end{aligned}$$

$$\begin{aligned}&2\psi^{\mathrm T}(t)\bar Ag(\phi(t))\\ &\le \psi^{\mathrm T}(t)\bar Ak_j\psi(t)+\phi^{\mathrm T}(t)\bar Ak_j\phi(t), \quad (14)\end{aligned}$$

$$\begin{aligned}&2\alpha g^{\mathrm T}(\phi(t))CB\psi(t)\\ &\le \alpha k_j\phi_i^{\mathrm T}(t)CB\phi(t)+\alpha k_j\psi^{\mathrm T}(t)CB\psi(t), \quad (15)\end{aligned}$$

it follows from Eqs.(12)–(15) that

$$\begin{aligned}&\dot V(t)\\ &\le \phi^{\mathrm T}(t)\Big[k_j^2D^{\mathrm T}D+AB+\bar Ak_j\\ &\quad+\frac{1}{1-\mu}k_j(\gamma I+(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}D^{\tau}\mathrm e^{\varepsilon\tau})k_j\Big]\phi(t)\\ &\quad-\alpha\phi^{\mathrm T}(t)(-\omega)\phi(t)\\ &\quad+\psi^{\mathrm T}(t)\Big[AB-2\bar A(1-\varepsilon)+\bar Ak_j+k_j\alpha CB\Big]\psi(t)\\ &\le \lambda_{\mathrm M}\Big[k_j^2D^{\mathrm T}D+AB+\bar Ak_j\\ &\quad+\frac{1}{1-\mu}k_j(\gamma I+(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}D^{\tau}\mathrm e^{\varepsilon\tau})k_j\Big]\\ &\quad\times\|\phi(t)\|_2^2-\alpha\lambda_m(-\omega)\|\phi(t)\|_2^2\\ &\quad+\Big[AB-2\bar A(1-\varepsilon)+\bar Ak_j+\alpha CB\Big]\|\psi(t)\|_2^2. \quad (16)\end{aligned}$$

The choice

$$\begin{aligned}\alpha >{}& \frac{\lambda_M}{\lambda_m(-\omega)}\Big(k_j^2D^{\mathrm T}D+AB+\bar Ak_j\\ &+\frac{(\gamma I+(D^{\tau}\mathrm e^{\varepsilon\tau})^{\mathrm T}D^{\tau}\mathrm e^{\varepsilon\tau})}{1-\mu}k_j^2\Big)\\ >{}& 0,\end{aligned}$$

ensures that $\dot{V}(t) < 0$, $\forall g(\phi(t)) \neq 0$. Note that $\forall g(\phi(t)) \neq 0$ implies that $\phi(t) \neq 0$. Now let $\forall g(\phi(t)) = 0$, $\psi(t) = 0$ and $\phi(t) \neq 0$. In this case, $\dot{V}(t)$ is in the following form:

$$\begin{aligned}\dot{V}(t) \leq & -\phi^{\mathrm{T}}(t)A^2\phi(t) - \phi^{\mathrm{T}}(t)A^2\phi(t)\\ & +2\phi^{\mathrm{T}}(t)AD^{\tau}\mathrm{e}^{\varepsilon\tau}g(\phi(t-\tau(t)))\\ & -\alpha\eta g^{\mathrm{T}}(\psi(t-\tau(t)))(D^{\tau}\mathrm{e}^{\varepsilon\tau})^{\mathrm{T}}\\ & \times PD^{\tau}\mathrm{e}^{\varepsilon\tau}g(\psi(t-\tau(t)))\\ & -\frac{1-\dot{\tau}(t)}{1-\mu}g^{\mathrm{T}}(\phi(t-\tau(t)))\\ & \times(\gamma I + (D^{\tau}\mathrm{e}^{\varepsilon\tau})^{\mathrm{T}}D^{\tau}\mathrm{e}^{\varepsilon\tau})\\ & \times g(\phi(t-\tau(t)). \end{aligned} \tag{17}$$

Since

$$(D^{\tau})^{\mathrm{T}}PD^{\tau} \geq 0, \eta > 0, \alpha > 0,$$

$\dot{V}(t)$ satisfies

$$\begin{aligned}\dot{V}(t) \leq & -\phi^{\mathrm{T}}(t)A^2\phi(t) - \phi^{\mathrm{T}}(t)A^2\phi(t)\\ & +2\phi^{\mathrm{T}}(t)AD^{\tau}\mathrm{e}^{\varepsilon\tau}g(\phi(t-\tau(t)))\\ & -g^{\mathrm{T}}(\phi(t-\tau(t)))(D^{\tau}\mathrm{e}^{\varepsilon\tau})^{\mathrm{T}}\\ & \times D^{\tau}\mathrm{e}^{\varepsilon\tau}g(\phi(t-\tau(t)). \end{aligned} \tag{18}$$

Using Eq.(7) in the above inequality, we obtain

$$\dot{V}(t) \leq -\phi^{\mathrm{T}}(t)A^2\phi(t) < 0. \tag{19}$$

Now consider the case where $g(\phi(t)) = \phi(t) = 0$, $\psi(t) = 0$. In this case, $\dot{V}(t)$ takes the form:

$$\begin{aligned}\dot{V}(t) \leq & -\alpha\eta g^{\mathrm{T}}(\psi(t-\tau(t)))(D^{\tau}\mathrm{e}^{\varepsilon\tau})^{\mathrm{T}}\\ & \times PD^{\tau}\mathrm{e}^{\varepsilon\tau}g(\psi(t-\tau(t)))\\ & -\frac{1-\dot{\tau}(t)}{1-\mu}g^{\mathrm{T}}(\phi(t-\tau(t)))(\gamma I\\ & +(D^{\tau}\mathrm{e}^{\varepsilon\tau})^{\mathrm{T}}D^{\tau}\mathrm{e}^{\varepsilon\tau})g(\phi(t-\tau(t)). \end{aligned} \tag{20}$$

Since $(D^{\tau})^{\mathrm{T}}D^{\tau} > 0$, it directly follows that

$$\dot{V}(t) < 0, \forall g(\phi(t-\tau(t))) \neq 0.$$

Hence, we have proved that $\dot{V}(t) = 0$ if and only if $g(\phi(t)) = \phi(t) = \psi(t) = g(\phi(t) - \tau(t)) = 0$, otherwise $\dot{V}(t) < 0$. On the other hand, $V(t)$ is radially unbounded, that is $V(t) \longrightarrow \infty$ as $\|\phi\|(t) \longrightarrow \infty$. Thus, from standard Lyapunov stability theorems (see, e.g., Ref.[9], Corollary 3.2, Chapter 3), it follows that the origin of system (3) is globally exponentially stable.

Remark 3.1 In the Theorem 3.1, the constraint conditions on the competitive parameters of system (3) is imposed. Furthermore, it should be noted that the functions satisfying the condition (H2) are less conservative than those satisfying condition (H2*).

Theorem 3.2 Assume that (H1)–(H3) hold. If the system parameters $d_i, D_{ij}, D_{ij}^{\tau}$ satisfy

$$\begin{aligned}& -d_i + \sum_{i=1}^{N} D_{ij}k_j + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}|D_{ij}|^{2\gamma_j}k_j^{2\delta_j}\mathrm{e}^{\varepsilon\tau}\frac{1}{1-\mu}\\ & +\frac{1}{2}\sum_{i=1}^{N}|D_{ij}^{\tau}|^{2\gamma_j'}k_j^{2\delta_j'}\mathrm{e}^{\varepsilon\tau} + \frac{1}{2}k_j + \frac{1}{2}B_j \leq 0,\\ & -1 + \frac{1}{2}k_j + \frac{1}{2}B_j \leq 0, \end{aligned}$$

where $\delta_j + \delta_j{}' = 1, \gamma_j + \gamma_j{}' = 1, i, j = 1, 2, \cdots, N$, then the equilibrium for system (3) is globally exponentially stable.

Proof Consider the following Lyapunov functional:

$$\begin{aligned}V(t) = & \sum_{i=1}^{N}(|\phi(t)^2| + |\psi_i(t)^2|)\\ & +\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{1-\mu}|D_{ij}^{\tau}|^{2\gamma_j}k_j^{2\delta_j}\mathrm{e}^{\varepsilon\tau}\\ & \times\int_{t-\tau(t)}^{t}|\phi(t)^2|\mathrm{d}t. \end{aligned} \tag{21}$$

Then, we compute the derivative of $V(t)$ along the trajectories of Eq.(3),

$$\begin{aligned}\dot{V}(t) \leq & \sum_{i=1}^{N}\Bigg[-d_i + \varepsilon + \sum_{i=1}^{N}D_{ij}k_j\\ & +\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}|D_{ij}|^{2\gamma_j}k_j^{2\delta_j}\mathrm{e}^{\varepsilon\tau}\frac{1}{1-\mu}\Bigg]2\phi(t)^2\\ & +[-1+\varepsilon]2\psi_i(t)^2 + \sum_{i=1}^{N}2\phi(t)B_j\psi_i(t)\\ & +2\phi(t)\sum_{i=1}^{N}|D_{ij}^{\tau}|\mathrm{e}^{\varepsilon\tau}k_j(\phi(t-\tau(t)))\\ & +2\psi_i(t)k_j\phi(t)\\ & -\sum_{i=1}^{N}\sum_{j=1}^{N}|D_{ij}|^{2\gamma_j}k_j^{2\delta_j}\mathrm{e}^{\varepsilon\tau}|\psi_i(t-\tau(t))^2|. \end{aligned} \tag{22}$$

Since

$$\begin{aligned}& 2\phi(t)\sum_{i=1}^{N}|D_{ij}^{\tau}|\mathrm{e}^{\varepsilon\tau}k_j\phi(t-\tau(t))\\ \leq & \sum_{i=1}^{N}|D_{ij}^{\tau}|^{2\gamma_j'}k_j^{2\delta_j'}\mathrm{e}^{\varepsilon\tau}\phi(t)^2\\ & +\sum_{i=1}^{N}|D_{ij}^{\tau}|^{2\gamma_j}\mathrm{e}^{\varepsilon\tau}k_j^{2\delta_j}\phi(t-\tau(t))^2, \end{aligned}$$

$$2\psi_i(t)k_j\phi(t) \le k_j\psi_i(t)^2 + k_j\phi(t)^2,$$
$$2\phi(t)B_j\psi_i(t) \le B_j\phi(t)^2 + B_j\psi_i(t)^2, \quad (23)$$

from Eq.(22) we can show that

$$\dot{V}(t) \ \le 0. \quad (24)$$

To this end, from Theorem 3.2, we can conclude that the dynamics of the competitive delayed neural network Eq.(3) is globally exponentially stable, and the proof of Theorem 3.2 is completed.

Remark 3.2 In this paper, our results impose milder constraints on model Eq.(3) and the new criterion can be applied to other neural networks. On the other hand, In Theorem 3.2, the exponential of the parameters $|D_{ij}^{\tau}|$, k_j are variable. According to Theorem 3.2, when the conditions $\delta_j + \delta_j{}' = 1$, $\gamma_j + \gamma_j{}' = 1$, $i = j = 1, 2, \cdots, n$, are satisfied, then the equilibrium for system (3) is globally stable. Obviously, the constraints on the parameters of the networks can be relaxed. Moreover, the time-varying delays are investigated in this model, the earlier results are extended and improved.

Theorem 3.3 For the competitive neural networks with different time scales system (3), assume that (H1)–(H3) hold. If the system parameters satisfy

$$-d_i + \sum_{i=1}^{N} D_{ij}k_j + \frac{1}{p}\sum_{i=1}^{N}\sum_{j=1}^{N}|D_{ij}^{\tau}|k_j e^{\varepsilon\tau}\frac{1}{1-\mu}$$
$$+(p-1)\frac{1}{p}\sum_{i=1}^{N}|D_{ij}^{\tau}|e^{\varepsilon\tau} + \frac{1}{p}k_j + (p-1)\frac{1}{p}B_j \le 0,$$
$$-1 + (p-1)\frac{1}{p}k_j + \frac{1}{p}B_j \le 0, i, j = 1, 2, \cdots, N,$$

for $p \ge 1$, then the equilibrium for system (3) is globally exponentially stable.

Consider the following Lyapunov functional

$$V(t) = \sum_{i=1}^{N}(|\phi(t)^p| + |\psi_i(t)^p|) + \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{1-\mu}|D_{ij}^{\tau}|k_j e^{\varepsilon\tau} \times \int_{t-\tau(t)}^{t}|\phi(t)^p|, \quad (25)$$

then the derivative of this functional along the solution of system (3) is

$$\dot{V}(t) \le \sum_{i=1}^{N}\Bigg[-d_i + \varepsilon + \sum_{i=1}^{N}D_{ij}k_j + \frac{1}{p}\sum_{i=1}^{N}\sum_{j=1}^{N}|D_{ij}^{\tau}|k_j e^{\varepsilon\tau}\frac{1}{1-\mu}\Bigg]p|\phi(t)^p|$$
$$+[-1+\varepsilon]p|\psi_i(t)^p| + \sum_{i=1}^{N}p\phi(t)^{p-1}B_j\psi_i(t)$$
$$+p\phi(t)^{p-1}\sum_{i=1}^{N}|D_{ij}^{\tau}|e^{\varepsilon\tau}k_j(\phi(t-\tau(t)))$$
$$+p\psi_i(t)^{p-1}k_j\phi(t)$$
$$-\sum_{i=1}^{N}\sum_{j=1}^{N}|D_{ij}^{\tau}|k_j e^{\varepsilon\tau}|\psi_i(t-\tau(t))^p|. \quad (26)$$

According to the inequality $pa^{p-1}b \le (p-1)a^p + b^p$, we can easily conclude that

$$p\phi(t)^{p-1}\sum_{i=1}^{N}|D_{ij}^{\tau}|e^{\varepsilon\tau}k_j\phi(t-\tau(t)) \le (p-1)\sum_{i=1}^{N}|D_{ij}^{\tau}|e^{\varepsilon\tau}\phi(t)^p + \sum_{i=1}^{N}|D_{ij}^{\tau}|e^{\varepsilon\tau}k_j\phi(t-\tau(t))^p,$$

$$p\psi_i(t)^{p-1}k_j\phi(t) \le (p-1)k_j\psi_i(t)^p + k_j\phi(t)^p,$$
$$p\phi(t)^{p-1}B_j\psi_i(t) \le (p-1)B_j\phi(t)^p + B_j\psi_i(t)^p. \quad (27)$$

From Eqs.(26) and (27), we can have the following inequality

$$\dot{V}(t) \ \le 0. \quad (28)$$

By Theorem 3.3, we see that the equilibrium point of Eq.(3) with the above parameters is globally exponentially stable. This completes the proof.

Remark 3.3 It is well known, few authors have considered the global exponential stability of competitive neural networks with different time scales Eq.(3) and time-varying delays. Furthermore, in Theorem 3.3, our exponential stability results when $p = 1$ can be included into those results obtained by the Ref.[5]. We improve predecessor's conclusions via the proper Lyapunov functions and the Jacobsthal inequality $pa^{n-1}b \le (p-1)a^n + b^n$. For any integer $p \ge 1$, our results provide the neural network with a wide selection. Therefore, this means that our results are applicable to networks with a much broader connection structure. Hence, the earlier results Refs.[3,5] are extended and improved.

4. An example

In this section, an example will be used to illustrate our conclusions.

Example 4.1 Consider the competitive neural networks with different time scales Eq.(3) and time-varying delays

$$\begin{aligned}\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} &= A\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + D\begin{bmatrix}g(x_1(t))\\ g(x_2(t))\end{bmatrix}\\ &\quad + D^\tau\begin{bmatrix}g(x_1(t))\\ g(x_2(t))\end{bmatrix} + B\begin{bmatrix}s_1(t)\\ s_2(t)\end{bmatrix},\\ \begin{bmatrix}\dot{s}_1(t)\\ \dot{s}_2(t)\end{bmatrix} &= -\begin{bmatrix}s_1(t)\\ s_2(t)\end{bmatrix} + \begin{bmatrix}g(x_1(t))\\ g(x_2(t))\end{bmatrix}, \end{aligned} \tag{29}$$

when $g_j(x) = \tanh(x)$, $\tau(t) = (1 - \mathrm{e}^{-t})/(1 + \mathrm{e}^{-t})$, $\mu = 0.5, \mathrm{e}^{\varepsilon\tau} = 1$. where $k_j = \alpha = P = \eta = I$,

$$A = \begin{pmatrix}2 & 0\\ 1 & 1\end{pmatrix}, \quad D = (d_{ij})_{n\times n} = \begin{pmatrix}-3 & 2\\ 5 & 1\end{pmatrix},$$

$$D^\tau = (d_{ij})^\tau_{n\times n} = \begin{pmatrix}1 & 5\\ 0 & -2\end{pmatrix}, \quad C = \begin{pmatrix}1 & 0\\ 0 & 2\end{pmatrix},$$

$$B = \begin{pmatrix}2 & -1\\ -1 & 1\end{pmatrix}, \quad \bar{A} = \begin{pmatrix}2 & 1\\ 1 & 5\end{pmatrix},$$

According to Theorem 3.1, we can conclude as follows:

$$\begin{aligned}&-2k_jCA + 2k_j^2CD + k_j^2CB\\ &+\alpha\eta(D^\tau\mathrm{e}^{\varepsilon\tau})^{\mathrm{T}}PD^\tau\mathrm{e}^{\varepsilon\tau}k_j^2 + \eta^{-1}\frac{1}{1-\mu}k_j^2CP^{-1}C < 0,\\ &AB - 2\bar{A} + \bar{A}k_j + k_j\alpha CB = \begin{pmatrix}4 & -4\\ -2 & -3\end{pmatrix} < 0.\end{aligned}$$

So it is easy to verify that Eq.(29) satisfies all the conditions of Theorem 3.1 and Lemma 2.1, the two competitive neural networks defined above which satisfy the initial conditions $\phi(s) = (-1, -0.5)^{\mathrm{T}}$, $\varphi(s) = (0.2, 0)^{\mathrm{T}}$ are globally exponentially stable. On the other hand, Fig.1 shows the transient response of state variables with time-varying delay, and we can conclude that our new criterion is independent of delays.

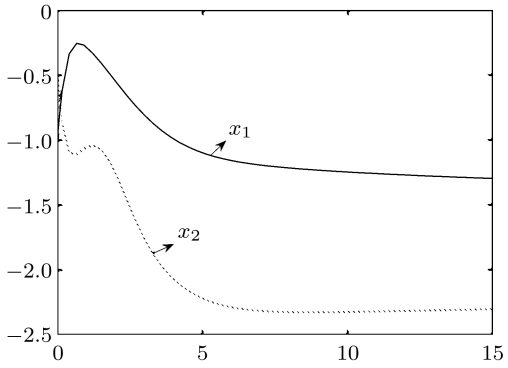


Fig.1. The time response of state variables $x_1(t)$ and $x_2(t)$.

5. Conclusions

In this paper, some new criteria for global exponential stability of the competitive neural networks with different time scales and time-varying delays have been derived. Moreover, the sufficient criteria obtained here are computationally more flexible and more efficient than many other existing criteria. And it is shown that the new results impose weaker conditions for the exponential stability. So these possess an important leading significance in the design and applications of globally exponentially stable competitive neural networks with different time scales and time-varying delays.

References

[1] Amari S 1983 *IEEE Trans. Systems, Man, and Cybernetics* **13** 741

[2] Cohen A M and Grossberg S 1983 *IEEE Trans. Systems, Man, and Cybernetics* **13** 815

[3] Lu H T and He Z Y 2005 *Neural Networks* **18** 243

[4] Meyer-Baese A, Pilyugin S S and Chen Y 2003 *IEEE Trans. Neural Networks* **14** 716

[5] Lou X Y and Cui B T 2007 *Physica* A **380** 563

[6] Lu H T and Chen G R 2005 *IEEE Trans. Circuits Systems* II **52** 761

[7] Tolga Ensari and Sabri Arik 2005 *IEEE Trans. Circuits Systems* II **52** 126

[8] Qi H and Qi Li 2004 *IEEE Trans. Neural Networks* **15** 99

[9] Khalil H K 2002 *Nonlinear Systems* (third edition) (Englewood Cliffs, NJ: Prentice-Hall)

[10] Galicld M, Istritz L and Wire H 1999 *IEEE Trans. Neural Networks* **10** 741
[11] Arik S and Tavsanoglu V 2000 *IEEE Trans. Circuits Systems* I **47** 571
[12] Cao J D and Wang J 2005 *IEEE Trans. Circuits Systems* I **52** 920
[13] Pourciau B H 1982 *Journal of Mathematical Analysis and Applications* **85** 279
[14] Jin L and Gupta M 1999 *IEEE Trans. Neural Networks* **10** 1321
[15] Lou X Y and Cui B T 2007 *IEEE Trans. Systems, Man and Cybernet.:* Part B **37** 713
[16] Wu W and Cui B T 2007 *Chin. Phys.* **16** 1009
[17] Tan W and Wang Y N 2005 *Chin. Phys.* **14** 72
[18] Zhang Q, Ma R N, Wang C and Xu J 2003 *Chin. Phys.* **12** 22
[19] Zhang Q, Gao L, Wang C, Yuan T and Xu J 2003 *Acta Phys. Sin.* **52** 1606 (in Chinese)
[20] Lu H T 2002 *Phys. Lett.* A **298** 109
[21] Liu X W, Huang Q Z, Gao X and Shao S Q 2007 *Chin. Phys.* **16** 2272
[22] Wang M S, Hou Zhong H and Xin H W 2006 *Chin. Phys.* **15** 2553
[23] Wang S, Cai L, Li Q and Wu G 2007 *Chin. Phys.* **16** 2631