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Hawking radiation from gravity's rainbow via gravitational anomaly*

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Based on the anomaly cancellation method, initiated by Robinson and Wilczek, we investigates Hawking radiation from the modified Schwarzschild black hole from gravity's rainbow from the anomaly point of view. Unlike the general Schwarzschild space–time, the metric of this black hole depends on the energies of probes. The obtained result shows to restore the underlying general covariance at the quantum level in the effective field, the covariant compensating flux of energy–momentum tensor, which is related to the energies of the probes, should precisely equal to that of a $(1+1)$-dimensional blackbody at the Hawking temperature.

Keywords: Hawking radiation, anomaly, gravity's rainbow, event horizon
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1. Introduction

Since Stephen Hawking first proved that black hole can emit particles in the form of purely thermal spectrum from the black hole horizon,[1] many approaches to derive Hawking radiation and its corresponding temperature have been proposed, such as Euclidean gravity,[2] string theory,[3] temperature Green function,[4] tunnelling picture,[5−15] and so on during the past thirty years. Recently, Robinson and Wilczek, motivated by the method of trace anomaly,[16] proposed a novel method on Hawking radiation from the viewpoint of anomaly at the event horizon (EH).[17] According to their viewpoint, the scalar field theory in any dimensional space–time can be reduced to that on (t, r) section of the original background by a universal dimensional reduction technique. Then in the 2-dimensional reduction, to describe the observable physics, they integrated out the classically insignificant ingoing modes to construct the effective field outside the black hole horizon. However quantum mechanically, the contributions of these ingoing modes cannot be ignored. Hence the effective field becomes chiral and exhibits gravitational anomaly with respect to the general coordinate symmetry. The underlying theory of course should be covariant under the infinitesimal coordinate transition. Therefore to restore the covariance at the quantum level, they introduced a flux to eliminate the anomaly at the EH and found this compensating flux precisely equalled that of a (1+1)-dimensional blackbody at the Hawking temperature eventually.

Up to now, based on their paradigm, investigations on charged space–time,[18] rotating space–time,[19] de-Sitter[20−22] and BTZ space–time[23] (for others see Ref.[24–27]) have been done extensively. To comprehensively understand Hawking radiation and some pertinent problems, such as the information problem and the origin of black hole entropy etc, investigations on the Planck scale physics,[28] particularly in Hawking radiation from the anomaly point of view, is of great importance and significance. Therefore, in this paper, we adopt the anomaly cancellation method to investigate the Hawking radiation from the modified black hole from gravity's rainbow which stems from the incorporation of deformed special relativity (DSR) with the curvature of space–time.[29] The Schwarzschild solution of the rainbow metric has been presented in Ref.[30]. The main feature of this metric is that the geometry of space–time depends on the energies of probe particles falling in it. That is to say, there are different space–time geometries for

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probe particles with different energies. Because of this quantum effect, the related energy–momentum tensor fluxes that obtained in this context, with covariance at the EH after the gravitational anomaly at the quantum level is cancelled, are also closely related to the energies of the probe particles.

The remainders are arranged as follows. We introduce the modified black hole and some thermodynamic properties in the next section. In Section 3, the method of Robinson and Wilczek is applied to investigate the Hawking radiation from the modified black hole from gravity's rainbow. Finally, a brief discussion is given in the last section.

2. The modified Schwarzschild black hole from gravity's rainbow

In the light of the energy independent coordinates and the energy independent mass, the Schwarzschild solution from the rainbow metric can be written as[30]

$$\mathrm{d}s^2 = -A\mathrm{d}t_s^2 + B^{-1}\mathrm{d}r^2 + r^2/f_2^2\mathrm{d}\Omega^2, \tag{1}$$

where $A = \left(1 - \dfrac{2m}{r}\right)/f_1^2$, $B = f_2^2\left(1 - \dfrac{2m}{r}\right)$, m is the mass of the modified black hole from gravity's rainbow, f_1 and f_2 are energy functions which satisfy

$$E^2 f_1^2(E;\lambda) - p^2 f_2^2(E;\lambda) = m_0^2, \tag{2}$$

where E is the energy of probe particles, λ is a parameter of order of the Planck length. To go back to the special relativity as $E/E_{\mathrm{P}} \ll 1$ (E_{P} is the Planck energy), the correspondence principle will impose $f_1 = f_2 = 1$. Moreover, Eq.(2) relates the metric to the energies and momenta of the test particles, that is, a given infinity observer with different energy quantum to probe the space–time obtains different space–time geometries. Because of this, the present space–time is endowed with a Planck scale modification and has some quantum effects.[29]

The Arnowitt–Deser–Misner (ADM) mass of this black hole can be calculated via the Komar integral

$$M = \frac{-1}{8\pi}\int \xi^{\mu;\nu}_{(t)}\mathrm{d}^2\Sigma_{\mu\nu} = \frac{m}{f_1 f_2}, \tag{3}$$

where $\xi^{\mu}_{(t)}$ is the normalized time–like Killing vector. Obviously, for the modified Schwarzschild black holes, the ADM mass is not equivalent to the mass parameter m strictly, but depends on the energies of the probe particles. This is because the quantum correction effect of the gravity's rainbow space–time stemming from DSR has energy dependence. For the metric (1), we obtain that the EH is located at $r_{\mathrm{h}} = 2m$ as the usual Schwarzschild black hole for all observers, however the entropy $S = 4\pi m^2/f_2^2$ does not have the usual form, but depends on particle's energy.

3. Quantum anomaly and Hawking radiation of the modified Schwarzschild black hole

Now we focus on investigating Hawking radiation of the modified Schwarzschild black hole from gravity's rainbow space–time by the anomaly cancellation method proposed by Robinson and Wilczek. We find for metric Eq.(1) $g_{tt}.g_{rr} \neq 1$, so its surface gravity does not have the usual form, but $\kappa = 1/2\sqrt{A'(r)B'(r)}\,|_{r_{\mathrm{h}}}$, where the prime denotes the derivative with respect to r. As mentioned above,[17−20] before the effective field theory is formulated, the higher scalar field in the gravity's rainbow background should be reduced to that in a (1+1)-dimensional space–time by the dimensional reduction technique. However, note that in the modified Schwarzschild space–time, if we take into account the interaction between the probe and emitted particle, the reduced effective metric will be very complicated, which is adverse for our next discussion. Therefore to study Hawking radiation from the modified Schwarzschild black hole from the anomaly point of view, we should deal with it approximately. That is in the low order approximation, we can ignore that interaction and the back reaction of the emitted particles on the background space–time. Then the action for the scalar field in the modified black hole from gravity's rainbow is

$$\begin{aligned} S[\varphi] =& \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\varphi\nabla^2\varphi \\ =& \frac{1}{2}\int \mathrm{d}^4x\frac{r^2\sin\theta}{f_1 f_2^3}\varphi\Big[-\frac{1}{A(r)}\partial_t^2 + \frac{1}{r^2}\partial_r r^2 B(r)\partial_r \\ &+ \frac{f_2^2}{r^2}\partial_\theta^2 + \frac{f_2^2}{r^2\sin^2\theta}\Big]\varphi. \end{aligned} \tag{4}$$

After the partial wave decomposition in terms of the spherical harmonics $\varphi = \sum_{l,m}\phi_\omega(t,r)Y_{l,m}$ and the tortoise coordinate transformation $\mathrm{d}r_*/\mathrm{d}r = f(r)^{-1}$, we find that the effective radial potential for partial wave modes of the scalar field vanishes exponentially

as $r \to r_{\rm H}$. Thus after the subordinate terms are omitted, physics that near the EH can be described effectively by an infinite collection of (1+1)-dimensional scalar field with the metric

$$\mathrm{d}s^2 = -A\mathrm{d}t_{\rm s}^2 + B^{-1}\mathrm{d}r^2. \tag{5}$$

Moreover the background dilaton factor $\Psi = r^2/f_1 f_2^3$, which is often ignored in the static background, is also related to the energies of probe particles. The reason for this arises from the changes of ADM mass and the area at the EH. In the two-dimensional space–time, since the horizon is a null hypersurface, ingoing modes classically cannot affect the dynamics outside the black hole horizon. So we integrate out the classically irrelevant ingoing modes to define the effective field theory outside the horizon that makes the effective theory becoming chiral and suffering from a gravitational anomaly with respect to the general coordinate symmetry. However the underlying theory should be covariant, hence we shall introduce an energy–momentum tensor flux to cancel this nonexistent anomaly at the quantum level. In the near-horizon region $[r_{\rm H},\, r_{\rm H}+\varepsilon]$, when the ingoing modes are ignored, the effective field is chiral and suffers from a gravitational anomaly which satisfies the anomalous equation[31]

$$\nabla_\mu T^\mu_{({\rm H})\nu} = A_\nu = \frac{1}{\sqrt{-g}}\partial_\mu N^\mu_\nu, \tag{6}$$

where

$$N^\mu_\nu = \frac{1}{96\pi}\varepsilon^{\beta\mu}\partial_\alpha \Gamma^\alpha_{\nu\beta}. \tag{7}$$

In which, $\varepsilon^{\beta\mu} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $\Gamma^\alpha_{\nu\beta}$ is the Christoffel connection. The covariant form, on the other hand, also takes the form as

$$\nabla_\mu \tilde{T}^\mu_{({\rm H})\nu} = \tilde{A}_\nu = \frac{1}{\sqrt{-g}}\partial_\mu \tilde{N}^\mu_\nu. \tag{8}$$

While in the region $[r_{\rm H}+\varepsilon,\, +\infty]$, there is not anomaly and the energy–momentum tensor satisfies the conservation equation $\nabla_\mu T^\mu_{\nu(0)} = 0$. So now, we focus on discussing the anomaly in the region $[r_{\rm H},\, r_{\rm H}+\varepsilon]$. Incorporating Eqs.(5) and (7) with the Christoffel connection we obtain

$$N^r_t = \frac{1}{96\pi}\partial_r \Gamma^r_{tt} = \frac{1}{192\pi}(A'B' + A''B). \tag{9}$$

We now consider the variation of the effective action that induced by the infinitesimal general coordinate transformation $x^\mu \to x^\mu + \lambda^\mu$ with a gauge parameter λ

$$\begin{aligned} -\delta_\lambda W &= \int \mathrm{d}x^2\sqrt{-g}\lambda^\nu \nabla_\mu T^\mu_\nu \\ &= \int \mathrm{d}t\mathrm{d}r\sqrt{-g}\lambda^t\nabla_\mu(T^u_{t(0)}\,\Theta_+ + T^\mu_{t({\rm H})}H) \\ &= \int \mathrm{d}t\mathrm{d}r\lambda^t\{\partial_r(N^r_t H) + [\sqrt{-g}(T^r_{t(0)} - T^\mu_{t({\rm H})}) \\ &\quad + N^r_t]\delta(r - r_h - \varepsilon)\}, \end{aligned} \tag{10}$$

where the current outside the horizon has been written as the sum of two regions, $T^\mu_\nu = T^u_{t(0)}\,\Theta_+(r) + T^\mu_{t({\rm H})}H(r)$ by the scalar step function $\Theta_+ = \Theta\,(r - r_{\rm h} - \varepsilon)$ and scalar top hat function $H = 1 - \Theta_+$ (in the region $[r_{\rm H},\, r_{\rm H}+\varepsilon]$, $H=1$; while in $[r_{\rm H}+\varepsilon, +\infty]$, $H=0$). The total effective action must be gauge invariant and the first term would be cancelled by the quantum effect of the classically irrelevant ingoing modes that we have ignored. Therefore, to hold the general coordinate invariance at the quantum level, the coefficient of the delta function should vanish, namely

$$c_0 = c_{\rm h} - N^r_t\,(H)\,, \tag{11}$$

where $c_0 = \sqrt{-g}T^r_{t(0)}$, $c_{\rm h} = \sqrt{-g}T^r_{t({\rm H})} - \int^r_{r_{\rm h}} \mathrm{d}r\partial_r N^r_t$ are the energy–momentum tensor fluxes observed by the observer located at infinity and the EH respectively. To determine c_0 precisely, we impose the covariant energy–momentum tensor to be vanished at the horizon (regularity condition[32]), that is

$$\sqrt{-g}\tilde{T}^r_t = \sqrt{-g}T^r_t + \frac{B}{192\pi A}(AA'' - 2A'^2) = 0. \tag{12}$$

Then we obtain that at the horizon

$$c_{\rm h} = \sqrt{-g}T^r_{t({\rm H})} = \frac{\kappa_h^2}{24\pi} = 2N^r_t(r_{\rm H}), \tag{13}$$

where

$$\kappa = \frac{1}{2}\sqrt{A'(r)B'(r)}\,|_{r_{\rm h}} = 2\pi/\beta = \frac{f_2}{f_1}\frac{1}{4m}$$

is the surface gravity of the black hole. Therefore, the total energy–momentum tensor flux, which is required to cancel the gravitational anomaly at the quantum level to save the underlying general coordinate invariance at the EH, can be expressed as

$$c_0 = 2N^r_t(r_{\rm H}) - N^r_t(r_{\rm H}) = \frac{\pi T_{\rm h}^2}{12} = \frac{1}{768\pi m^2}\frac{f_2^2}{f_1^2}, \tag{14}$$

where $T_{\rm h} = \dfrac{\kappa}{2\pi} = \dfrac{f_2}{f_1}\dfrac{1}{8\pi m}$ is the Hawking temperature, which is related to the energies of the test particles, at the EH of the modified Schwarzschild black hole from gravity's rainbow.

Now we turn to derive the total energy–momentum tensor flux from the blackbody radiation.

The Planckian distribution with the inverse temperature takes the form as $N(\omega) = 1/(\exp(\beta\omega) \pm 1)$. To avoid super-radiance, we only discuss fermions and obtain

$$F_{\mathrm{E}} = \int_0^{\infty} \frac{\omega}{\pi} N(\omega)\, \mathrm{d}\omega = \frac{\pi}{12} T_{\mathrm{H}}^2. \tag{15}$$

Comparing Eq.(14) with Eq.(15), we find that the compensating flux of energy–momentum tensor, required cancelling the gravitational anomaly at the EH to hold the underlying general coordinate invariance at the quantum level, is equivalent to the Hawking radiation flux from the EH. That is, Hawking radiation can be determined by the compensating flux with covariance at the EH.

4. Discussion

In this paper, we find that Hawking radiation can save the underlying general covariance at the quantum level. However, in the background of gravity's rainbow, an observer with different energies to probe the background space–time will get different conditions to restore this covariance. In the low energy realm[29] i.e. $E/E_{\mathrm{P}} \ll 1$ ($E_{\mathrm{P}} = \sqrt{1/8\pi}$ is the Planck energy), to save the general coordinate covariance, the black hole must radiate out at the Hawking temperature $T_{\mathrm{h}} = 1/8\pi m f$, where $f = f_1 = \mathrm{e}^{-\frac{1}{2}E^2/E_{\mathrm{P}}^2}$, $f_2 = 1$ are obtained by the imposition of correspondence principle. While for large mass black holes $E = T$, $f = \mathrm{e}^{-\frac{1}{2}T^2/E_{\mathrm{P}}^2}$, where T is the characteristic temperature of the emitted particles from the black hole, to hold the underlying covariance at the EH, the black hole must radiate at the Hawking temperature

$$T^2 \simeq \frac{1}{64\pi^2 M^2} + \frac{T^2}{8\pi M^2}.$$

Of course, if $f_1 = f_2 = 1$, to restore the general invariance at the EH, the modified Schwarzschild black hole from gravity's rainbow should do as the classical Schwarzschild black hole, which is consistent with the initial work of Robinson and Wilczek.

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