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A scheme for approximate conditional teleportation of entangled two-mode cavity state without Bell state measurement*

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An alternative scheme to approximately conditionally teleport entangled two-mode cavity state without Bell state measurement in cavity QED is proposed. The scheme is based on the resonant interaction of a ladder-type three-level atom with two bimodal cavities. The entangled cavity state is reconstructed with only one atom interacting with the two cavities successively.

Keywords: teleportation, entangled, two-mode state
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Since its first proposal,[1] a large number of theoretical investigations have been proposed to show that teleportation of atomic state or cavity field state can be accomplished.[2−8] In the first step of the teleportation process, two spin-1/2 particles are prepared in a maximally entangled state. Then, a joint measurement is performed on the particle to be teleported and one of the correlated pair. Finally, the information about the joint measurement is transmitted to the other observer through a classical channel, so that he can reconstruct the initial state of the teleported particle on the second particle of the correlated pair. Davidovich *et al*[2] have presented a scheme for the teleportation of an unknown atomic state between two high-Q cavities, initially prepared in an entangled photon-number state. Cirac and Parkins[3] have made another cavity QED proposal for the realization of the quantum teleportation of an atomic state by using two additional atomic levels of one of the correlated pair. Zheng and Guo have proposed a scheme for the teleportation of an unknown atomic state based on the Raman atom-cavity interaction.[7]

Recently, Zheng[9] has given a simplified scheme for approximately conditional teleportation of an unknown atomic state without Bell-state measurement. In the scheme, only two two-level resonant atoms are involved, thus an additional atom is unnecessary. Other proposals also have been brought forward to teleport an unknown atomic state without Bell-state measurement in cavity QED.[10,11] On the other hand, schemes to teleport a cavity state, such as a superposition of zero-, one-and two-photon states,[12] a superposition of two coherent states,[13] and an entangled zero- and one-photon states[14−16] have been presented. In Ref.[16], a scheme was proposed to teleport the entangled zero- and one-photon states from a bimodal cavity to another one without using Bell-state measurement. The scheme needs two two-level atoms to interact with the cavity mode respectively.

In this paper, based on the resonant interaction of a ladder-type three-level atom with two bimodal cavities, we propose an alternative scheme for the teleportation of entangled zero- and one-photon states of a two-mode cavity without Bell state measurement. A measurement on the atom may directly collapse cavity 2 to the initial state of cavity 1 with cavity 1 left in the vacuum state. The probability of success and the fidelity of our scheme are 0.25 and 0.99, respectively. The advantage of our scheme is that we only use one atom to resonantly interact with the two cavities in succession, thus it is more simple and more effective than that of Ref.[16].

We consider the resonant interaction of a ladder-type three-level atom with a two-mode field, as shown

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in Fig.1. In the interaction picture, the Hamiltonian for such a system is described as ($\hbar = 1$)[17,18]

$$\begin{aligned} H =& \omega_e|e\rangle\langle e| + \omega_g|g\rangle\langle g| + \omega_i|i\rangle\langle i| + \omega_1 a_1^+ a_1 \\ &+\omega_2 a_2^+ a_2 + \ g_1(a_1^+|g\rangle\langle e| + a_1|e\rangle\langle g|) \\ &+g_2(a_2^+|i\rangle\langle g| + a_2|g\rangle\langle i|), \end{aligned} \tag{1}$$

where $|e\rangle$ is the upper level of the atom, $|g\rangle$ is the middle level and $|i\rangle$ is the lower level of the atom, a_i^+ and $a_i (i = 1, 2)$ are the creation and annihilation operators for the ith cavity mode, $g_1(g_2)$ is the coupling constant of the transition between $|e\rangle$ ($|i\rangle$) and $|g\rangle$ with the ith cavity mode. ω_1, ω_2 are the frequencies of cavity modes, ω_i, ω_g and ω_e are the energy-levels for states $|i\rangle$, $|g\rangle$ and $|e\rangle$, respectively.

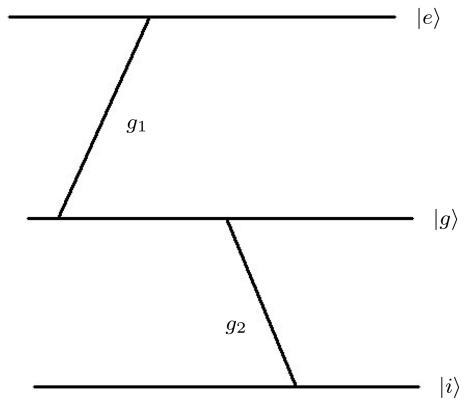


Fig.1. Schematic diagram of the Ξ-type three level atom resonantly coupling to a two-mode field ($|e\rangle, |g\rangle$ and $|i\rangle$ denote the states of the atom, g_1, g_2 are the coupling coefficients).

In the interaction picture, the interaction Hamiltonian is given by

$$\begin{aligned} H_{\rm I} =& \ g(a_1^+|g\rangle\langle e| + a_1|e\rangle\langle g|) \\ &+g(a_2^+|i\rangle\langle g| + a_2|g\rangle\langle i|). \end{aligned} \tag{2}$$

For simplicity, we here assume that $g_1 = g_2 = g$. Suppose that the state (cavity 1) to be teleported is prepared in the state

$$|\Psi\rangle_{c1} = c_0\,|0,0\rangle_1 + c_1\,|1,1\rangle_1 \tag{3}$$

with $|c_0|^2 + |c_1|^2 = 1$, the state can be prepared by sending a ladder-type atom initially prepared in the $|e\rangle$ through a bimodal cavity to resonantly interact with the two-mode vacuum state $|0,0\rangle$ for a definite time.[19] In order to teleport this state, firstly we send a ladder-type three-level atom (atom A) prepared in state

$$|\Psi\rangle_{\rm a} = |e\rangle\,, \tag{4}$$

through a bimodal cavity (cavity 2) in two-mode vacuum state

$$|\Psi\rangle_{c2} = |0,0\rangle_2\,. \tag{5}$$

After an interaction time τ_1, the atom–cavity system evolves into

$$\begin{aligned} |\Psi\rangle_{a-c2} =& \ \frac{1}{2}(\cos(\sqrt{2}g\tau_1)+1)\,|e\rangle\,|0,0\rangle_2 \\ &+\frac{\sqrt{2}}{2}(\cos(\sqrt{2}g\tau_1)-1)\,|i\rangle\,|1,1\rangle_2 \\ &-\frac{\rm i}{\sqrt{2}}\sin(\sqrt{2}g\tau_1)\,|g\rangle\,|1,0\rangle_2\,. \end{aligned} \tag{6}$$

Therefore, the state of whole system becomes

$$\begin{aligned} |\Psi\rangle =& \ |\Psi\rangle_{a-c2}\otimes|\Psi\rangle_{c1} = (c_0\,|0,0\rangle_1 + c_1\,|1,1\rangle_1) \\ &\times\left[\frac{1}{2}(\cos(\sqrt{2}g\tau_1)+1)\,|e\rangle\,|0,0\rangle_2\right. \\ &+\frac{\sqrt{2}}{2}(\cos(\sqrt{2}g\tau_1)-1)\,|i\rangle\,|1,1\rangle_2 \\ &\left.-\frac{\rm i}{\sqrt{2}}\sin(\sqrt{2}g\tau_1)\,|g\rangle\,|1,0\rangle_2\right]. \end{aligned} \tag{7}$$

Then atom A interacts with cavity 1, after an interaction time τ_2 the system evolves into

$$\begin{aligned} |\Psi\rangle =& \ \frac{c_0}{2}(\cos(\sqrt{2}g\tau_1)+1)\,|0,0\rangle_2\left[\frac{1}{2}(\cos(\sqrt{2}g\tau_2)+1)\,|e\rangle\,|0,0\rangle_1 + \frac{\sqrt{2}}{2}(\cos(\sqrt{2}g\tau_2)-1)\,|i\rangle\,|1,1\rangle_1\right. \\ &\left.-\frac{\rm i}{\sqrt{2}}\sin(\sqrt{2}g\tau_2)\,|g\rangle\,|1,0\rangle_1\right] + \frac{c_1}{2}(\cos(\sqrt{2}g\tau_1)+1)\,|0,0\rangle_2\left[\frac{1}{2}(\cos(\sqrt{4}g\tau_2)+1)\,|e\rangle\,|1,1\rangle_1\right. \\ &\left.+\frac{\sqrt{6}}{4}(\cos(\sqrt{4}g\tau_2)-1)\,|i\rangle\,|2,2\rangle_1 - \frac{{\rm i}\sqrt{2}}{\sqrt{2}}\sin(\sqrt{4}g\tau_2)\,|g\rangle\,|2,1\rangle_1\right] + \frac{\sqrt{2}c_0}{2}(\cos\sqrt{2}g\tau_1 - 1) - |1,1\rangle_2\,|i\rangle\,|0,0\rangle_1 \end{aligned}$$

$$+\frac{\mathrm{i}}{\sqrt{2}}\sin(\sqrt{2}g\tau_1)\,|1,0\rangle_2\,[c_0\,|g\rangle\,|0,0\rangle_1+c_1(\cos(\sqrt{3}g\tau_2)\,|g\rangle\,|1,1\rangle_1\,\frac{\mathrm{i}\sin(\sqrt{3}g\tau_2)}{3}(|e\rangle\,|0,1\rangle_1+|i\rangle\,|1,2\rangle_1)\Big]$$
$$+\frac{\sqrt{2}c_1}{2}(\cos(\sqrt{2}g\tau_1)-1)\,|1,1\rangle_2\,\Big[\frac{1}{2}(\cos(\sqrt{2}g\tau_2)+1)\,|i\rangle\,|1,1\rangle_1+\frac{1}{2}(\cos(\sqrt{2}g\tau_2)-1)\,|e\rangle\,|0,0\rangle_1$$
$$-\frac{\mathrm{i}}{\sqrt{2}}\sin(\sqrt{2}g\tau_2)\,|g\rangle\,|1,0\rangle_1\Big]. \tag{8}$$

Now we perform a measurement on the atom. If this atom is detected in the state $|e\rangle$, the cavity field collapses to the state

$$\begin{aligned}|\,\Psi\rangle\;=\;&\frac{c_0}{2}(\cos(\sqrt{2}g\tau_1)+1)\,|0,0\rangle_2\,\frac{1}{2}(\cos(\sqrt{2}g\tau_2)+1)\,|0,0\rangle_1+\frac{c_1}{2}(\cos(\sqrt{2}g\tau_1)+1)\,|0,0\rangle_2\\&\times\frac{1}{2}(\cos(\sqrt{4}g\tau_2)+1)\,|1,1\rangle_1+\frac{\mathrm{i}c_1}{\sqrt{2}}\sin(\sqrt{2}g\tau_1)\,|1,0\rangle_2\,\frac{\mathrm{i}\sin(\sqrt{3}g\tau_2)}{3}\,|0,1\rangle_1\\&+\frac{\sqrt{2}c_1}{2}(\cos(\sqrt{2}g\tau_1)-1)\,|1,1\rangle_2\,\frac{1}{2}(\cos(\sqrt{2}g\tau_2)-1)\,|0,0\rangle_1\,.\end{aligned} \tag{9}$$

If the interaction time is chosen as $g\tau_2=0.55\pi$, $(\sqrt{2}g\tau_1)=\dfrac{\pi}{2}$, then we have $\cos(\sqrt{4}g\tau_2)=-0.953$, $\cos(\sqrt{3}g\tau_2)=-0.98$. Then the state $|\,\Psi\rangle$ collapses to the state

$$|\,\Psi'\rangle=N_{\mathrm{f}}\,(c_0\,|0,0\rangle_2+c_1\,|1,1\rangle_2)\,|0,0\rangle_1\,, \tag{10}$$

where N_{f} is a normalization factor. In this case the cavity 2 is approximately in the initial state of cavity 1, and the cavity 1 is left in the two-mode vacuum state. The probability of success is 0.25. The fidelity is

$$F=\frac{\dfrac{1}{2}}{\dfrac{1}{2}+\dfrac{c_1^2\sin^2(\sqrt{3}g\tau_2)}{18}+\dfrac{c_1^2(\cos(\sqrt{4}g\tau_2)+1)^2}{16}}\simeq 0.99. \tag{11}$$

We briefly address the experimental feasibility of the proposed scheme. For Rydberg atoms with principal quantum numbers 50, 51 and 52, the radiative time is $T_{\mathrm{r}}=30\,\mathrm{ms}$. In the experiment reported in Ref.[20], the photon lifetime in the cavity is $T_{\mathrm{c}}=\dfrac{Q}{2\pi\upsilon}\simeq 130\,\mathrm{ms}$. Set $g=2\pi\times 25\,\mathrm{kHz}$. Thus, we have the interaction time $\tau_1=\dfrac{\pi}{2\sqrt{2}g}=7\,\mu\mathrm{s}$, $\tau_2=\dfrac{0.55\pi}{g}=11\,\mu\mathrm{s}$. Considering the travelling time of atom, the total time to complete the procedure is about $180\,\mu\mathrm{s}$, which is much shorter than T_{r} and T_{c}. Therefore, there is sufficient time to complete the procedure. Moreover, with the improvement of technology, higher Q cavities might be available, the proposed scheme might be realizable.

In summary, we have presented a scheme for approximate conditional teleportation of entangled zero- and one-photon states of a two-mode cavity based on the resonant interaction of a ladder-type three-level atom with two bimodal cavities without Bell state measurement in cavity QED. A measurement on atom may directly collapse cavity 2 to the initial state of cavity 1 with cavity 1 left in the vacuum state. The probability of success and the fidelity of our scheme are 0.25 and 0.99, respectively. Compared with the scheme in Ref.[16], our scheme is more simplified and also more effective since only one atom is needed to interact with the two cavities, respectively, whereas the scheme in Ref.[16] requires two atoms, each of them must interact with one mode of the two bimodal cavities.

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