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Remote atom entanglement in a fibre-connected three-atom system*

Guo Yan-Qing(郭彦青)[a)†], Chen Jing(陈 菁)[b)], and Song He-Shan(宋鹤山)[c)]

[a)] *Department of Physics, Dalian Maritime University, Dalian* 116026, *China*
[b)] *College of Science, Beijing Forestry University, Beijing* 100083, *China*
[c)] *Department of Physics, Dalian University of Technology, Dalian* 116024, *China*

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An Ising-type atom–atom interaction is obtained in a fibre-connected three-atom system. The interaction is effective when $\Delta \approx \gamma_0 \gg g$. The preparations of remote two-atom and three-atom entanglements governed by this interaction are discussed in a specific parameter region. The overall two-atom entanglement is very small because of the existence of the third atom. However, the three-atom entanglement can reach a maximum very close to 1.

Keywords: three-atom system, Ising model, remote atom entanglement
PACC: 0367

1. Introduction

Generating the entanglement between spatially separated atoms plays an important role in quantum information processing and quantum computation, such as quantum storage,[1] quantum key distribution[2] and quantum states swapping.[3] To efficiently entangle two or more distant atoms, one must create some kind of direct or indirect interaction between them, such as by adopting appropriate measurement in optical fields that conditionally interact with atoms and thereby the atoms (as a subsystem) can be projected to an entangled state, or by using quantum-correlated fields interacting with atoms and thereby the entanglement among the fields can be transferred to atoms. Based on this, a variety of schemes for entangling distant atoms or distant photons have been proposed recently.[4−14] For example, fascinating schemes have been presented to efficiently entangle distant atoms, where the single-photon interference effect was applied with[4] or without[5] a weak driving laser pulse. Recently, Mancini and Bose proposed a novel scheme to directly entangle two atoms trapped in distant cavities[6] which were connected via optical fibres. Using the input–output theory, in the adiabatic approximation, the authors obtained an effective Ising model for two atoms. In their scheme, photon acted as an intermediate quantum information carrier and mapped the quantum information from the atom in one cavity to an atom in another. Such systems are meaningful not only to quantum measurement or testing Bell's inequalities but also to potential applications such as quantum encryption[15] or constructing universal quantum gates[16] that are essential for designing quantum network. Nevertheless, in discussing quantum networking with trapped atoms and photons in a cavity quantum electrodynamics (QED) system,[17] two problems should be overcome: how to generate the entanglement of an N-atom system, and what the exact influence of the collective interaction on the entanglement shared by remote atoms is. These problems have been discussed intensively, for instance in the scheme proposed by Cabrillo *et al.*[4] The simplest multi-atom case is a three-atom system which might be an intuitive extension from a two-atom case. In our scheme, we extend the model of two-atom circumstance in Ref.[6] to the case of three-atom which turns out to be a three-atom Ising model. Such an approach might be meaningful to discussing the above problems for multiple distant atoms. We first investigate the dependence of the effective Ising coupling coefficients on the atom–cavity detuning and cavity leakage. Then, we discuss the influence of an atom on the entanglement properties of other two atoms. Furthermore, we study the characters of remote three-atom entanglement and the tangle between one atom and the rest two atoms.

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†Corresponding author. E-mail: yqguo@newmail.dlmu.edu.cn

2. Optical fibres connected three-atom system

The schematic setup for our system is shown in Fig.1. Three identical two-level atoms 1, 2 and 3 are trapped in spatially distant cavities C_1, C_2 and C_3 respectively. All the cavities are assumed to be single-sided ones. Three off-resonant external driving fields ε_1, ε_2 and ε_3 are applied to cavity C_1, C_2 and C_3 respectively. In each cavity, a local laser field that is resonantly coupled to the atom is applied. Two neighbouring cavities are connected via optical fibres. Apparently, the subsystem comprised of cavities C_1 and C_2 or C_2 and C_3 is just the setup proposed in Ref.[6].

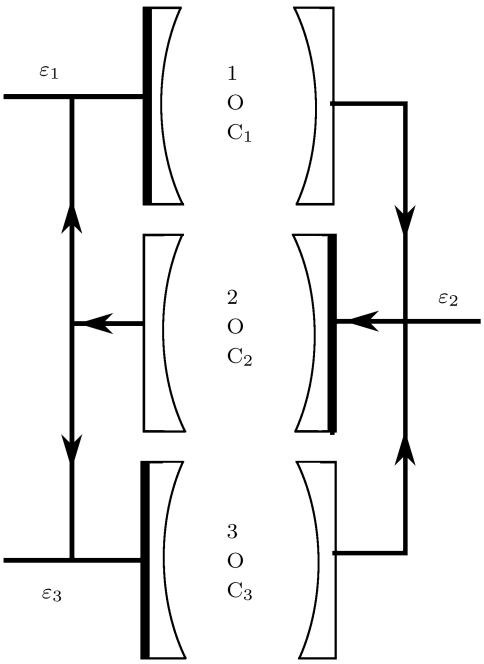


Fig.1. Schematic setup of the system. Three two-level atoms located in spatially separated single-sided optical cavities that are connected via optical fibres.

In the interaction picture, using the cavity input–output theory[18] and taking the adiabatic approximation,[19] we obtain an effective Hamiltonian for this system as (see Appendix A)

$$H_{\rm eff} = J_{12}\sigma_1^z\sigma_2^z + J_{23}\sigma_2^z\sigma_3^z + J_{31}\sigma_3^z\sigma_1^z + \sum_i \Gamma_i(\sigma_i^+ + \sigma_i^-), \quad (1)$$

which is a three-particle Ising chain with magnetic fields perpendicular to the z direction,[20] J_{12} and J_{23} represent coupling coefficients of the nearest-neighbouring (NN) atoms, while J_{31} represents the interaction strength between the next-nearest-neighbouring (NNN) atoms. $\sigma_i^z (i = 1, 2, 3)$ is spin operator of atom i, $\sigma_i^+ (\sigma_i^-)$ is atomic raising (lowering) operators. Γ_i is the magnitude of the locally applied laser field interacting with atom i. We define

$$\begin{aligned} J_{12} &= 2\gamma_0\chi^2 {\rm Im}\left\{\alpha_1\alpha_2^* {\rm e}^{{\rm i}\phi_{21}}/[M^2 - W^2]\right\}, \\ J_{23} &= 2\gamma_0\chi^2 {\rm Im}\left\{\alpha_3\alpha_2^* {\rm e}^{{\rm i}\phi_{32}}/[M^2 - W^2]\right\}, \\ J_{31} &= 2\gamma_0\chi^2 {\rm Im}\left\{\gamma_0\alpha_3\alpha_1^* {\rm e}^{{\rm i}(\phi_{23}+\phi_{12})}/[M(M^2 - W^2)]\right\}, \end{aligned} \quad (2)$$

where γ_0 is the cavity leakage rate, $\chi = g^2/\Delta$, with g being the coupling strength between the atom and the cavity field in cavity C_i, and Δ being the detuning between the internal transition and the cavity field frequency, where, a large detuning approximation has been assumed, i.e. $\Delta \gg g$; $M = {\rm i}\Delta + \gamma_0$; $W^2 = \gamma_0^2\left[{\rm e}^{{\rm i}(\phi_{21}+\phi_{12})} + {\rm e}^{{\rm i}(\phi_{32}+\phi_{23})}\right]$. The phase factors $\phi_{ij} (ij = 12, 21, 23, 32)$ are caused from the photons transmission along optical fibres from cavity C_j to cavity C_i.[21] Physically, they depend on the frequency of the photons and the distance between cavities. And

$$\begin{aligned} \alpha_1 &= \frac{\varepsilon_1 M^2 + \varepsilon_2 M\gamma_0 {\rm e}^{{\rm i}\phi_{12}} + \gamma_0^2[\varepsilon_3 {\rm e}^{{\rm i}\Theta_{13}} - \varepsilon_1 {\rm e}^{{\rm i}\Phi_1}]}{M(M^2 - W^2)}, \\ \alpha_2 &= \frac{\varepsilon_2 M + \gamma_0(\varepsilon_1 {\rm e}^{{\rm i}\phi_{21}} + \varepsilon_3 {\rm e}^{{\rm i}\phi_{23}})}{M^2 - W^2}, \\ \alpha_3 &= \frac{\varepsilon_3 M^2 + \varepsilon_2 M\gamma_0 {\rm e}^{{\rm i}\phi_{32}} + \gamma_0^2[\varepsilon_1 {\rm e}^{{\rm i}\Theta_{31}} - \varepsilon_3 {\rm e}^{{\rm i}\Phi_3}]}{M(M^2 - W^2)}, \end{aligned} \quad (3)$$

where $\Theta_{13} = \phi_{12} + \phi_{23}$, $\Phi_1 = \phi_{23} + \phi_{32}$, $\Theta_{31} = \phi_{32} + \phi_{21}$, and $\Phi_3 = \phi_{21} + \phi_{12}$. The global system is now determined by a series of independent parameters as ε_1, ε_2, ε_3, Δ, γ_0, ϕ_{12}, ϕ_{32}, Γ_1, Γ_2 and Γ_3. In the next section, we discuss the optimal region of the parameters for the preparation of remote atom entanglement.

3. Parameter space description of Ising coupling coefficients

From Eq.(3), the condition $M^2 \approx W^2$ leads to large Ising coupling coefficients. This condition also keeps the validity of the adiabatic approximation in the case of weak local laser fields, i.e. $J_{12}(J_{23}, J_{31}) \gg \Gamma_i$, $i = 1, 2, 3$. We can further simplify the condition

into

$$\phi_{21} + \phi_{12} \approx \phi_{23} + \phi_{32} \approx \frac{\pi}{2}, \quad \Delta \approx \gamma_0. \tag{4}$$

For simplicity, we assume $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_0$, and $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_0$. The parameter space can now be expressed in units of g as $(\Delta/g, \gamma_0/g)g$. In Figs.2 and 3, we give the description of Ising coupling coefficients for NN and NNN atoms in the parameter space, where we assume $\varepsilon_0 = 2g$. We can see that the coupling coefficients for NN atoms as well as NNN atoms can be divided into two regions: (a) the region where $\Delta > \gamma_0$, and (b) the region where $\Delta < \gamma_0$. In most area of the two regions, the coupling coefficients are very small, only in the regions just besides the line $\Delta = \gamma_0$ are they large enough so that the validity of the adiabatic approximation can be kept.

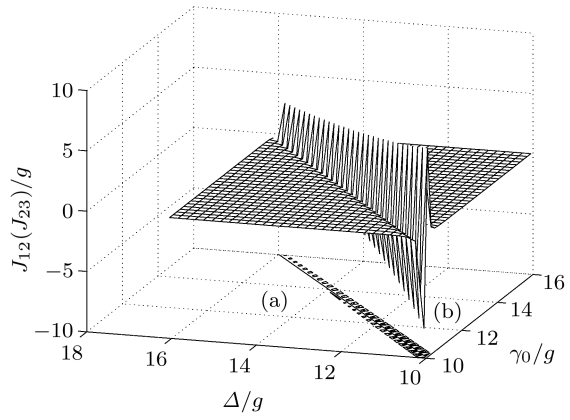


Fig.2. The coupling coefficients ($J_{12}(J_{23})$) for NN atoms under different values of the detuning (Δ) and the cavity leakage rate (γ_0). All the variables are presented in units of g.

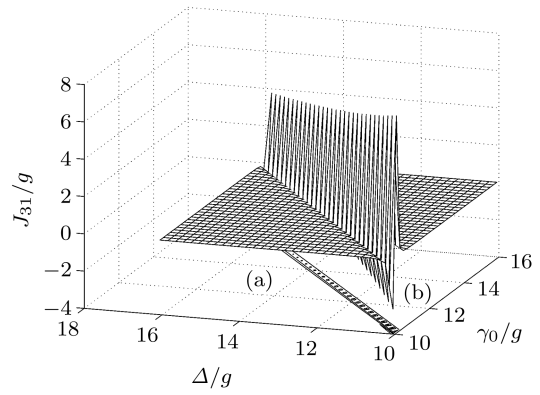


Fig.3. The coupling coefficients (J_{31}) for NN atoms under different values of the detuning (Δ) and the cavity leakage rate (γ_0). All the variables are presented in units of g.

In the following discussion, we will study two-atom entanglement nature and three-atom entanglement properties based on the parameter space.

4. The nearest-neighbouring and next-nearest-neighbouring remote two-atom entanglement

In this section, we discuss the nature of remote two-atom subsystem entanglement which is generated in our system. Wootters proposed a general measure of the amount of two-qubit (noted as 1 and 2) entanglement. It is named Concurrence[22]

$$C_{12} = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{5}$$

where λ_i $(i = 1, 2, 3, 4)$ is one of the non-negative square roots of the four eigenvalues of non-Hermitian matrix $\rho_{12}\tilde{\rho}_{12}$ with $\tilde{\rho}_{12}$ being defined as $(\sigma_y \otimes \sigma_y)\rho_{12}^*(\sigma_y \otimes \sigma_y)$ and ρ_{12} being the density matrix of the two-qubit system.

We depict the two-atom entanglement situations in Figs.4 and 5 for different parameter spaces. We assume that all the atoms are initially in their ground state, so that $|\psi(0)\rangle = |g\rangle_1 \otimes |g\rangle_2 \otimes |g\rangle_3$.

To start with, we investigate the influence of J_{31} on the entanglement of NN atoms. In Fig.4, we adopt appropriate values of Δ and γ_0 (which satisfy the condition in Eq.(4)) and assume $\Gamma_0 = 0.1g$. The Ising coupling coefficients are $J_{12} = J_{23} \approx -2.4g$, $J_{31} \approx 1.2g$ for solid line, and $J_{31} \approx -1.2g$ for dotted line. If all the signs of the coefficients are reversed, the resulting concurrences are not changed. Evidently, relatively large entanglement for NN atoms can be obtained when $J_{12}(J_{23})J_{31} < 0$. While, compared with the result in Ref.[6], the overall entanglement is very weak, for the two-atom subsystem is in a mixed state during the evolution.

In addition, the entanglement for NN atoms can be manipulated through the alternating of the locally applied laser fields. In Fig.5, we adopt the same parameters as those in Fig.4 but for $\Gamma_0 = 0.2g$. Figure 5 indicates that, the increase of Γ_0 remarkably improves the entanglement for NN atoms. The period is depressed, but the amount of entanglement is much enhanced. The amount of entanglement for NNN atoms, under the parameters we assumed, is generally much weaker than NN atoms. To improve the entanglement for NNN atoms, the Ising coupling coefficient between

NNN atoms must be enhanced. In Fig.6, we depict the entanglement for NNN atoms. Correspondingly, $J_{12} = J_{23} = -2.4g$, and $J_{31} = 1.2g$. To modulate the entanglement, we let $\Gamma_2 = 0$. Under this circumstance, the entanglement for NNN atoms can be compared with that for NN atoms (see Fig.4).

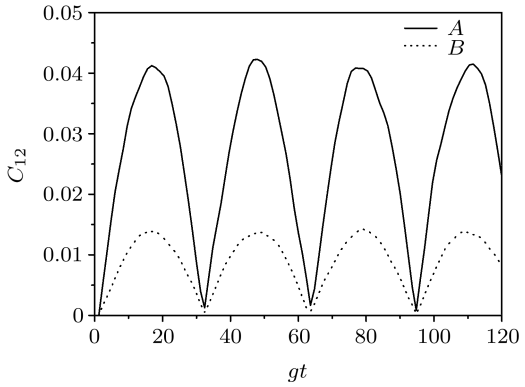


Fig.4. Entanglement of atoms 1 and 2 versus time for A: (solid line) $J_{31} \approx 1.2g$, and B: (dotted line) $J_{31} \approx -1.2g$, where we assume $\Gamma_0 = 0.1g$.

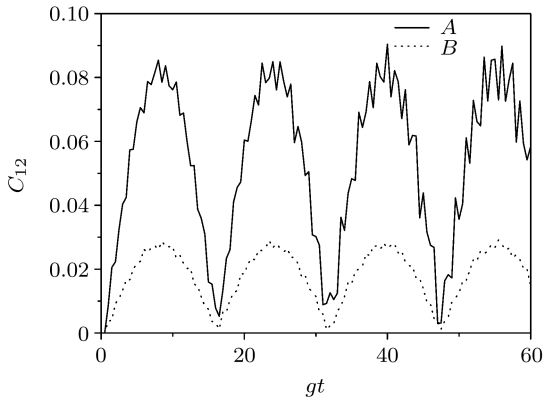


Fig.5. Descriptions are the same as those in Fig.4 but for $\Gamma_0 = 0.2g$. Note that the whole time scale is half of that in Fig.4.

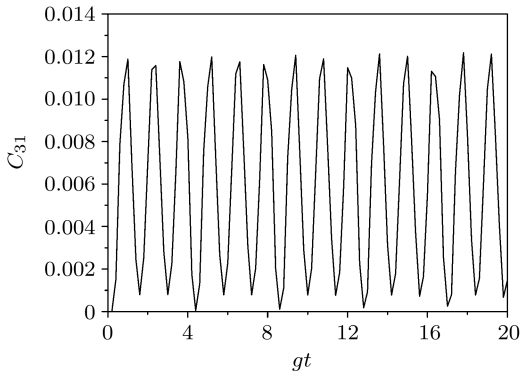


Fig.6. Entanglement of atoms 1 and 3 versus time when the laser field applied to cavity C_2 is turned off, that is, $\Gamma_1 = \Gamma_3 = 0.3g$, and $\Gamma_2 = 0$.

5. The remote three-atom entanglement properties

The intrinsic three-partite entanglement which is widely used for measuring the three-partite entanglement of pure states is defined as[23]

$$C_{123} = C_{1(23)} - C_{12}^2 - C_{13}^2, \tag{6}$$

where $C_{1(23)}$, which represents the tangle between subsystem 1 and the rest of the global system (denoted as (23)), is written as

$$C_{1(23)} = 4\mathrm{Det}\rho_1 = 2(1 - \mathrm{Tr}\rho_1^2). \tag{7}$$

In Fig.7, we plot the remote three-atom entanglement C_{123}(the solid line), the tangle $C_{1(23)}$(the dotted line), and the Concurrence C_{12}(the dashed line) for $|\psi(0)\rangle = |g\rangle_1 \otimes |g\rangle_2 \otimes |g\rangle_3$, where the corresponding parameters are the same as those represented by the dotted line in Fig.5. In fact, there is only very little difference between C_{123} and $C_{1(23)}$. To distinguish $C_{1(23)}$ from C_{123}, the line of $C_{1(23)}$ is raised to $0.1 + C_{1(23)}$.

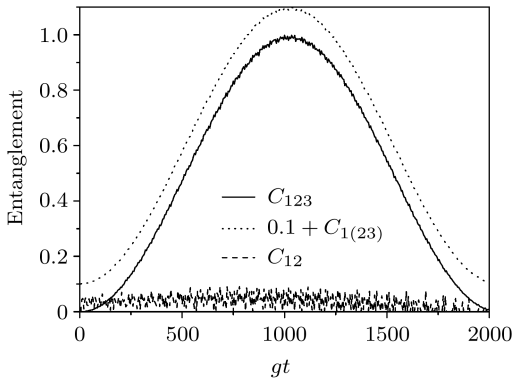


Fig.7. The three-atom entanglement C_{123}(solid line), the tangle $C_{1(23)}$(dotted line), and the concurrence C_{12}(dashed line) versus time, where $J_{12}(J_{23}) = -2.4g$, $J_{31} = -1.2g$ and $\Gamma_0 = 0.2g$. To distinguish $C_{1(23)}$ from C_{123}, the line for $C_{1(23)}$ is raised to $0.1 + C_{1(23)}$.

It has been pointed in the last section that the Concurrence that represents the bipartite entanglement between atom 1 and atom 2 is very small. While, the tangle, which expresses the entanglement of atom 1 and the rest of the global system, and the remote three-atom entanglement can reach maximum values of almost 1 for intermediate values of gt. Under the condition of strong Ising coupling coefficients ($J_{12}(J_{23}, J_{31}) \gg \Gamma_0$), if we express the tangle between

atom 1 and the rest of the global system as a sum of the remote three-atom entanglement and the bipartite entanglement, the three-atom entanglement will act as the largest contribution. It has been concluded that in a system of N spin-half particles, under the condition of strong Ising coupling coefficients, the N-partite entanglement will be dominant.[24]

6. Conclusion

We have obtained a three-atom Ising chain in a cavity QED system by connecting three distant cavities via optical fibres. The Ising coupling coefficients are found to be large in the region where $\Delta \approx \gamma_0 \gg g$, which keeps the validity of the adiabatic approximation. We have discussed the generation of remote atom entanglement. The overall two-atom entanglement is very small because of the existence of the third atom. While, the NN atom entanglement can be improved when the coupling coefficient of NNN atoms has a contrary sign with respect to that of NN atoms. The locally applied laser fields play an important role in modulating the entanglement quantitatively and qualitatively not only for NN atoms but also for NNN atoms. Furthermore, we have studied the remote three-atom entanglement and the tangle. It is shown that three-atom entanglement, which has a much longer period than the two-atom entanglement, can reach a maximum very close to 1.

In addition, it should be noted that the dissipation of the photon information along the fibres should be investigated, while, the dissipation can be included in the Ising coupling coefficients and act as a decaying exponential factor $\mathrm{e}^{-\nu L}$, where ν is the dissipation rate per metre, L is the total length of the fibre.[25] The phase factors $\mathrm{e}^{\mathrm{i}\phi_{12}}$ and $\mathrm{e}^{\mathrm{i}\phi_{23}}$ are then replaced by $\mathrm{e}^{\mathrm{i}\phi_{12}-\nu L_{12}}$ and $\mathrm{e}^{\mathrm{i}\phi_{23}-\nu L_{23}}$. In fact, the dissipative effect along fibres can be compensated by lowering the detuning Δ. One can obtain large Ising coupling coefficients by adopting the parameters in the regions just besides the line $\Delta \approx \sqrt{2\mathrm{e}^{-\nu(L_{12}+L_{23})}-1}\gamma_0$.

Appendix A

In the interaction picture, the Hamiltonian of the global system can be written as

$$H_{\mathrm{int}} = H_{\mathrm{int1}} + H_{\mathrm{int2}} + H_{\mathrm{int3}} + H_{\mathrm{int4}}, \tag{A1}$$

where H_{int1} represents the effective interaction between atoms and cavity fields, H_{int2} is the coupling between external driving fields and cavity fields, H_{int3} represents the interaction between locally applied laser fields and atoms, H_{int4} is the interaction between cavity fields and their environment which is described as a superposition of series of harmonic oscillators. Under the condition of large detuning, we have[26]

$$H_{\mathrm{int1}} = \chi \sum_i A_i^+ A_i \sigma_i^z, \tag{A2}$$

where $i = 1, 2, 3$, $A_i(A_i^+)$ represents cavity fields annihilation (creation) operator in cavity C_i. And[18]

$$H_{\mathrm{int2}} = \sum_i \varepsilon_i (A_i^+ + A_i), \tag{A3}$$

$$H_{\mathrm{int3}} = \sum_i \Gamma_i (\sigma_i^+ + \sigma_i^-). \tag{A4}$$

We assume Γ_i to be weak enough so that the quantum adiabatic theory[19] can be applied to the following calculations:

$$H_{\mathrm{int4}} = i \int_{-\infty}^{+\infty} d\omega \sum_i \kappa_{\mathrm{C}_i} [b_{\mathrm{C}_i}(\omega) A_i^+ + \mathrm{h.c.}], \tag{A5}$$

where $b_{\mathrm{C}_i}(\omega)$, $i = 1, 2, 3$, is the annihilation operator of the harmonic oscillator with the frequency ω. κ_{C_i} is the interaction strength between the cavity C_i and the harmonic oscillator. The kinetic equations for cavity field operators turn out to be[18]

$$\begin{aligned}
\dot{A}_1 &= -\left(\mathrm{i}\Delta + \mathrm{i}\chi\sigma_1^z + \frac{\gamma_{C_1}}{2}\right) A_1 + \sqrt{\gamma_{C_1}} A_{1,\mathrm{in}} + \varepsilon_1, \\
\dot{A}_2 &= -\left(\mathrm{i}\Delta + \mathrm{i}\chi\sigma_2^z + \frac{\gamma_{C_1}}{2}\right) A_2 + \sqrt{\gamma_{C_2}} A_{2,\mathrm{in}} + \varepsilon_2, \\
\dot{A}_3 &= -\left(\mathrm{i}\Delta + \mathrm{i}\chi\sigma_3^z + \frac{\gamma_{C_1}}{2}\right) A_3 + \sqrt{\gamma_{C_3}} A_{3,\mathrm{in}} + \varepsilon_3,
\end{aligned} \tag{A6}$$

where $\gamma_{\mathrm{C}_i} = 2\pi[k_{\mathrm{C}_i}(\omega)]^2$ $(i = 1, 2, 3)$. If cavities C_1 and C_2 are connected via optical fibres (as shown in Fig.1), so are cavities C_2 and C_3, the input–output conditions should be included, so that[21]

$$\begin{aligned}
\dot{A}_1 &= -\frac{\gamma_{\mathrm{C}_1}}{2} A_1 + \sqrt{\gamma_{\mathrm{C}_1}} A_{2,\mathrm{out}} \mathrm{e}^{\mathrm{i}\phi_{12}}, \\
\dot{A}_2 &= -\frac{\gamma_{\mathrm{C}_2}}{2} A_2 + \sqrt{\gamma_{\mathrm{C}_2}} A_{1,\mathrm{out}} \mathrm{e}^{\mathrm{i}\phi_{21}} \\
&\quad + \sqrt{\gamma_{\mathrm{C}_2}} A_{3,\mathrm{out}} \mathrm{e}^{\mathrm{i}\phi_{23}}, \\
\dot{A}_3 &= -\frac{\gamma_{\mathrm{C}_3}}{2} A_3 + \sqrt{\gamma_{\mathrm{C}_3}} A_{2,\mathrm{out}} \mathrm{e}^{\mathrm{i}\phi_{32}}.
\end{aligned} \tag{A7}$$

For simplicity, assuming the decay rates $\gamma_{C_1} = \gamma_{C_2} = \gamma_{C_3} = \gamma_0$ and taking into account the usual boundary conditions[18]

$$A_{i,\text{out}} + A_{i,\text{in}} = \sqrt{\gamma_0} A_i, \tag{A8}$$

where $i = 1, 2, 3$, we can rewrite the kinetic equations for cavity field operators as

$$\begin{aligned}
\dot{A}_1 &= -MA_1 - \mathrm{i}\chi A_1 \sigma_1^z + \sqrt{\gamma_0} A_{1,\text{in}} \\
&\quad + \mathrm{e}^{\mathrm{i}\phi_{12}} (\gamma_0 A_2 - \sqrt{\gamma_0} A_{2,\text{in}}) + \varepsilon_1, \\
\dot{A}_2 &= -MA_2 - \mathrm{i}\chi A_2 \sigma_2^z + \sqrt{\gamma_0} A_{2,\text{in}} \\
&\quad + \sum_{j=1,3} \mathrm{e}^{\mathrm{i}\phi_{2j}} (\gamma_0 A_j - \sqrt{\gamma_0} A_{j,\text{in}}) + \varepsilon_2, \\
\dot{A}_3 &= -MA_3 - \mathrm{i}\chi A_3 \sigma_3^z + \sqrt{\gamma_0} A_{3,\text{in}} \\
&\quad + \mathrm{e}^{\mathrm{i}\phi_{32}} (\gamma_0 A_2 - \sqrt{\gamma_0} A_{2,\text{in}}) + \varepsilon_3.
\end{aligned} \tag{A9}$$

To solve these equations explicitly, we first obtain the expectation values of cavity field operators through

$$\frac{\mathrm{d}\langle A_1 \rangle}{\mathrm{d}t} = \frac{\mathrm{d}\langle A_2 \rangle}{\mathrm{d}t} = \frac{\mathrm{d}\langle A_3 \rangle}{\mathrm{d}t} = 0. \tag{A10}$$

The steady states for cavity fields in C_1, C_2 and C_3 can be obtained as

$$\begin{aligned}
\alpha_1 &= \frac{\varepsilon_1 M^2 + \varepsilon_2 M \gamma_0 \mathrm{e}^{\mathrm{i}\phi_{12}} + \gamma_0^2 [\varepsilon_3 \mathrm{e}^{\mathrm{i}\Theta_{13}} - \varepsilon_1 \mathrm{e}^{\mathrm{i}\Phi_1}]}{M(M^2 - W^2)}, \\
\alpha_2 &= \frac{\varepsilon_2 M + \gamma_0 (\varepsilon_1 \mathrm{e}^{\mathrm{i}\phi_{21}} + \varepsilon_3 \mathrm{e}^{\mathrm{i}\phi_{23}})}{M^2 - W^2}, \\
\alpha_3 &= \frac{\varepsilon_3 M^2 + \varepsilon_2 M \gamma_0 \mathrm{e}^{\mathrm{i}\phi_{32}} + \gamma_0^2 [\varepsilon_1 \mathrm{e}^{\mathrm{i}\Theta_{31}} - \varepsilon_3 \mathrm{e}^{\mathrm{i}\Phi_3}]}{M(M^2 - W^2)},
\end{aligned} \tag{A11}$$

where $\Theta_{13} = \phi_{12} + \phi_{23}$, $\Phi_1 = \phi_{23} + \phi_{32}$, $\Theta_{31} = \phi_{32} + \phi_{21}$, and $\Phi_3 = \phi_{21} + \phi_{12}$.

Then, in the regime of strong cavity leakage and large detuning (which lead to $\gamma_0, \Delta \gg \chi$), kinetic equation (A9) is reformed into the following homogeneous linear equations:

$$\begin{aligned}
\dot{a_1} &= -Ma_1 - \mathrm{i}\chi \alpha_1 \sigma_1^z + \sqrt{\gamma_0} a_{1,\text{in}} \\
&\quad + \mathrm{e}^{\mathrm{i}\phi_{12}} (\gamma_0 a_2 - \sqrt{\gamma_0} a_{2,\text{in}}), \\
\dot{a_2} &= -Ma_2 - \mathrm{i}\chi \alpha_2 \sigma_2^z + \sqrt{\gamma_0} a_{2,\text{in}} \\
&\quad + \sum_{j=1,3} \mathrm{e}^{\mathrm{i}\phi_{2j}} (\gamma_0 a_j - \sqrt{\gamma_0} a_{j,\text{in}}), \\
\dot{a_3} &= -Ma_3 - \mathrm{i}\chi \alpha_3 \sigma_3^z + \sqrt{\gamma_0} a_{3,\text{in}} \\
&\quad + \mathrm{e}^{\mathrm{i}\phi_{32}} (\gamma_0 a_2 - \sqrt{\gamma_0} a_{2,\text{in}}),
\end{aligned} \tag{A12}$$

where we have replaced field operators A_i with $a_i + \alpha_i$ $(i = 1, 2, 3)$. In solving Eq.(A12), one can adiabatically eliminate the effect of vacuum input noise. The resulting cavity field operators are now represented by linear combinations of atomic spin operators σ_i^z $(i = 1, 2, 3)$. Substituting the resulting field operators into Eq.(A1), we obtain the effective Hamiltonian of the global system in the interaction picture as

$$\begin{aligned}
H_{\text{eff}} &= J_{12} \sigma_1^z \sigma_2^z + J_{23} \sigma_2^z \sigma_3^z + J_{31} \sigma_3^z \sigma_1^z \\
&\quad + \sum_i \Gamma_i (\sigma_i^+ + \sigma_i^-),
\end{aligned} \tag{A13}$$

where J_{12}, J_{23} and J_{31} are expressed by expression (2).

In deriving expression (A13), we neglect self-energy terms including σ_i^z and self-interaction terms including $(\sigma_i^z)^2$ and $(\sigma_i^z)^3$ that do not change the initial system state. Also, we eliminate higher order terms that include $\chi^3 \sigma_1^z \sigma_2^z \sigma_3^z$, for the corresponding coupling coefficients are much weaker than J_{12}, J_{23} and J_{31}. The typical difference between this Hamiltonian and that in Ref.[6] lies in the third term in expression (A13).

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