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Scheme for implementing quantum dense coding with W-class state in cavity QED*

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An experimentally feasible protocol for realizing dense coding by using a class of W-state in cavity quantum electrodynamics (QED) is proposed in this paper. The prominent advantage of our scheme is that the successful probability of the dense coding with a W-class state can reach 1. In addition, the scheme can be implemented by the present cavity QED techniques.

Keywords: quantum dense coding, W-class state, cavity quantum electrodynamics (QED)
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1. Introduction

Quantum entanglement, one of the most fascinating features of quantum mechanics, not only provides an important tool for distinguishing the quantum mechanics from the classical physics, but also plays a key role in quantum information processing and quantum computation. With the wide application of entanglement in quantum information processing, more and more physicists do their best to prepare entangled states in experiment. Recently, a number of schemes for preparing various entangled states have been proposed.[1−10]

There have been some typical examples utilizing the characteristic of quantum entanglement, such as quantum teleportation,[11] quantum dense coding,[12] quantum cryptography,[13] and so on. Here, we pay our attention to the quantum dense coding. It is well known that the quantum dense coding is the process of transmitting two-bit classical information by sending only one qubit with the aid of long-range Einstein–Podolsky–Rosen (EPR) pairs.[14] In experiment, two-particle quantum dense coding has been demonstrated by using optical system and nuclear magnetic resonance (NMR).[15−17] On the other hand, cavity quantum electrodynamics (QED) is a system of atoms interacting with a quantized electromagnetic field and has been proved to be an ideal system for experimentally implementing quantum communication protocols. So, all kinds of protocols that have been proposed carry out the dense coding by using bipartite or tripartite entangled states in cavity QED.[18−23] As far as the two-qubit entangled states are concerned, Lin *et al*[24] proposed a scheme to implement the quantum dense coding for two three-level atoms in cavity QED. Ye and Guo[25] suggested an alternative scheme for implementing the quantum dense coding in cavity QED. The scheme was insensitive to both the cavity decay and the thermal field, and its successful probability was 1. As for the three-particle side, Ye and Yu[26] proposed a feasible scheme for the dense coding in cavity QED by using a tripartite entangled Greenberger–Horne–Zeilinger (GHZ) state[27] and a W state.[28] We note that the W state has a distinct property in contrast with the GHZ state, that is, when any one of the three qubits is lost, the remaining two qubits are still in entangled states.[2] Therefore, it is natural to assert that the W state is a better option for quantum channel in the quantum dense coding. However, using W state as quantum channel for dense coding is probabilistic. As indicated in Ref.[26], the dense coding with W state is a probabilistic scheme and the probability is 2/3. But in Ref.[29], Agrawal and Pati proved that a class of W-state could be used for perfect teleportation and dense coding with a probability of 100%. Nevertheless, they mentioned nothing about

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how to implement the quantum information process in experiment.

The main purpose of our paper is to realize the dense coding with a class of W-states in cavity QED. The W-class state can be prepared by only one step, i.e. just by sending three atoms simultaneously interacting with a single-mode cavity field. Our scheme need not carry out a three-qubit measurement which was depicted in Ref.[29] and the four orthogonal states can be differentiated completely only by using the interaction of two atoms with a single-mode nonresonant cavity. Thus, our protocol can realize the quantum dense coding completely.

2. Interaction of atoms with a single-mode cavity field

Consider N identical two-level atoms simultaneously interacting with a single-mode cavity field. In the interaction picture and the rotating-wave approximation, the interaction Hamiltonian is expressed as[30]

$$H_{\mathrm{I}} = \sum_{j=1}^{N} g_j (\mathrm{e}^{-\mathrm{i}\,\delta t} a^{+} S_j^{-} + \mathrm{e}^{\mathrm{i}\,\delta t} a S_j^{+}), \tag{1}$$

where $S_j^{-} = |g_j\rangle\langle e_j|$ and $S_j^{+} = |e_j\rangle\langle g_j|$ are spin flip operators down and up of the jth atom respectively; a^{+} and a are the creation and annihilation operators of the cavity mode respectively; g_j is the coupling constant of the jth atom to the cavity mode; δ is the detuning between the atomic transition frequency ω_0 and cavity frequency ω_{a}.

If in the case of resonance $\delta = 0$, the atom–cavity interaction Hamiltonian is described as follow:

$$H_{\mathrm{I}} = \sum_{j=1}^{N} g_j (a^{+} S_j^{-} + a S_j^{+}). \tag{2}$$

If in the case of large detuning $\delta \gg g$, there is no energy exchange between the atoms and the cavity field. Suppose that the initial state of cavity field is in the vacuum state, then the effective Hamiltonian[30] of the system will become

$$H_{\mathrm{eff}} = \lambda \left(\sum_{j=1,}^{N} |e_j\rangle\langle e_j| + \sum_{i,j=1}^{N} S_j^{+} S_i^{-} \right) i \neq j, \tag{3}$$

where $\lambda = g_j^2/\delta$.

3. The generation of W-class state

Consider that three identical atoms resonantly interact with a single-mode cavity, assume that atoms 1, 2 and 3 are initially in state $|egg\rangle_{321}$ and the cavity mode is in vacuum state $|0\rangle_{\mathrm{c}}$ and let atoms1, 2 and 3 interact with the cavity, which is governed by Hamiltonian (2), then after an interaction time t, the state of the system of the three atoms and the cavity will evolve to[31]

$$\begin{aligned} &|\Psi\rangle \\ &= \left[(g_3^2/E^2)\cos(Et) + (E^2 - g_3^2)/E^2\right] |gge\rangle_{123} |0\rangle_{\mathrm{c}} \\ &\quad +(g_3/E^2)[\cos(Et) - 1]\, |g\rangle_3 (g_2 |ge\rangle_{12} \\ &\quad +g_1 |eg\rangle_{12}) |0\rangle_{\mathrm{c}} - (\mathrm{i}g_3/E)\sin(Et)\, |ggg\rangle_{123} |1\rangle_{\mathrm{c}}, \end{aligned} \tag{4}$$

where $E = \sqrt{g_1^2 + g_2^2 + g_3^2}$. Choosing $Et = \pi$ and $g_3 = g_2 = \dfrac{1}{\sqrt{2}} g_1$, state (4) becomes

$$|W\rangle_{123} = \frac{1}{2}(|gge\rangle_{123} - |geg\rangle_{123} - \sqrt{2}\,|egg\rangle_{123}), \tag{5}$$

Performing a rotation operation $|e\rangle \to \mathrm{i}\,|e\rangle$ on atom 1 and $|g\rangle \to -|g\rangle$ on atom 3, then we can obtain the W-class state[29]

$$|W\rangle_{123} = \frac{1}{2}(|gge\rangle_{123} + |geg\rangle_{123} + \mathrm{i}\sqrt{2}\,|egg\rangle_{123}). \tag{6}$$

Thus using three atoms simultaneously interacting resonantly with a single-mode cavity field, we can prepare the W-class state.

4. Dense coding with W-class state

Next, we will consider implementing the quantum dense coding by using the above W-class state. Suppose that Alice and Bob share an entangled state $|W\rangle_{123}$. Alice owns particle 1, while Bob has particles 2 and 3.

To realize the quantum dense coding, Alice performs one of the four local operations $\{\mathrm{I}, \sigma_x, \mathrm{i}\sigma_y, \sigma_z\}$ (I is the identity operator and σ_i are three Pauli operators) on her atom (atom 1). Thus, the operations denote 2 bits of classical information, and will transform the state of three atoms into one of the following four states:

$$|\psi^{+}\rangle_{123} = \frac{1}{2}(|gge\rangle_{123} + |geg\rangle_{123} + \mathrm{i}\sqrt{2}\,|egg\rangle_{123}), \tag{7}$$

$$\left|\phi^{+}\right\rangle_{123}=\frac{1}{2}(|ege\rangle_{123}+|eeg\rangle_{123}+\mathrm{i}\sqrt{2}\,|ggg\rangle_{123}), \quad (8)$$

$$\left|\phi^{-}\right\rangle_{123}=\frac{1}{2}(|ege\rangle_{123}+|eeg\rangle_{123}-\mathrm{i}\sqrt{2}\,|ggg\rangle_{123}), \quad (9)$$

$$\left|\psi^{-}\right\rangle_{123}=\frac{1}{2}(|gge\rangle_{123}+|geg\rangle_{123}-\mathrm{i}\sqrt{2}\,|egg\rangle_{123}). \quad (10)$$

Then, Alice sends her atom 1 to Bob. The more important problem is how Bob differentiates the four states, i.e. how to realize the quantum dense coding. In the following, we introduce an experimentally feasible scheme for distinguishing the four states in detail.

Firstly, Bob performs a trivial single-qubit rotation $|e\rangle \to \mathrm{i}\,|e\rangle$ on atom 2. Expressions (7)–(10) can be, respectively, rewritten as

$$\left|\psi^{+}\right\rangle_{123}=\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|g\rangle_1(|ge\rangle_{23}+\mathrm{i}\,|eg\rangle_{23})\right] +\mathrm{i}\frac{1}{\sqrt{2}}\,|e\rangle_1 gg\rangle_{23}\,, \quad (11)$$

$$\left|\phi^{+}\right\rangle_{123}=\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|e\rangle_1(|ge\rangle_{23}+\mathrm{i}\,|eg\rangle_{23})\right] +\mathrm{i}\frac{1}{\sqrt{2}}\,|g\rangle_1 gg\rangle_{23}\,, \quad (12)$$

$$\left|\phi^{-}\right\rangle_{123}=\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|e\rangle_1(|ge\rangle_{23}+\mathrm{i}\,|eg\rangle_{23})\right] -\mathrm{i}\frac{1}{\sqrt{2}}\,|g\rangle_1 gg\rangle_{23}\,, \quad (13)$$

$$\left|\psi^{-}\right\rangle_{123}=\frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}|g\rangle_1(|ge\rangle_{23}+\mathrm{i}\,|eg\rangle_{23})\right] -\mathrm{i}\frac{1}{\sqrt{2}}\,|e\rangle_1 gg\rangle_{23}\,. \quad (14)$$

Then, he sends atoms 2 and 3 simultaneously into a vacuum cavity, in the case of large detuning, according to expression (3), by choosing the interaction time $t=\dfrac{\pi}{4\lambda}$, he obtains the following expressions:

$$\left|\psi^{+}\right\rangle_{123}=\frac{1}{\sqrt{2}}(|ge\rangle_{13}+\mathrm{i}|eg\rangle_{13})|g\rangle_2, \quad (15)$$

$$\left|\phi^{+}\right\rangle_{123}=\frac{1}{\sqrt{2}}(|ee\rangle_{13}+\mathrm{i}|gg\rangle_{13})|g\rangle_2, \quad (16)$$

$$\left|\phi^{-}\right\rangle_{123}=\frac{1}{\sqrt{2}}(|ee\rangle_{13}-\mathrm{i}|gg\rangle_{13})|g\rangle_2, \quad (17)$$

$$\left|\psi^{-}\right\rangle_{123}=\frac{1}{\sqrt{2}}(|ge\rangle_{13}-\mathrm{i}|eg\rangle_{13})|g\rangle_2, \quad (18)$$

which are the four Bell states about atoms 1 and 3. Using a simple protocol in Ref.[25], Bob can tell the four bases exactly. Let atoms 1 and 3 simultaneously interact with a single-mode cavity and are driven by a classical field in the same time. Choose interaction time $\tau=\dfrac{\pi}{4\lambda}$ and adjusting Rabi frequency $\Omega\tau=\pi$, the four states will evolve to

$$\left|\psi^{+}\right\rangle_{13}\to\frac{1}{2}[(|ge\rangle_{13}-\mathrm{i}|eg\rangle_{13})+\mathrm{i}(|eg\rangle_{13}-\mathrm{i}|ge\rangle_{13})]=|ge\rangle_{13}, \quad (19)$$

$$\left|\phi^{+}\right\rangle_{13}\to\frac{1}{2}[(|ee\rangle_{13}-\mathrm{i}|gg\rangle_{13})+\mathrm{i}(|gg\rangle_{13}-\mathrm{i}|ee\rangle_{13})]=|ee\rangle_{13}, \quad (20)$$

$$\left|\phi^{-}\right\rangle_{13}\to\frac{1}{2}[(|ee\rangle_{13}-\mathrm{i}|gg\rangle_{13})-\mathrm{i}(|gg\rangle_{13}-\mathrm{i}|ee\rangle_{13})]=-\mathrm{i}|gg\rangle_{13}, \quad (21)$$

$$\left|\psi^{-}\right\rangle_{13}\to\frac{1}{2}[(|ge\rangle_{13}-\mathrm{i}|eg\rangle_{13})-\mathrm{i}(|eg\rangle_{13}-\mathrm{i}|ge\rangle_{13})]=-\mathrm{i}|eg\rangle_{13}. \quad (22)$$

Hence, it is easy to differentiate the Bell-state measurement exactly by detecting atoms 1 and 3 separately. In terms of the measurement result, Bob can tell Alice's operation on atom 1. In other words, Alice only sends Bob one qubit, but Bob obtains two bits of classical information. So, the target of dense coding is achieved. What is more, the probability of implementing quantum dense coding is 100%.

Secondly, we give a brief discussion on the experimental feasibility of the proposed scheme. The values of radiative time of Rydberg atoms, T_{r}, with principal quantum numbers 49, 50, and 51 are all about $3\times10^{-2}\,\mathrm{s}$; the decay time of the cavity, T_{c}, is approximately 1 ms; the atom–cavity coupling strength g is about $2\pi\times24\,\mathrm{kHz}$.[30,32] With the choice of $\delta=10\,g$, the required interaction time is on the order of $T\approx1.1\times10^{-4}\,\mathrm{s}$, then the time required to perform the whole procedure is on the order of $10^{-3}\,\mathrm{s}$, which is much shorter than T_{r}. Therefore, based on the present cavity QED techniques, the proposed scheme might be realizable.

5. Conclusion

We have proposed a simple experimental scheme to realize the quantum dense coding with a special

W state. Compared with the previous schemes, our scheme has some prominent advantages. Firstly, the successful probability of dense coding via a W-class state is equal to 1. Secondly, a W-class state used as quantum channel can be prepared by only one step, i.e. just by sending three atoms simultaneously interacting resonantly with a single-mode cavity field, and four orthonormal states can be differentiated completely only by using the interaction of two atoms with a single-mode nonresonant cavity. In addition, the scheme can be implemented by the present cavity QED techniques.

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