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One step to generate quantum controlled phase-shift gate using a trapped ion*

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This paper presents a very simple scheme for generating quantum controlled phase-shift gate with only one step by using the two vibrational modes of a trapped ion as the two qubits. The scheme couples two vibration degrees of freedom coupled with a suitable chosen laser excitation via the ionic states.

Keywords: one step, controlled phase-shift gate, trapped ion
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1. Introduction

Recently, much attention has been focused on the realization of quantum computer. Some investigations to the realization of quantum computer are important, such as quantum entanglement, which can not find its counterpart in the classical world and is the fundament of the new field in quantum information theory and plays a key role in the quantum information processing.[1] Quantum entanglement has been applied to quantum dense coding,[2] quantum teleportation,[3] and quantum cryptography.[4] One of the quantum entanglement states is the so-called cluster state,[5−8] which can be used to implement a quantum computer on a network. The quantum logical gates, also, are necessary for the realization of quantum computer. Up to now, many physical systems such as nuclear magnetic resonances,[9,10] trapped ions,[11,12] cavity QED[13−21] and optical systems,[22,23] have been proposed for quantum logical gate. During those physical systems, the trapped ion has been found to have many applications to quantum information processing. In Ref.[11], ions confined in a linear RF (Paul) trap are cooled and form a stable spatial array whose motion is described by the normal modes. Two internal levels in each atom form a qubit. The spacing between the ions is typically of the order of 1 μm, large enough that the direct coupling of the ions is negligible.

Most researches related to logical gates using ions have been focused on one-dimensional ionic motions. Our scheme eliminates the complexities of using two ions for the two-qubit gate. In this paper we present a very simple scheme for generating quantum controlled phase-shift gate with one step using the two vibrational modes of a trapped ion as the two qubits. The scheme couples two vibrational degrees of freedom coupled with a suitable chosen laser excitation via the ionic states. Our protocol is very simple and thus can be probably realized in experiment. The controlled phase-shift gate can be exploited to implement the communication of the different trapped ions, and it is important for quantum computation

The paper is organized as follows. In Section 2, we briefly review the Hamiltonian of the system. In Section 3, we make use of the Hamiltonian to implement the controlled phase-shift gate. A conclusion is given in Section 4.

2. The property of the Hamiltonian

Assume that the ion is confined to a bi-dimensional harmonic potential, so the motion of centre of mass is completely equivalent to a two-

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dimensional oscillator characterized by the oscillation frequencies $\upsilon_{\rm a}$ and $\upsilon_{\rm b}$ in two orthogonal directions, X and Y, with the corresponding creation {annihilation} operators $a^{\dagger}(a)$ and $b^{\dagger}(b)$ in the corresponding X and Y directions.

Consider a three-level ion confined in a two-dimensional harmonic potential in a configuration as shown in Fig.1, the ion is excited by two laser beams linearly polarized in the X and Y directions, corresponding to the energy levels $|g\rangle \leftrightarrow |f\rangle$ and $|f\rangle \leftrightarrow |e\rangle$. The two lasers are detuned to the excited state $|f\rangle$ respectively, thus generating a Raman transition between $|g\rangle$ and $|e\rangle$ states. After performing the rotating-wave and dipole approximations, one can obtain

$$\begin{aligned} H \;=\; & \hbar\omega_g \left|g\right\rangle \left\langle g\right| + \hbar\omega_f \left|f\right\rangle \left\langle f\right| + \hbar\omega_e \left|e\right\rangle \left\langle e\right| \\ & + \hbar\upsilon_a a^{\dagger}a + \hbar\upsilon_b b^{\dagger}b + \hbar\Omega_{gf} \left|g\right\rangle \left\langle f\right| \mathrm{e}^{-\mathrm{i}(k_{gf}x-\omega_{gf}t)} \\ & + \hbar\Omega_{gf}^{*} \left|f\right\rangle \left\langle g\right| \mathrm{e}^{\mathrm{i}(k_{gf}x-\omega_{gf}t)} \\ & + \hbar\Omega_{fe} \left|e\right\rangle \left\langle f\right| \mathrm{e}^{-\mathrm{i}(k_{fe}x-\omega_{fe}t)} \\ & + \hbar\Omega_{fe}^{*} \left|f\right\rangle \left\langle e\right| \mathrm{e}^{\mathrm{i}(k_{fe}x-\omega_{fe}t)}, \end{aligned} \tag{1}$$

where k_{gf} and k_{fe} are the wavenumbers, ω_{gf} and ω_{fe} are the laser frequencies, the frequencies ω_g, ω_f and ω_e are the ones associated with the levels $|g\rangle$, $|f\rangle$ and $|e\rangle$ respectively, and the operators $a^{\dagger}(a)$ and $b^{\dagger}(b)$ are the creation (annihilation) operators of vibrational states corresponding to the X and Y directions respectively. x and y are the quadratures corresponding to X and Y respectively, which can be written as $x = \Delta x_0(a + a^{\dagger})$, $y = \Delta y_0(b + b^{\dagger})$, where $\Delta x_0 = (\hbar/(2\upsilon_{\rm a}m))^{1/2}$ and $\Delta y_0 = (\hbar/(2\upsilon_b m))^{1/2}$ are the widths of the bi-dimensional potential ground states in the X and Y directions respectively, m is the mass of the ion.

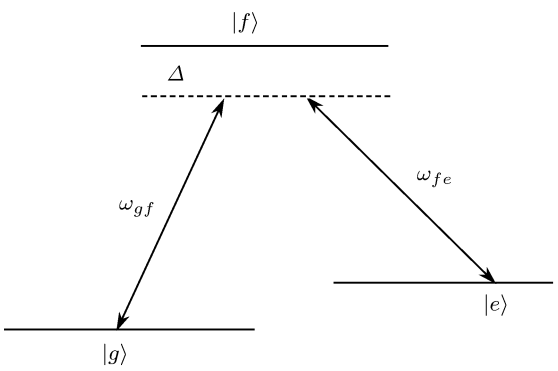


Fig.1. Energy level diagram of a three-level trapped ion interacting with two classical laser fields at Rabi frequencies Ω_{gf} and Ω_{fe}, and Δ is the detuning between lasers and the excited state. The two lasers are polarized in the X and Y directions respectively and therefore will excite vibrations in those directions.

Consider in the dispersive case of large detunings as follows:

$$\left|\Delta_{gf}\right|, \left|\Delta_{fe}\right| \gg \left|\Omega_{gf}\right|, \left|\Omega_{fe}\right|, \left|\Delta_{gf} - \Delta_{fe}\right|, \tag{2}$$

where $\Delta_{fe} \equiv \omega_f - \omega_e - \omega_{fe}$, $\Delta_{gf} \equiv \omega_f - \omega_g - \omega_{gf}$, Ω_{gf} and Ω_{fe} are the coupling constants corresponding to X and Y respectively. Performing an adiabatic elimination, one can obtain the effective Hamiltonian[24,25]

$$\begin{aligned} H \;=\; & \hbar\widetilde{\omega}_g \left|g\right\rangle \left\langle g\right| + \hbar\widetilde{\omega}_e \left|e\right\rangle \left\langle e\right| + \hbar\upsilon_a a^{\dagger}a + \hbar\upsilon_b b^{\dagger}b \\ & + \hbar\Omega_{ge} \left|g\right\rangle \left\langle e\right| \mathrm{e}^{-\mathrm{i}(k_{gf}x-k_{fe}y-(\omega_{gf}-\omega_{fe})t)} \\ & + \hbar\Omega_{ge}^{*} \left|e\right\rangle \left\langle g\right| \mathrm{e}^{\mathrm{i}(k_{gf}x-k_{fe}y-(\omega_{gf}-\omega_{fe})t)}, \end{aligned} \tag{3}$$

where the state $|f\rangle$ is adiabatically eliminated. The effective Raman coupling constant is $\Omega_{ge} = \Omega_{gf}\Omega_{fe}^{*}(1/\Delta_{gf} + 1/\Delta_{fe})$, the $\hbar\widetilde{\omega}_g$ and $\hbar\widetilde{\omega}_e$ are the Stark shifted atomic energies as a result of the adiabatic process, given by $\widetilde{\omega}_g = \omega_g - 2\left|\Omega_{gf}\right|^2/\Delta_{gf}$ and $\widetilde{\omega}_e = \omega_e - 2\left|\Omega_{fe}\right|^2/\Delta_{fe}$ respectively, the Hamiltonian in the Raman regime and in the interaction picture is

$$\begin{aligned} H_{\rm I} \;=\; & -\hbar\omega_{ge}\mathrm{e}^{-\frac{1}{2}(\eta_{gf}^2+\eta_{fe}^2)} \left|g\right\rangle \left\langle e\right| \\ & \times \sum_{m,\mu,n,\upsilon} \frac{(-\mathrm{i}\eta_{gf})^{m+\mu}}{m!\mu!} \frac{(-\mathrm{i}\eta_{fe})^{n+\upsilon}}{n!\upsilon!} a^{+m}a^{\mu}b^{+\upsilon}b^{n} \\ & \times \mathrm{e}^{\mathrm{i}(\upsilon_a[m-\mu]+\upsilon_b[\upsilon-n]+\Delta_{ge})t} + \mathrm{h.c.}, \end{aligned} \tag{4}$$

where we have defined the Raman detuning as $\Delta_{ge} = \omega_{gf} - \omega_{fe} - (\widetilde{\omega}_g - \widetilde{\omega}_e)$ and the Lamb–Dicke parameter directions as $\eta_{gf} = \Delta x_0 k_{gf}$ and $\eta_{fe} = \Delta y_0 k_{fe}$ respectively in the X and Y directions.

Now, one can choose the detuning parameter by appropriately selecting the frequencies of the two lasers Eq.(4), so one can obtain resonance between two specific vibrational levels. For illustrating this phenomenon, we explore a virtual level $|c\rangle$ with an energy $\hbar\omega_c$, which makes the Raman transition between different vibrational levels. If choose $\omega_{gf} = (\omega_c + \widetilde{\omega}_g) - k_a\upsilon_a$ and $\omega_{fe} = (\omega_c - \widetilde{\omega}_e) + l_b\upsilon_a$, then with respect to the $|c\rangle$ level, the first laser is tuned to the k_a red sideband of the X vibration and the second laser is tuned to the l_b red sideband corresponding to the Y vibration. One can obtain the Raman detuning

$$\Delta_{ge} = -k_a\upsilon_a - l_b\upsilon_b, \tag{5}$$

where k_a and l_b are positive integers, which can be controlled as well as the phase of the coupling constant g. Considering only the resonant terms Eq.(4) and having $m = \mu + k_a$ and $\upsilon = n + l_b$, we find that the Hamiltonian (4) becomes as follows:

$$H_{\rm I} = \hbar\left|g\right\rangle \left\langle e\right| \sum_{\mu,n} \Omega(\mu,n) a^{\dagger k_a} a^{\dagger\mu} a^{\mu} b^{\dagger l_b} b^{\dagger n} b^{n} + \mathrm{h.c.}, \tag{6}$$

where the coupling constants is defined as

$$\Omega(\mu,n)=-\Omega_{ge}\mathrm{e}^{-\frac{1}{2}(\eta_{gf}^2+\eta_{fe}^2)}\frac{(-\mathrm{i}\eta_{gf})^{2\mu+k_a}}{\mu!(\mu+k_a)!}\frac{(\mathrm{i}\eta_{fe})^{2n+l_b}}{n!(n+l_b)!}. \tag{7}$$

Under the Lamb–Dicke limit η_{gf}, $\eta_{fe}\ll 1$ and keeping only the lowest order terms in η_{gf} and η_{fe}, from Eqs.(6) and (7) one can obtain ($\mu=n=0$)

$$H_{\mathrm{I}}=\hbar\left\{|g\rangle\langle e|\,\Omega a^{\dagger k_a}b^{\dagger l_b}+|e\rangle\langle g|\,\Omega^* a^{k_a}b^{l_b}\right\}. \tag{8}$$

3. Generation of quantum controlled phase-shift gate

In this section, we use Eq.(8) to implement quantum controlled phase-shift gate. By adjusting the lasers we can obtain suitable time evolutions of our systems. Choosing the two combinations $k_a=1$ and $l_b=1$, one can obtain

$$\begin{aligned}&|g\rangle|m\rangle_a|n\rangle_b\\ \rightarrow\ &\cos(\sqrt{mn}\Omega t)|g\rangle|m\rangle_a|n\rangle_b\\ &-\mathrm{i}\sin(\sqrt{mn}\Omega t)|e\rangle|m-1\rangle_a|n-1\rangle_b,\end{aligned} \tag{9}$$

where m and n are the vibrational states of the ion in the X and Y directions respectively. If the initial internal ionic state is $|g\rangle$, we can implement the controlled phase-shift gate with only one step.

Choosing the interval time $t=\dfrac{\pi}{\Omega}$ and different numbers m and n, then from Eq.(9) we can obtain the evolutions of the different initial states

$$\begin{aligned}&|g\rangle|0\rangle_a|0\rangle_b\rightarrow|g\rangle|0\rangle_a|0\rangle_b,\\ &|g\rangle|0\rangle_a|1\rangle_b\rightarrow|g\rangle|0\rangle_a|1\rangle_b,\\ &|g\rangle|1\rangle_a|0\rangle_b\rightarrow|g\rangle|1\rangle_a|0\rangle_b,\\ &|g\rangle|1\rangle_a|1\rangle_b\rightarrow-|g\rangle|1\rangle_a|1\rangle_b.\end{aligned} \tag{10}$$

We have implemented the controlled phase-shift gate with one step, in which the vibration state m is the control bit and the vibration state n is the target bit.

As we know, a C-NOT gate can be used as a controlled phase-shift gate by three steps as follows:

$$\begin{aligned}|1\rangle_a|1\rangle_b&\xrightarrow{H_b}|1\rangle_a(|0\rangle_b-|1\rangle_b)/\sqrt{2}\\ &\xrightarrow{\Phi_{a,b}}|1\rangle_a(|0\rangle_b+|1\rangle_b)/\sqrt{2}\\ &\xrightarrow{H_b}|1\rangle_a|0\rangle_b,\end{aligned} \tag{11}$$

where H is a Hadamard operation ($|0\rangle\rightarrow(|0\rangle+|1\rangle)/\sqrt{2}, |1\rangle\rightarrow(|0\rangle-|1\rangle)/\sqrt{2}$), $\Phi_{a,b}$ is a C-NOT gate where qubit a is the control bit and b is the target bit. In our scheme only one step is required. Therefore, our protocol is important. Moreover, most researches related to logical gates using ions have been focused on one-dimensional ionic motions. Our scheme eliminates the complexities of using two ions for the two-qubit gate, and we assume that the ion is confined to a bi-dimensional harmonic potential. Now, we demonstrate the experimental side. Our scheme uses the two vibrational modes of a trapped ion as the two qubits, and will probably meet some difficulties in experiment. Making use of a cavity, different trapped ions can communicate and it is useful for computational purposes. In Ref.[26], Mundt *et al* proposed a protocol with a single calcium ion stored in a spherical trap and placed in the centre of a confocal resonator, and achieved deterministic coupling of a cavity mode to the ion's vibrational state by selective tuning the cavity field to a vibrational sideband of the $S_{1/2}-D_{5/2}$ electronic transition.

4. Conclusions

In this paper, we have implemented quantum controlled phase-shift gate with one step by using the two vibrational modes and the internal states of a trapped ion. Our scheme is very simple and eliminates the complexities of using two ions for the two-qubit gate. Especially, our scheme only uses one step, so it is very important. It is useful for quantum computation by exploring the different trapped ions.

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