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Quantum teleportation of an arbitrary superposition of atomic states

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This paper proposes a scheme to teleport an arbitrary multi-particle two-level atomic state between two parties or an arbitrary zero- and one-photon entangled state of multi-mode between two high-Q cavities in cavity QED. This scheme is based on the resonant interaction between atom and cavity and does not involve Bell-state measurement. It investigates the fidelity of this scheme and find out the case of this unity fidelity of this teleportation. Considering the practical case of the cavity decay, this paper finds that the condition of the unity fidelity is also valid and obtains the effect of the decay of the cavity on the successful probability of the teleportation.

Keywords: teleportation, cavity QED, fidelity, cavity decay
PACC: 0365

1. Introduction

Quantum teleportation first introduced by Bennett *et al*[1] has received great attention because of its esoteric characteristic and its wide application in quantum computing and quantum communication. To implement teleportation between two separated observers namely Alice and Bob, they have to take four steps: (i) to prepare a state to be teleported (Alice); (ii) to prepare the nonlocal channel; (iii) to do a Bell measurement (Alice) and (iv) to make a classical communication from Alice to Bob. Since this proposal was suggested, a number of feasible schemes based on the original idea have been proposed for the teleportation of atomic states or field states in the realm of cavity QED.[2−12] The first experimental realization of teleportation of atomic qubits was demonstrated by Riebe *et al*[13] and Barrett *et al*[14] in a trap recently. There are also some protocols for teleportation differing from the original scheme for the reason of no Bell-state measurement.[15−17] In Ref.[16], Zheng introduces the approximate teleportation of an unknown atomic state with the fidelity near 99%. In his scheme no shared particles between Alice and Bob, because of the resonant interaction between atom and cavity, the cavity is used to store Alice's information and transfer the information to Bob after conditional dynamics.

But these schemes are used for a few qubits, we need the system of multi-qubit states in quantum communication and quantum computing such as factorizing a very large number[18] or searching an unordered quantum database.[19] Therefore, the teleportation of a multi-qubit state is significant. There are some schemes for this kind of teleportation.[20−23] However, the multi-qubit states that they teleported are in special form such as atomic Dicke states in Ref.[23]. We improve their studies to teleport an arbitrary superposition of two-level atomic states in this paper.

There are two characteristics in our scheme: One is that no Bell-state measurement is employed. The other is that an arbitrary superposition of two-level atomic states can be transferred between two separated parties. The scalability of the two-qubit case in Ref.[17] to teleport the zero- and one-photon entangled state has also been obtained. To discuss the experimental possibility of our proposal, we consider the case of the cavity decay and find its effect on the success probability. In addition, the case of the unity fidelity applied to not only the ideal case but also the case of the cavity decay is obtained in this paper.

2. Two-atom teleportation

We discuss the two-atom case first. The atomic state to be teleported is assumed to be a superposition

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state

$$|\Psi\rangle_A = \sum_{p_1,p_2=0}^{1} \alpha_{p_1,p_2}|q_{p_1}\rangle_{\mathrm{A}_1}|q_{p_2}\rangle_{\mathrm{A}_2}, \quad (1)$$

where the state $|q_0\rangle = |g\rangle$ and $|q_1\rangle = |e\rangle$ are the ground and excited state of an atom respectively, α_{p_1,p_2} is an unknown coefficient. At the beginning of the teleportation the two atoms and the two well separated modes of an empty resonant cavity are prepared in the state $|e\rangle_{\mathrm{B}_1}|e\rangle_{\mathrm{B}_2}|0\rangle_{\mathrm{C}_1}|0\rangle_{\mathrm{C}_2}$. Bob drives the atom B_1 flying through the initially empty resonant cavity. During its flying in the cavity, the interaction of the atom B_1 and the mode C_1 can be described as the Jaynes–Cummings Hamiltonian, for the resonant interaction it is expressed as

$$H_{\mathrm{I}} = g(a^{\dagger}S^{-} + aS^{+}) \quad (2)$$

with $a^{\dagger}$ and a being the creation and annihilation operator for the cavity field, S^{+} and S^{-} being the raising and lowering operator for atoms, g being the atom–field coupling constant. We note that when the atom interacts with one mode of the resonant cavity, the other mode remains unaffected. After simple calculation, the evolution of the state of the atoms (B_1, B_2) and modes (C_1, C_2) is given by

$$|\Psi(t)\rangle_{\mathrm{BC}} = [\cos(gt)|e\rangle_{\mathrm{B}_1}|0\rangle_{\mathrm{C}_1} - \mathrm{i}\sin(gt)|g\rangle_{\mathrm{B}_1}|1\rangle_{\mathrm{C}_1}]|e\rangle_{\mathrm{B}_2}|0\rangle_{\mathrm{C}_2}. \quad (3)$$

Bob controls the velocity of atom B_1 flying through the cavity, which keeps $gt = \pi/4$, then the state of Eq.(3) becomes

$$|\Psi(\pi/4)\rangle_{\mathrm{BC}} = \frac{1}{\sqrt{2}}(|e\rangle_{\mathrm{B}_1}|0\rangle_{\mathrm{C}_1} - \mathrm{i}|g\rangle_{\mathrm{B}_1}|1\rangle_{\mathrm{C}_1})|e\rangle_{\mathrm{B}_2}|0\rangle_{\mathrm{C}_2}. \quad (4)$$

Now the state of the whole system is

$$|\Psi\rangle_{\mathrm{ABC}} = \frac{1}{\sqrt{2}}(|e\rangle_{\mathrm{B}_1}|0\rangle_{\mathrm{C}_1} - \mathrm{i}|g\rangle_{\mathrm{B}_1}|1\rangle_{\mathrm{C}_1})|e\rangle_{\mathrm{B}_2}|0\rangle_{\mathrm{C}_2} \times \sum_{p_1,p_2=0}^{1} \alpha_{p_1,p_2}|q_{p_1}\rangle_{\mathrm{A}_1}|q_{p_2}\rangle_{\mathrm{A}_2}. \quad (5)$$

Here we discuss the evolution of $|q_{p_i}\rangle_{\mathrm{A}_i}(|e\rangle_{\mathrm{B}_i}|0\rangle_{\mathrm{C}_i} - \mathrm{i}|g\rangle_{\mathrm{B}_i}|1\rangle_{\mathrm{C}_i})$, for simplicity later, which is labelled as $|\phi_{p_i}\rangle_i$, $i \in \{1,2\}$: Alice sends the atom A_i flying through the initially empty resonant cavity. During its flying in the cavity, the interaction of the atom A_i and the mode C_i governed by the Hamiltonian Eq.(2) as the same as the interaction of atom B_1 and mode C_1 leads the state $|\phi_{p_i}\rangle_i$ to be

$$|\phi_{p_i}(gt')\rangle_i = \begin{cases} |g\rangle_{\mathrm{A}_i}|e\rangle_{\mathrm{B}_i}|0\rangle_{\mathrm{C}_i} - \mathrm{i}[\cos(gt')|g\rangle_{\mathrm{A}_i}|g\rangle_{\mathrm{B}_i}|1\rangle_{\mathrm{C}_i} - \mathrm{i}\sin(gt')|e\rangle_{\mathrm{A}_i}|g\rangle_{\mathrm{B}_i}|0\rangle_{\mathrm{C}_i}] & \text{if} \quad p_i = 0, \\ \cos(gt')|e\rangle_{\mathrm{A}_i}|e\rangle_{\mathrm{B}_i}|0\rangle_{\mathrm{C}_i} - \mathrm{i}\sin(gt')|g\rangle_{\mathrm{A}_i}|e\rangle_{\mathrm{B}_i}|1\rangle_{\mathrm{C}_i} & \\ -\mathrm{i}[\cos(\sqrt{2}gt')|e\rangle_{\mathrm{A}_i}|g\rangle_{\mathrm{B}_i}|1\rangle_{\mathrm{C}_i} - \mathrm{i}\sin(\sqrt{2}gt')|g\rangle_{\mathrm{A}_i}|g\rangle_{\mathrm{B}_i}|2\rangle_{\mathrm{C}_i}] & \text{if} \quad p_i = 1, \end{cases} \quad (6)$$

and then Alice detects the atom A_i, when she finds atom A_i in the state $|e\rangle_{\mathrm{A}_i}$, the state $|\phi_{p_i}(gt')\rangle_i$ is projected into the state

$$|\phi'_{p_i}(gt')\rangle_i \propto |\phi''_{p_i}(gt')\rangle_i = \begin{cases} -\sin(gt')|e\rangle_{\mathrm{A}_i}|g\rangle_{\mathrm{B}_i}|0\rangle_{\mathrm{C}_i} & \text{if} \quad p_i = 0, \\ |e\rangle_{\mathrm{A}_i}[\cos(gt')|e\rangle_{\mathrm{B}_i}|0\rangle_{\mathrm{C}_i} - \mathrm{i}(\cos(\sqrt{2}gt')|g\rangle_{\mathrm{B}_i}|1\rangle_{\mathrm{C}_i}] & \text{if} \quad p_i = 1. \end{cases} \quad (7)$$

Insert $|\phi'_{p_1}(gt')\rangle_1$ into the Eq.(5), the state of the whole system collapses in the form

$$|\Psi(gt')\rangle_{\mathrm{ABC}} = N_1 \sum_{p_1,p_2=0}^{1} \alpha_{p_1,p_2}|\phi''_{p_1}(gt')\rangle_1|q_{p_2}\rangle_{\mathrm{A}_2}|e\rangle_{\mathrm{B}_2}|0\rangle_{\mathrm{C}_2}, \quad (8)$$

where N_1 is the normalization factor.

As the same as he controls atom B_1 interacting with C_1, Bob controls the atom B_2 interacting with the mode C_2 for $gt = \pi/4$ resonantly, and then Alice sends the atom A_2 interacting with the mode C_2 for the time t' in the same way as stated be-

fore, finally Alice measures the atom A_2, when the atom A_2 is in the state $|e\rangle_{A_2}$, setting $\sin(gt') = M_1(t')$, $\cos(gt') = M_2(t')$, $\sin(gt')\cos(gt') = L(t')$ and $\cos(\sqrt{2}gt') = T(t')$ the whole system directly collapses to be in the state

$$\begin{aligned}
&|\Phi(gt')\rangle_{ABC}\\
=&N_2\sum_{p_1,p_2=0}^{1}\alpha_{p_1,p_2}|\phi''_{p_1}(gt')\rangle_1|\phi''_{p_2}(gt')\rangle_2\\
=&N_2|e\rangle_{A_1}|e\rangle_{A_2}\{\alpha_{0,0}M_1^2(t')|g\rangle_{B_1}|g\rangle_{B_2}|0\rangle_{C_1}|0\rangle_{C_2}\\
&+\alpha_{0,1}[-L(t')|g\rangle_{B_1}|e\rangle_{B_2}|0\rangle_{C_1}|0\rangle_{C_2}\\
&+\mathrm{i}M_1(t')T(t')|g\rangle_{B_1}|g\rangle_{B_2}|0\rangle_{C_1}|1\rangle_{C_2}]\\
&+\alpha_{1,0}[-L(t')|e\rangle_{B_1}|g\rangle_{B_2}|0\rangle_{C_1}|0\rangle_{C_2}\\
&+\mathrm{i}M_1(t')T(t')|g\rangle_{B_1}|g\rangle_{B_2}|1\rangle_{C_1}|0\rangle_{C_2}]\\
&+\alpha_{1,1}[M_2^2(t')|e\rangle_{B_1}|e\rangle_{B_2}|0\rangle_{C_1}|0\rangle_{C_2}\\
&-\mathrm{i}M_2(t')T(t')|g\rangle_{B_1}|e\rangle_{B_2}|1\rangle_{C_1}|0\rangle_{C_2}\\
&-\mathrm{i}M_2(t')T(t')|e\rangle_{B_1}|g\rangle_{B_2}|0\rangle_{C_1}|1\rangle_{C_2}\\
&-T^2(t')|g\rangle_{B_1}|g\rangle_{B_2}|1\rangle_{C_1}|1\rangle_{C_2}]\},
\end{aligned}\tag{9}$$

where N_2 is a normalization factor. From Eq.(9) we know that when $\cos(\sqrt{2}gt') \longrightarrow 0$, the state $|\Phi(gt')\rangle_{BC}$ reduces approximately to

$$\begin{aligned}
|\Phi'(gt')\rangle_{BC} = \{&\alpha_{0,0}M_1^2(t')|g\rangle_{B_1}|g\rangle_{B_2}\\
&-L(t')[\alpha_{0,1}|g\rangle_{B_1}|e\rangle_{B_2}+\alpha_{1,0}|e\rangle_{B_1}|g\rangle_{B_2}]\\
&+\alpha_{1,1}M_2^2(t')|e\rangle_{B_1}|e\rangle_{B_2}\}|0\rangle_{C_1}|0\rangle_{C_2}.
\end{aligned}\tag{10}$$

Let $gt' = 7\pi/4$, then $\cos(\sqrt{2}gt') \simeq 0.078 \simeq 0$ and $\cos(gt') = -\sin(gt') = \dfrac{1}{\sqrt{2}}$, the Eq.(10) becomes

$$\begin{aligned}
|\Phi'(7\pi/4)\rangle_{BC} = (&\alpha_{0,0}|g\rangle_{B_1}|g\rangle_{B_2}\\
&+\alpha_{0,1}|g\rangle_{B_1}|e\rangle_{B_2}+\alpha_{1,0}|e\rangle_{B_1}|g\rangle_{B_2}\\
&+\alpha_{1,1}|e\rangle_{B_1}|e\rangle_{B_2})|0\rangle_{C_1}|0\rangle_{C_2},
\end{aligned}\tag{11}$$

in which we realize the teleportation of the initial state from Alice to Bob, namely

$$|\Psi\rangle_B = \sum_{p_1,p_2=0}^{1}\alpha_{p_1,p_2}|q_{p_1}\rangle_{B_1}|q_{p_2}\rangle_{B_2}.\tag{12}$$

With the probability of success 0.25, the fidelity between $|\Phi'(gt')\rangle_{BC}$ and $|\Phi(gt')\rangle_{BC}$ can be obtained as

$$F = \frac{\alpha_{0,0}^2M_1^4(t') + (\alpha_{0,1}^2+\alpha_{1,0}^2)L^2(t') + \alpha_{1,1}^2M_2^4(t')}{\alpha_{0,0}^2M_1^4(t') + (\alpha_{0,1}^2+\alpha_{1,0}^2)[L^2(t')+T^2(t')M_1^2(t')] + \alpha_{1,1}^2[M_2^4(t')+T^4(t')+2T^2(t')M_2^2(t')]}.\tag{13}$$

The fidelity depends on α_{p_1,p_2}, when $gt' = 7\pi/4$, $F \geq 0.976$. When the teleported state is in the form $|\Psi\rangle_A = \alpha_{0,1}|e\rangle_{A_1}|g\rangle_{A_2} + \alpha_{1,0}|g\rangle_{A_1}|e\rangle_{A_2}$, the fidelity has nothing to do with α_{p_1,p_2}. We obtain $T(t') = \cos(\sqrt{2}gt') = 0$ exactly with $gt' = \pi/(2\sqrt{2})$ or its any odd multiple. Then Eq.(9) is

$$|\Phi'(\pi/(2\sqrt{2}))\rangle_{BC} = N_2[-\sin(\pi/(2\sqrt{2}))\cos(\pi/(2\sqrt{2}))][\alpha_{0,1}|e\rangle_{B_1}|g\rangle_{B_2} + \alpha_{1,0}|g\rangle_{B_1}|e\rangle_{B_2}]|0\rangle_{C_1}|0\rangle_{C_2}\tag{14}$$

and hence the protocol has the fidelity 1 and the success probability $\cos^2\left(\dfrac{\pi}{2\sqrt{2}}\right)\sin^2\left(\dfrac{\pi}{2\sqrt{2}}\right) = 0.158$. Moreover, the new choice also make the interaction time about five times shorter than the choice $gt' = 7\pi/4$.

3. N-atom and N-mode teleportation

To appreciate the scalability of the protocol, we discuss the generalization of our scheme to teleport an arbitrary multi-qubit superposition of two-level atomic state. The initial state of atoms to be teleported is assumed as

$$|\Psi\rangle_A = \sum_{p_1,\cdots,p_N=0}^{1}\alpha_{p_1,\cdots,p_N}|q_{p_1}\rangle_{A_1}\cdots|q_{p_N}\rangle_{A_N}.\tag{15}$$

The N atoms and the empty cavity are prepared in the state $|e\rangle_{B_1}|0\rangle_{C_1}\cdots|e\rangle_{B_N}|0\rangle_{C_N}$. For each qubit, Alice and Bob only do three operations: (i) the atom B_i $[i \in (1,\cdots,N)]$ is driven to interact with the cavity mode C_i for $gt = \pi/4$ by Bob, (ii) the atom A_i is sent to interact with the same mode C_i for t' by Alice, and (iii) the atom A_i is detected by Alice whether it is in the excited state $|e\rangle_{A_i}$. Similarly as before, $|q_{p_i}\rangle_{A_i}(|e\rangle_{B_i}|0\rangle_{C_i} - \mathrm{i}|g\rangle_{B_i}|1\rangle_{C_i})$, which is labelled as $|\phi_{p_i}\rangle_i$ $[i \in (1,\cdots,N)]$ evolves to $|\phi_{p_i}(gt')\rangle_i$ after the

second operation and $|\phi'_{p_i}(gt')\rangle_i$ after the third operation, that is just the same as the two atoms case.

All these operations for every qubit are accomplished, that leads the state of the whole system to be

$$|\Phi(gt')\rangle_{\rm ABC} = M_N \sum_{p_1,\cdots,p_N=0}^{1} \alpha_{p_1,\cdots,p_N} |\phi''_{p_1}(gt')\rangle_1 \cdots \times |\phi''_{p_i}(gt')\rangle_i \cdots |\phi''_{p_N}(gt')\rangle_N. \quad (16)$$

Let $gt' = 7\pi/4$, then $\cos(\sqrt{2}gt') \simeq 0$ and $\cos(gt') = -\sin(gt') = \dfrac{1}{\sqrt{2}}$. The state of Bob's atoms and cavity approximately is

$$|\Phi'(7\pi/4)\rangle_{\rm BC} = \sum_{p_1,\cdots,p_N=0}^{1} \alpha_{p_1,\cdots,p_N} |q_{p_1}\rangle_{\rm B_1}|0\rangle_{\rm C_1} \cdots \times |q_{p_N}\rangle_{\rm B_N}|0\rangle_{\rm C_N}. \quad (17)$$

The state of Bob's atoms is approximately in the state of Alice's with the cavity left in the vacuum state. The success probability $\left(\dfrac{1}{2}\right)^N$ decreases polynomially with the number of qubits. When every superposition item has l atoms in the state $|e\rangle$, we find that setting $gt = \pi/(2\sqrt{2})$ or its any odd multiple, $\cos(\sqrt{2}gt) = 0$ exactly. Then the unity fidelity is obtained.

The idea can also be used to teleport the entanglement state of N modes of a high-Q cavity to another with N atoms. We assume that the cavity (cavity E) of Alice to be teleported is initially in the state

$$|\Psi\rangle_{\rm E} = \sum_{n_1,\cdots,n_N=0}^{1} \alpha_{n_1,\cdots,n_N} |n_1\rangle_{\rm E_1} \cdots |n_N\rangle_{\rm E_N}, \quad (18)$$

where the modes $\rm E_1$, $\rm E_2, \cdots$, $\rm E_N$ are well separated. The well separated modes in cavity D of Bob and the N atoms are also prepared in the state $|e\rangle_{\rm F_1}|0\rangle_{\rm D_1} \cdots |e\rangle_{\rm F_i}|0\rangle_{\rm D_i} \cdots |e\rangle_{\rm F_N}|0\rangle_{\rm D_N}$, each qubit also needs to be operated three times. The atom $\rm F_i$ $[i \in (1, \cdots, N)]$ is driven to interact with the mode $\rm D_i$ for $gt = \pi/4$ by Alice and then is made to interact with the mode $\rm E_i$ in another cavity for t' by Bob, finally Alice detects the atom $\rm F_i$ to find if it is in the excited state $|e\rangle_{\rm F_i}$. To begin with $i = 1$, the atom $\rm F_1$ interacts with the mode $\rm D_1$ for $gt = \pi/4$. The whole state is as follows:

$$|\Psi\rangle_{\rm FDE} = \frac{1}{\sqrt{2}} \sum_{n_1,\cdots,n_N=0}^{1} \alpha_{n_1,\cdots,n_N} (|e\rangle_{\rm F_1}|0\rangle_{\rm D_1} - {\rm i}|g\rangle_{\rm F_1}|1\rangle_{\rm D_1})|n_1\rangle_{\rm E_1} \cdots |e\rangle_{\rm F_i}|0\rangle_{\rm D_i}|n_i\rangle_{\rm E_i} \cdots |e\rangle_{\rm F_N}|0\rangle_{\rm D_N}|n_N\rangle_{\rm E_N}. \quad (19)$$

Similarly for simplicity, suppose $(|e\rangle_{\rm F_i}|0\rangle_{\rm D_i} - {\rm i}|g\rangle_{\rm F_i}|1\rangle_{\rm D_i})|n_i\rangle_{\rm E_i} = |\varphi_{n_i}\rangle_i$, $[i \in (1, \cdots, N)]$, the interaction of the atom $\rm F_i$ and the cavity mode $\rm E_i$ leads $|\varphi_{n_i}\rangle_i$ to be

$$|\varphi_{n_i}(gt')\rangle_i = \begin{cases} (\cos(gt')|e\rangle_{\rm F_i}|0\rangle_{\rm D_i}|0\rangle_{\rm E_i} - {\rm i}\sin(gt')|g\rangle_{\rm F_i}|0\rangle_{\rm D_i}|1\rangle_{\rm E_i}) - {\rm i}|g\rangle_{\rm F_i}|0\rangle_{\rm D_i}|0\rangle_{\rm E_i}, & \text{if} \quad n_i = 0, \\ \cos(\sqrt{2}gt')|e\rangle_{\rm F_i}|0\rangle_{\rm D_i}|1\rangle_{\rm E_i} - {\rm i}\sin(\sqrt{2}gt')|g\rangle_{\rm F_i}|0\rangle_{\rm D_i}|2\rangle_{\rm E_i}, & \\ -{\rm i}(\cos(gt')|g\rangle_{\rm F_i}|1\rangle_{\rm D_i}|1\rangle_{\rm E_i} - {\rm i}\sin(gt')|e\rangle_{\rm F_i}|1\rangle_{\rm D_i}|0\rangle_{\rm E_i}), & \text{if} \quad n_i = 1. \end{cases} \quad (20)$$

Then the atom $\rm F_i$ is detected in the state $|e\rangle_{\rm F_i}$. The state $|\varphi_{n_i}(gt')\rangle_i$ evolves to

$$|\varphi'_{n_i}(gt')\rangle_i \propto |\varphi''_{n_i}(gt')\rangle_i = \begin{cases} \cos(gt')|e\rangle_{\rm F_i}|0\rangle_{\rm D_i}|0\rangle_{\rm E_i}, & \text{if} \quad n_i = 0 \\ \cos(\sqrt{2}gt')|e\rangle_{\rm F_i}|0\rangle_{\rm D_i}|1\rangle_{\rm E_i} - \sin(gt')|e\rangle_{\rm F_i}|1\rangle_{\rm D_i}|0\rangle_{\rm E_i}, & \text{if} \quad n_i = 1. \end{cases} \quad (21)$$

Now it is clearly that when the three operations for every qubit are finished, the whole system collapses to be in the state

$$|\Phi(gt')\rangle_{\rm FDE} = L_N \sum_{n_1,\cdots,n_N=0}^{1} \alpha_{n_1,\cdots,n_N} |\varphi''_{n_1}(gt')\rangle_1 \cdots \varphi''_{n_N}(gt')\rangle_N, \quad (22)$$

where L_N is the normalization factor. Just follow the above scheme, we choose $gt' = 7\pi/4$, the state of cavity D is approximately in the initial entangled state of cavity E,

$$|\Phi(7\pi/4)\rangle_{\rm D} \simeq \sum_{n_1,\cdots,n_N=0}^{1} \alpha_{n_1,\cdots,n_N} |n_1\rangle_{\rm D_1} \cdots |n_N\rangle_{\rm D_N} \quad (23)$$

Once every superposition item has the same number l modes of the cavity D in the state $|0\rangle$, $gt = \pi/(2\sqrt{2})$ or its any odd multiple is our optimized choice to obtain the unity fidelity. The Ref.[17] has presented two-mode case to teleport entangled zero- and one-photon states $|\Psi\rangle_{\rm E} = \alpha_{1,0}|1\rangle_{\rm E_1}|0\rangle_{\rm E_2} + \alpha_{0,1}|0\rangle_{\rm E_1}|1\rangle_{\rm E_2}$ with 0.25 successful probability. We find that the success probability to teleport the multi-mode state $\left(\frac{1}{2}\right)^N$ decreases polynomially with the number of qubits in our case.

4. The effect of the cavity decay

Considering the real experimental condition, we discuss the case of cavity decay. Assume that the cavity decay rate is κ, the evolution of the system is governed by the non-Hermitian Hamiltonian (assuming $\hbar = 1$)

$$H' = H_I - \mathrm{i}\frac{\kappa}{2}a^\dagger a. \tag{24}$$

Including the cavity decay the Eq.(3) becomes

$$\begin{aligned}|\Psi(t)\rangle_{\rm BC} =& \exp\left(-\frac{\kappa t}{4}\right)\left\{\left[\cos\left(\frac{\lambda t}{4}\right) - \frac{k}{\lambda}\sin\left(\frac{\lambda t}{4}\right)\right]|e\rangle_{\rm B_1}|0\rangle_{\rm C_1} - \mathrm{i}\frac{4g}{\lambda}\sin\left(\frac{\lambda t}{4}\right)|g\rangle_{\rm B_1}|1\rangle_{\rm C_1}\right\}|e\rangle_{\rm B_2}|0\rangle_{\rm C_2}\\ =& \exp\left(-\frac{\kappa t}{4}\right)\frac{4g}{\lambda}\left[\cos\left(\frac{\lambda t}{4}+\delta\right)|e\rangle_{\rm B_1}|0\rangle_{\rm C_1} - \mathrm{i}\sin\left(\frac{\lambda t}{4}\right)|g\rangle_{\rm B_1}|1\rangle_{\rm C_1}\right]|e\rangle_{\rm B_2}|0\rangle_{\rm C_2},\end{aligned} \tag{3'}$$

where $\lambda = \sqrt{16g^2 - \kappa^2}$. If the interaction time t is chosen to satisfy $\cos\left(\frac{\lambda t}{4}+\delta\right) = \sin\left(\frac{\lambda t}{4}\right)$, where $\cos(\delta) = \frac{\lambda}{4g}$, the Eq.(4) becomes

$$|\Psi(\pi/4)\rangle_{\rm BC} = \exp\left(-\frac{\kappa t}{4}\right)\frac{4g}{\lambda}\sin\left(\frac{\lambda t}{4}\right)(|e\rangle_{\rm B_1}|0\rangle_{\rm C_1} - \mathrm{i}|g\rangle_{\rm B_1}|1\rangle_{\rm C_1})|e\rangle_{\rm B_2}|0\rangle_{\rm C_2}. \tag{4'}$$

Similarly, we can label $|q_{p_i}\rangle_{\rm A_i}(|e\rangle_{\rm B_i}|0\rangle_{\rm C_i} - \mathrm{i}|g\rangle_{\rm B_i}|1\rangle_{\rm C_i})$ as $|\phi_{p_i}\rangle_i$, and get the expression corresponding to Eq.(6) as

$$|\phi_{p_i}(gt')\rangle_i = \begin{cases} |g\rangle_{\rm A_i}|e\rangle_{\rm B_i}|0\rangle_{\rm C_i} - \mathrm{i}\frac{4g}{\lambda}\mathrm{e}^{-\frac{\kappa t'}{4}}\left[\cos\left(\frac{\lambda t'}{4}+\delta\right)|g\rangle_{\rm A_i}|g\rangle_{\rm B_i}|1\rangle_{\rm C_i}\right.\\ \left.-\mathrm{i}\sin\left(\frac{\lambda t'}{4}\right)|e\rangle_{\rm A_i}|g\rangle_{\rm B_i}|0\rangle_{\rm C_i}\right], & \text{if} \quad p_i = 0,\\ \frac{4g}{\lambda}\mathrm{e}^{-\frac{\kappa t'}{4}}\left[\cos\left(\frac{\lambda t'}{4}+\delta\right)|e\rangle_{\rm A_i}|e\rangle_{\rm B_i}|0\rangle_{\rm C_i} - \mathrm{i}\sin\left(\frac{\lambda t'}{4}\right)|g\rangle_{\rm A_i}|e\rangle_{\rm B_i}|1\rangle_{\rm C_i}\right]\\ -\mathrm{i}\frac{4g\sqrt{2}}{\lambda'}\mathrm{e}^{-\frac{\kappa t'}{4}}\left[\cos\left(\frac{\lambda' t'}{4}+\delta'\right)|e\rangle_{\rm A_i}|g\rangle_{\rm B_i}|1\rangle_{\rm C_i} - \mathrm{i}\sin\left(\frac{\lambda' t'}{4}\right)|g\rangle_{\rm A_i}|g\rangle_{\rm B_i}|2\rangle_{\rm C_i}\right], & \text{if} \quad p_i = 1, \end{cases} \tag{6'}$$

where $\lambda' = \sqrt{32g^2 - \kappa^2}$, $\cos(\delta') = \frac{\lambda'}{4g\sqrt{2}}$. We complete the above teleportations by setting the interaction time t' of atom and cavity to satisfy the two conditions: $\cos\left(\frac{\lambda t'}{4}+\delta\right) = -\sin\left(\frac{\lambda t'}{4}\right)$, $\cos\left(\frac{\lambda'}{4}+\delta'\right) \approx 0$ where $\cos(\delta') = \frac{\lambda'}{4g\sqrt{2}}$. And the corresponding success probability of the teleportation is

$$P_{\rm suc} = \left[\frac{4g}{\lambda}\exp\left(-\frac{\kappa t'}{4}\right)\sin\left(\frac{\lambda t'}{4}\right)\right]^{2N} \tag{25}$$

When the condition of the unity fidelity of the ideal teleportation is fulfilled, we can also diminish the cavity decay's effects on the fidelity of the teleportation absolutely with the chose time which matches $\cos\left(\frac{\lambda' t'}{4}+\delta'\right) = 0$ exactly, and obtain the case of the unity fidelity corresponding to the success probability $\left\{\left[\frac{4g}{\lambda}\exp\left(-\frac{\kappa t'}{4}\right)\right]^2\cos\left(\frac{\lambda t'}{4}+\delta\right)\sin\left(\frac{\lambda t'}{4}\right)\right\}^N$.

5. Discuss and conclusion

In summary, we propose a scheme to teleport an arbitrary multi-particle two-level atomic states be-

tween atoms or an arbitrary zero- and one-photon entangled states of multi-mode between two high-Q cavities. By the limit of the conditional teleportation (the dictation of Alice is limited in a certain state), the ideal successful probability of our scheme decreases polynomially with the number of qubits. The condition of the unity fidelity obtained in our paper is applied to not only the ideal case but also the case of the cavity decay. In the case of the cavity decay, the effect of the decay of the cavity on the successful probability of the teleportation is also found.

To discuss the experimental possibility of our proposal we consider Rydberg atoms with principal quantum number 50 and 51, the radiative lifetime is $T_{\rm r} = 3\times10^{-2}$s. Assume that all the atoms go along the r direction through antinodes, then the atom–cavity coupling strength for each atom is identical. Given $g = 25 \times 2\pi$kHz, $\kappa \simeq 170.3 \times 2\pi$Hz,[24] we can show the dependence of the teleportation on cavity decay rate (atomic spontaneous decay is neglected because the interaction time t and t' of the order of 10^{-6}s and 10^{-5}s respectively much shorter than $T_{\rm r}$). When $\cos\left(\frac{\lambda t'}{4}+\delta\right) = -\sin\left(\frac{\lambda t'}{4}\right) = 0.7077$, we also have $\cos\left(\frac{\lambda' t'}{4}+\delta'\right) \approx 0.0789 \simeq 0$, so the teleportation will be successful with the corresponding success probability 0.48^N less than 0.5^N. Assume that the evolution in each qubit has no effects on others, we do not need to do operations for the qubits one-by-one. This makes it possible to reduce the time spent in the teleportation. In addition, the present scheme is more economical than others in the literature since there requires a less number of atoms, that diminishes the decoherence effect caused by unavoidable interactions between the system and its environment.

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