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Nonlinear H_∞ control of structured uncertain stochastic neural networks with discrete and distributed time varying delays*

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This paper is concerned with the problem of robust H_∞ control for structured uncertain stochastic neural networks with both discrete and distributed time varying delays. A sufficient condition is presented for the existence of H_∞ control based on the Lyapunov stability theory. The stability criterion is described in terms of linear matrix inequalities (LMIs), which can be easily checked in practice. An example is provided to demonstrate the effectiveness of the proposed result.

Keywords: delayed neural networks (DNNs), stochastic systems, Lyapunov functional, linear matrix inequality

PACC: 8710

1. Introduction

The stability problem of neural networks with discrete time delays has been investigated intensively during the past decades.[1−4] In order to deal with phoneme recognition, neural networks with distributed time delays are introduced.[5] Also, system uncertainties arise from many sources such as unavoidable approximation, data errors and ageing of systems. Recently, increasing attention has been paid to the problems of robust stability and robust stabilization of delayed neural networks (DNNs) with parameter uncertainty.[6−9] However, parameter uncertainty may cause more conservatism to DNNs than structured uncertainty.[10]

In practice, systems are almost always innately 'noisy'. Therefore, in order to model a system realistically, a degree of randomness must be incorporated into the model. Thus, a class of stochastic systems has received great attention in the past decade.[11] Consequently, the stability analysis problem for structured uncertain stochastic neural networks with both discrete and distributed time varying delays has practical significance for studying these networks. But there are few articles about such DNNs. In addition, the H_∞ filtering has received a lot of attention due to its advantage of the traditional Kalman filters.[12,13] The rest of this paper is organized as follows. In Section 2, the problems of stabilization and H_∞ control are formulated for structured uncertain stochastic neural networks with both discrete and distributed time varying delays. Established in Section 3 is a sufficient condition for the class of system we are treating to be robustly mean square stable. In Section 4 the H_∞ control problem to achieve an additional H_∞ performance index is addressed. An illustrative example is presented in Section 5 to demonstrate the applicability of the method.

2. Problem formulation

Consider the following stochastic neural network with structured uncertainties and multiple time-varying delays:

$$\begin{aligned} \mathrm{d}x(t) = {} & \big[-(A+\Delta A(t))x(t) + (W_0+\Delta W_0(t)) \\ & \times f(x(t)) + (W_1+\Delta W_1(t))f(x(t-h(t))) \end{aligned}$$

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$$
\begin{aligned}
&+[B+\Delta B(t)]u(t)+Cv(t)+(W_2+\Delta W_2)\\
&\times\int_{t-\tau(t)}^{t}f(x(s))\mathrm{d}s\big]\mathrm{d}t+\big[\Delta Ex(t)\\
&+\Delta E_{\mathrm{d}}x(t-h(t))\big]\mathrm{d}w(t),\\
z(t)\ &=\ Fx(t)+Du(t),\\
x(t)\ &=\ \phi(t),t\in[-H,0],
\end{aligned}\tag{1}
$$

where $x(t)\in R^n$ is the state; $u(t)\in R^u$ is the control input; $v(t)\in R^q$ is the disturbance input belonging to L_2; $z(t)\in R^z$ is the system output; $f(x(t))=(f_1(x_1(t)),f_2(x_2(t)),\ldots,f_n(x_n(t)))^{\mathrm{T}}$ is the neuron activation function with $f(0)=0$; the bounded functions $h(t)$ and $\tau(t)$ respectively represent discrete and distributed delays of systems with $0\le h(t)\le h_M$, and $\dot{h(t)}\le h_D<1, 0\le\tau(t)\le\tau_M$; $H=\max\{h_M,\tau_M\}$; $w(t)$ is an m-dimensional Brownian motion. It is assumed that $\phi^{\mathrm{T}}(t)\phi(t)\le v_0, t\in[-H,0]$, where $v_0>0$ can be seen as an upper bound in the initial states. The diagonal matrix $A=\mathrm{diag}(a_1,a_2,\ldots,a_n)$ has positive entries $a_i>0$. W_0, W_1, W_2, B, C, F, and D are constant matrices with appropriate dimensions. ΔA, ΔW_0, ΔW_1, ΔW_2, ΔB, ΔE, and ΔE_{d} represent the structured uncertainties in stochastic system (1).

Remark Stochastic system (1) can be understood as normal neural networks with discrete and distributed time varying delays.

In order to obtain our main results, the assumptions are made throughout this paper.

(H1) The activation functions $f_i(x_i)(i=1,2,\ldots,n)$ are bounded, and there exists a positive diagonal matrix $L=\mathrm{diag}(l_1,\ldots,l_n)$ such that

$$0\le\frac{f_i(\xi_1)-f_i(\xi_2)}{\xi_1-\xi_2}\le l_i,\forall\quad\xi_1\ne\xi_2,i=1,2,\ldots,n,$$

where $l_i>0(i=1,2,\ldots,n)$ are some given constants.

(H2) The perturbed matrices ΔA, ΔW_0, ΔW_1, ΔW_2, ΔB, $\Delta E(t)$, and $\Delta E_{\mathrm{d}}(t)$ are time -varying functions satisfying:[10] $\Delta W_0(t)=MF_0(t)N_{W_0}$, $\Delta W_1(t)=MF_1(t)N_{W_1}$, $\Delta W_2(t)=MF_2(t)N_{W_2}$, $[\Delta E(t),\ \Delta E_{\mathrm{d}}(t)]=MF_3(t)[N_E,N_{E_{\mathrm{d}}}]$, and $[\Delta A(t),\ \Delta B(t)]=MF_4(t)[N_A,N_B]$, where $M,N_A,\ N_B,\ N_{W_0}$, $N_{W_1},\ N_{W_2},\ N_E$ and $N_{E_{\mathrm{d}}}$ are given constant matrices. $F_l(t)(l=0,1,2,3,4)$ are unknown real time-varying matrices which have the following structure: $F_l(t)=\mathrm{blockdiag}\{\delta_{l_1}(t)I_{r_{l_1}},\ldots,\ \delta_{l_k}(t)I_{r_{l_k}},\ F_{l_1}(t),\ldots,\ F_{l_s}(t)\}$, $\delta_{l_i}\in R$, $|\delta_{l_i}|\le 1$, $1\le i\le k$ and $F_{l_j}^{\mathrm{T}}F_{l_j}\le I$, $1\le j\le s$. We define the set Δ_l as $\Delta_l=\{F_l^{\mathrm{T}}(t)F_l(t)\le I,\ F_lN_l=N_lF_l$ for all $N_l\in\Sigma_l\}$, where $\Sigma_l=\{N_l=\mathrm{blockdiag}\ [N_{l_1},\ldots,\ N_{l_k},n_{l_1}I_{f_{l_1}},\ldots,n_{l_s}I_{f_{l_s}}]\}$, N_{l_i} invertible for $1\le i\le k$, and $n_{l_j}\in R$, $n_{l_j}\ne 0$ for $1\le j\le s$.

The following useful lemmas will be used to derive the desired LMI-based stability criteria.

Lemma 1[11] The LMI $\begin{bmatrix}S_{11}&S_{12}\\ *&S_{22}\end{bmatrix}<0$, with $S_{11}=S_{11}^{\mathrm{T}}$, and $S_{22}=S_{22}^{\mathrm{T}}$, is equivalent to

$$S_{22}<0,\quad\text{and}\quad S_{11}-S_{12}S_{22}^{-1}S_{12}^{\mathrm{T}}<0.$$

Lemma 2[10] Let D, E, and F be real matrices of appropriate dimensions and $F_l\in\Delta_l$, then for any block-structured matrix $N_l\in\Sigma_l$ we will have

$$DF_lE+(DF_lE)^{\mathrm{T}}\le D(N_lN_l^{\mathrm{T}})D^{\mathrm{T}}+E^{\mathrm{T}}(N_lN_l^{\mathrm{T}})^{-1}E.$$

Lemma 3[10] Let A, D, E, F and P be real matrices of appropriate dimensions with $P=P^{\mathrm{T}}>0$ and $F_l\in\Delta_l$, then for any block-structured matrix $N_l\in\Sigma_l$ satisfying $P^{-1}-D(N_lN_l^{\mathrm{T}})D^{\mathrm{T}}>0$, we will have

$$
\begin{aligned}
&(A+DF_lE)^{\mathrm{T}}P(A+DF_lE)\\
\le\ &A^{\mathrm{T}}(P^{-1}-D(N_lN_l^{\mathrm{T}})D^{\mathrm{T}})^{-1}A\\
&+E^{\mathrm{T}}(N_lN_l^{\mathrm{T}})^{-1}E.
\end{aligned}
$$

Lemma 4[10] For any $z,y\in R^n$ and for any symmetric positive-definite matrix $X\in R^{n\times n}$, we have

$$-2z^{\mathrm{T}}y\le z^{\mathrm{T}}X^{-1}z+y^{\mathrm{T}}Xy.$$

Lemma 5[7] For any positive definite matrix $M>0$, scalar $\gamma>0$, vector function $\omega:[0,\gamma]\to R^n$ such that the integrations concerned are well defined, the following inequality holds:

$$
\begin{aligned}
&\left(\int_0^\gamma\omega(s)\mathrm{d}s\right)^{\mathrm{T}}M\left(\int_0^\gamma\omega(s)\mathrm{d}s\right)\\
\le\ &\gamma\left(\int_0^\gamma\omega(s)^{\mathrm{T}}M\omega(s)\mathrm{d}s\right)
\end{aligned}
$$

Definition 1 The trivial solution of stochastic time-delay system (1) is said to be mean-square asymptotically stable if, when $v(t)=0$, for all finite initial functions $\phi\in R^n$ defined on $[-H,0]$, the following equation holds:

$$\lim_{t\to\infty}E|x(t,\phi)|^2=0.$$

Definition 2 Uncertain time delay system (1) is robustly mean square stabilized if the trivial solution of the stochastic functional differential equation related to system (1) with $u(t) = Kx(t)$ is globally asymptotically stale for admissible uncertainties $\Delta A(t)$, $\Delta B(t)$, $\Delta E(t)$, $\Delta E_{\mathrm{d}}(t)$, $\Delta W_0(t)$, $\Delta W_1(t)$, and $\Delta W_2(t)$ in the mean square sense.

Definition 3 Given scalars $\gamma > 0$, and $\sigma > 0$, stochastic time-delay system (1) is said to be robustly closed-loop stable in the mean square sense with a disturbance attenuation rate γ if it is robustly stable in the sense of Definition 2 and for every $T \geq 0$, we have

$$\mathbb{E}\int_0^T z^{\mathrm{T}}(t)z(t)\mathrm{d}t < \gamma^2\left(\int_0^T v^{\mathrm{T}}(t)v(t)\mathrm{d}t + \sigma\right),$$

for any nonzero $v(t) \in L_2$ and any admissible uncertainties $\Delta A(t)$, $\Delta B(t)$, $\Delta E(t)$, $\Delta E_{\mathrm{d}}(t)$, $\Delta W_0(t)$, $\Delta W_1(t)$, and $\Delta W_2(t)$. Note that the scalar $\sigma > 0$ is induced by the initial condition of the system.

3. Robust stabilization in the mean square sense

Theorem 1 Structured uncertain time delay system (1) is robustly stabilized with $v(t) = 0$ in the mean square sense if there exist symmetric positive-definite matrices X, T_0, T_1, T_2, T_3, T_4, R_1, R_2, and a matrix Y satisfying the following LMIs:

$$\begin{bmatrix} -X & MT_3 \\ * & -T_3 \end{bmatrix} < 0, \tag{2}$$

$$\Pi = \begin{pmatrix} \Pi_{11} & W_0 & W_1 & W_2 & \Pi_{15} & \Pi_{16} & \Pi_{17} & XL & M & M & M \\ & \Pi_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & * & \Pi_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & * & * & \Pi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & * & * & * & -\frac{1}{2}T_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ & * & * & * & * & -\frac{1-h_D}{2}T_3 & 0 & 0 & 0 & 0 & 0 \\ & * & * & * & * & * & -T_4 & 0 & 0 & 0 & 0 \\ & * & * & * & * & * & * & -I & 0 & 0 & 0 \\ & * & * & * & * & * & * & * & -T_0 & 0 & 0 \\ & * & * & * & * & * & * & * & * & -T_1 & 0 \\ & * & * & * & * & * & * & * & * & * & -T_2 \end{pmatrix} < 0, \tag{3}$$

where $\Pi_{11} = -XA - AX + BY + Y^{\mathrm{T}}B^{\mathrm{T}} + MT_4M^{\mathrm{T}}$, $\Pi_{15} = XN_E^{\mathrm{T}}$, $\Pi_{16} = XN_{E_{\mathrm{d}}}^{\mathrm{T}}$, $\Pi_{17} = (N_AX - N_BY)^{\mathrm{T}}$, $\Pi_{22} = -I + R_1 + \tau_M^2R_2 + N_{W_0}^{\mathrm{T}}T_0N_{W_0}$, $\Pi_{33} = -(1 - h_D)R_1 + N_{W_1}^{\mathrm{T}}T_1N_{W_1}$, and $\Pi_{44} = -R_2 + N_{W_2}^{\mathrm{T}}T_2N_{W_2}$. In the above LMIs, T_0, T_1, T_2, T_3, and T_4 respectively take the forms of $(N_0N_0^{\mathrm{T}})^{-1}$, $(N_1N_1^{\mathrm{T}})^{-1}$, $(N_2N_2^{\mathrm{T}})^{-1}$, $N_3N_3^{\mathrm{T}}$, and $N_4N_4^{\mathrm{T}}$ for $N_l \in \Sigma_l(l = 1, \ldots, 4)$. Moreover, we can obtain the control law as $u(t) = YX^{-1}x(t)$.

Proof Applying the controller $u(t) = Kx(t)$ to system (1) with $v(t) = 0$, we have the following system:

$$\begin{aligned}\mathrm{d}x(t) = &\Big[-\overline{A}x(t) + \overline{W}_0 f(x(t)) + \overline{W}_1 f(x(t-h(t))) \\ &+\overline{W}_2\int_{t-\tau(t)}^{t} f(x(s))\mathrm{d}s\Big]\mathrm{d}t \\ &+\big[\Delta Ex(t) + \Delta E_{\mathrm{d}}x(t-h(t))\big]\mathrm{d}w(t),\end{aligned} \tag{4}$$

where $\overline{A} = A + \Delta A(t) - (B + \Delta B(t))K$, $\overline{W}_0 = W_0 + \Delta W_0(t)$, $\overline{W}_1 = W_0 + \Delta W_1(t)$, and $\overline{W}_2 = W_0 + \Delta W_2(t)$.

Choose a Lyapunov functional candidate $V(x(t), t)$ as

$$\begin{aligned}V(x(t)) =\ & x^{\mathrm{T}}(t)Px(t)+\int_{t-h(t)}^{t}x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s\\ &+\int_{t-h(t)}^{t}f^{\mathrm{T}}(x(s))R_1f(x(s))\mathrm{d}s\\ &+\tau_M\int_{t-\tau_M}^{t}(s-(t-\tau_M))f^{\mathrm{T}}(x(s))\\ &\times R_2f(x(s))\mathrm{d}s,\end{aligned}$$

where Q is given by $Q=\dfrac{2}{1-h_D}N_{E_{\mathrm{d}}}^{\mathrm{T}}T_3^{-1}N_{E_{\mathrm{d}}}$, then the averaged derivative will be given by the following expression:

$$\begin{aligned}\mathscr{L}V(x(t)) =\ & x^{\mathrm{T}}(t)[-P\overline{A}-\overline{A}^{\mathrm{T}}P]x(t)\\ &+2x^{\mathrm{T}}(t)P\overline{W}_0f(x(t))\\ &+2x^{\mathrm{T}}(t)P\overline{W}_1f(x(t-h(t)))\\ &+2x^{\mathrm{T}}(t)P\overline{W}_2\int_{t-h(t)}^{t}f(x(s))\mathrm{d}s\\ &+\big[\Delta Ex(t)+\Delta E_{\mathrm{d}}x(t-h(t))\big]^{\mathrm{T}}\\ &\times P\big[\Delta Ex(t)+\Delta E_{\mathrm{d}}x(t-h(t))\big]\\ &+x^{\mathrm{T}}(t)Qx(t)-(1-\dot{h}(t))x^{\mathrm{T}}(t-h(t))\\ &\times Qx(t-h(t))+f^{\mathrm{T}}(x(t))\\ &\times R_1f(x(t))-(1-\dot{h}(t))\\ &\times f^{\mathrm{T}}(x(t-h(t)))R_1f(x(t-h(t)))\\ &+\tau_M^2f^{\mathrm{T}}(x(t))R_2f(x(t))\\ &-\tau_M\int_{t-\tau_M}^{t}f^{\mathrm{T}}(x(t))R_2f(x(t))\mathrm{d}s\\ &+f^{\mathrm{T}}(x(t))f(x(t))\\ &-f^{\mathrm{T}}(x(t))f(x(t)).\end{aligned}\tag{5}$$

Let $X=P^{-1}$, then by Lemma 1, inequality (2) will be equivalent to

$$P^{-1}-MT_3M^{\mathrm{T}}>0.\tag{6}$$

Hence, it follows from Lemmas 3 and 4 that

$$\begin{aligned}&\big[\Delta Ex(t)+\Delta E_{\mathrm{d}}x(t-h(t))\big]^{\mathrm{T}}P\big[\Delta Ex(t)+\Delta E_{\mathrm{d}}x(t-h(t))\big]\\ &=\{MF_3(t)[N_Ex(t)+N_{E_{\mathrm{d}}}x(t-h(t))]\}^{\mathrm{T}}P\{MF_3(t)[N_Ex(t)+N_{E_{\mathrm{d}}}x(t-h(t))]\}\\ &\le[N_Ex(t)+N_{E_{\mathrm{d}}}x(t-h(t))]^{\mathrm{T}}T_3^{-1}[N_Ex(t)+N_{E_{\mathrm{d}}}x(t-h(t))]\\ &\le[N_Ex(t)]^{\mathrm{T}}T_3^{-1}[N_Ex(t)]+[N_{E_{\mathrm{d}}}x(t-h(t))]^{\mathrm{T}}T_3^{-1}[N_{E_{\mathrm{d}}}x(t-h(t))]\\ &\quad+[N_Ex(t)]^{\mathrm{T}}T_3^{-1}[N_Ex(t)]+[N_{E_{\mathrm{d}}}x(t-h(t))]^{\mathrm{T}}T_3^{-1}T_3T_3^{-1}[N_{E_{\mathrm{d}}}x(t-h(t))]\\ &=2[N_Ex(t)]^{\mathrm{T}}T_3^{-1}[N_Ex(t)]+2[N_{E_{\mathrm{d}}}x(t-h(t))]^{\mathrm{T}}T_3^{-1}[N_{E_{\mathrm{d}}}x(t-h(t))].\end{aligned}\tag{7}$$

By (H1) and Lemma 5, we have

$$\begin{aligned}&-\tau_M\int_{t-\tau_M}^{t}f^{\mathrm{T}}(x(t))R_2f(x(t))\mathrm{d}s\\ &\le-\tau(t)\int_{t-\tau(t)}^{t}f^{\mathrm{T}}(x(t))R_2f(x(t))\mathrm{d}s\\ &\le-\left[\int_{t-\tau(t)}^{t}f(x(s))\mathrm{d}s\right]^{\mathrm{T}}R_2\left[\int_{t-\tau(t)}^{t}f(x(s))\mathrm{d}s\right].\end{aligned}\tag{8}$$

$$f^{\mathrm{T}}(x(t))f(x(t))\le x^{\mathrm{T}}(t)LLx(t).\tag{9}$$

Substituting inequalities 7–9 into Eq.(5), we obtain

$$\mathscr{L}V(x(t))\le X^{\mathrm{T}}(t)\Xi X(t),\tag{10}$$

where $\Xi=\Xi_1+M_1F_0(t)N_0+N_0^{\mathrm{T}}F_0^{\mathrm{T}}(t)M_1^{\mathrm{T}}+M_1F_1(t)N_1+N_1^{\mathrm{T}}F_1^{\mathrm{T}}(t)M_1^{\mathrm{T}}+M_1F_2(t)N_2+N_2^{\mathrm{T}}F_2^{\mathrm{T}}(t)M_1^{\mathrm{T}}$, $X=[x^{\mathrm{T}}(t),f^{\mathrm{T}}(x(t)),f^{\mathrm{T}}(x(t-h(t))),\left(\int_{t-\tau(t)}^{t}f(x(s))\mathrm{d}s\right)^{\mathrm{T}}]^{\mathrm{T}}$,

$$\Xi_1=\begin{bmatrix}\Xi_{11}&PW_0&PW_1&PW_2\\ *&-I+R_1+\tau_M^2R_2&0&0\\ *&*&-(1-h_D)R_1&0\\ *&*&*&-R_2\end{bmatrix},$$

with $\Xi_{11}=-P(A-BK)-(A-BK)^{\mathrm{T}}P+2N_E^{\mathrm{T}}T_3^{-1}N_E+\dfrac{2}{1-h_D}N_{E_{\mathrm{d}}}^{\mathrm{T}}T_3^{-1}N_{E_{\mathrm{d}}}-PMF_4(t)(N_A-N_BK)-(MF_4(t)(N_A-N_BK))^{\mathrm{T}}P+LL$, $M_1=\begin{bmatrix}PM\\0\\0\\0\end{bmatrix}$, $N_0=\begin{bmatrix}0&N_{W_0}&0&0\end{bmatrix}$, $N_1=\begin{bmatrix}0&0&N_{W_1}&0\end{bmatrix}$, and $N_2=\begin{bmatrix}0&0&0&N_{W_2}\end{bmatrix}$. Using Lemmas 1 and 2, we have

$$\Xi \le \Xi' = \begin{pmatrix} \Xi'_{11} & PW_0 & PW_1 & PW_2 & N_E^{\mathrm{T}} & N_{E_{\mathrm{d}}}^{\mathrm{T}} & \Xi'_{17} & L & PM & PM & PM \\ & \Pi_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & * & \Pi_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & * & * & \Pi_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & * & * & * & -\frac{1}{2}T_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ & * & * & * & * & -\frac{1-h_D}{2}T_3 & 0 & 0 & 0 & 0 & 0 \\ & * & * & * & * & * & -T_4 & 0 & 0 & 0 & 0 \\ & * & * & * & * & * & * & -I & 0 & 0 & 0 \\ & * & * & * & * & * & * & * & -T_0 & 0 & 0 \\ & * & * & * & * & * & * & * & * & -T_1 & 0 \\ & * & * & * & * & * & * & * & * & * & -T_2 \end{pmatrix}, \tag{11}$$

where $\Xi'_{11} = -P(A-BK) - (A-BK)^{\mathrm{T}}P + (PM)T_4(PM)^{\mathrm{T}}$, and $\Xi'_{17} = (N_A - N_BK)^{\mathrm{T}}$.

Pre- and post-multiply the matrix Ξ' by diag (X, I, I, I, I, I, I, I) and let $Y = KX$, we can see that $\Xi' < 0$ is equivalent to LMI (3).

Note that LMI (3) implies that there exists a scalar $b > 0$ such that

$$\Pi + \mathrm{diag}(bI, 0, 0, 0, 0) < 0.$$

By this and inequality (10), we have

$$\mathscr{L}V(x(t)) \le -b|x(t)|^2.$$

According to Itô's formula, system (1) is robust stable in the mean square sense. This completes the proof.

4. H_∞ control problem

In this section, we derive a sufficient condition for robust H_∞ control of uncertain stochastic neural networks with both discrete and distributed delays. The main result is presented in the following theorem.

Theorem 2 Given scalars $\gamma > 0$ and $\sigma > 0$, if there exist symmetric positive-definite matrices X, T_0, T_1, T_2, T_3, T_4, R_1, R_2, and a matrix Y satisfying the following LMIs:

$$\begin{bmatrix} -X & MT_3 \\ * & -T_3 \end{bmatrix} < 0, \tag{12}$$

$$\begin{bmatrix} \Pi & C' & (XF^{\mathrm{T}} + Y^{\mathrm{T}}D^{\mathrm{T}})' \\ * & -\gamma^2 & 0 \\ * & * & -I \end{bmatrix} < 0, \tag{13}$$

$$P + h_MQ + R_1L^2h_M + \frac{\tau_M^3}{2}R_2L^2 \le \frac{\sigma}{v_0}\gamma^2, \tag{14}$$

where $C' = (\,C\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\,)^{\mathrm{T}}$, $(XF^{\mathrm{T}} + Y^{\mathrm{T}}D^{\mathrm{T}})' = (\,XF^{\mathrm{T}} + Y^{\mathrm{T}}D^{\mathrm{T}}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\,)^{\mathrm{T}}$, $\lambda_P = \lambda_{\max}(P)$, $\lambda_L = \lambda_{\max}(L)$, $\lambda_Q = \lambda_{\max}(Q)$, $\lambda_{R_1} = \lambda_{\max}(R_1)$, and $\lambda_{R_2} = \lambda_{\max}(R_2)$, then the closed-loop system is robustly stabilized with $u(t) = YX^{-1}x(t)$ in the mean square sense.

Proof For $t \in [-H, 0]$, introduce the cost function

$$J(t) = \mathbb{E}\left\{\int_0^t [z^{\mathrm{T}}(s)z(s) - \gamma^2 v^{\mathrm{T}}(s)v(s)]\mathrm{d}s\right\}. \tag{15}$$

Using the Dynkin formula yields

$$J(t) = \mathbb{E}\left\{\int_0^t [z^{\mathrm{T}}(s)z(s) - \gamma^2 v^{\mathrm{T}}(s)v(s) + \mathscr{L}V(x(t))]\mathrm{d}s\right\} + \mathbb{E}[V(x(0))] - \mathbb{E}[V(x(t))].$$

Since system(1) is robust mean square stable, we can obtain $\lim_{t\to\infty}\mathbb{E}[V(x(t))] = 0$. This implies that $J(t)$ is well defined as $t \to \infty$.

It can be easily seen that

$$J(t) \le \mathbb{E}\left\{\int_0^t \eta^{\mathrm{T}}(s)\Gamma\eta(s)\mathrm{d}s\right\} + \mathbb{E}[V(x(0))], \tag{16}$$

where

$$\eta(t) = \left[x^{\mathrm{T}}(t), f^{\mathrm{T}}(x(t)), f^{\mathrm{T}}(x(t-h(t))), \left(\int_{t-\tau(t)}^{t} f(x(s))\mathrm{d}s\right)^{\mathrm{T}}, v^{\mathrm{T}}(t)\right]^{\mathrm{T}},$$

and

$$\Gamma = \begin{bmatrix} \Xi_{11} + (F+DK)^{\mathrm{T}}(F+DK) & P\overline{W}_0 & P\overline{W}_1 & P\overline{W}_2 & PC \\ * & -I + R_1 + \tau_M^2 R_2 & 0 & 0 & 0 \\ * & * & -(1-h_D)R_1 & 0 & 0 \\ * & * & * & -R_2 & 0 \\ * & * & * & * & -\gamma^2 \end{bmatrix}.$$

Similarly to Theorem 1, inequality (13) implies $\Gamma < 0$, so

$$\mathbb{E}\left\{\int_0^t \left[z^{\mathrm{T}}(s)z(s) - \gamma^2 v^{\mathrm{T}}(s)v(s)\right]\mathrm{d}s\right\} \le \left(\lambda_P + h_M\lambda_Q + \lambda_{R_1}\lambda_L^2 h_M + \frac{\tau_M^3}{2}\lambda_{R_2}\lambda_L^2\right)v_0 \le \gamma^2\sigma.$$

This completes the proof.

5. An illustrative example

In this section, we demonstrate the theory developed in this paper by means of a simple example.

Consider a tow-mode structured uncertain stochastic neural network with time delays. The network data are given as follows:

$$A = \begin{bmatrix} 2.3 & 0 \\ 0 & 3.4 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ W_0 = \begin{bmatrix} 0.9 & -1.5 \\ 1.2 & 1 \end{bmatrix},\ W_1 = \begin{bmatrix} 0.8 & 0.6 \\ 0.5 & 0.7 \end{bmatrix},\ W_2 = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.2 \end{bmatrix},\ M = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$N_{W_0} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix},\ N_{W_1} = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.1 \end{bmatrix},\ N_{W_2} = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.1 \end{bmatrix},\ N_E = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.1 \end{bmatrix},\ N_{E_{\mathrm{d}}} = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.1 \end{bmatrix},$$

$$N_A = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.1 \end{bmatrix},\ N_B = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ L = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$F_0(t) = F_1(t) = F_2(t) = F_3(t) = F_4(t) = \begin{bmatrix} \sin t & 0 \\ 0 & \cos t \end{bmatrix}, \gamma = 2, h_D = 0, \text{ and } \tau_M = 1.$$

Using Matlab LMI control Toolbox to solve the LMIs (12) and (13), we obtain

$$X = \begin{bmatrix} 17.3848 & -0.0508 \\ -0.0508 & 11.8782 \end{bmatrix},\ Y = \begin{bmatrix} -15.8614 & -0.0556 \\ -0.0556 & -10.8255 \end{bmatrix},\ T_0 = \begin{bmatrix} 2.2873 & 0.0503 \\ 0.0503 & 8.8980 \end{bmatrix},\ T_1 = \begin{bmatrix} 1.6918 & 0.0411 \\ 0.0411 & 7.1462 \end{bmatrix},$$

$$T_2 = \begin{bmatrix} 43.1127 & 0.0688 \\ 0.0688 & 40.6526 \end{bmatrix}, T_3 = \begin{bmatrix} 85.6352 & -0.0476 \\ -0.0476 & 84.5660 \end{bmatrix},\ T_4 = \begin{bmatrix} 43.3861 & -0.0191 \\ -0.0191 & 43.2748 \end{bmatrix}, R_1 = \begin{bmatrix} 0.3272 & -0.0024 \\ -0.0024 & 0.3278 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 0.3272 & -0.0019 \\ -0.0019 & 0.3287 \end{bmatrix},\ \text{and } K = \begin{bmatrix} -0.9124 & -0.0086 \\ -0.0059 & -0.9114 \end{bmatrix}.$$

6. Conclusions

In this paper, we have investigated the stabilization problem as well as the H_∞ control problem for a class of structured uncertain stochastic neural networks with both discrete and distributed time varying delays. The stability criterion is expressed in terms of LMIs, which can be readily tested by some standard numerical packages. Therefore, the developed result is practical.

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