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Kinetics of aggregation growth with competition between catalyzed birth and catalyzed death*

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An aggregation growth model of three species A, B and C with the competition between catalyzed birth and catalyzed death is proposed. Irreversible aggregation occurs between any two aggregates of the like species with the constant rate kernels $I_n (n = 1, 2, 3)$. Meanwhile, a monomer birth of an A species aggregate of size k occurs under the catalysis of a B species aggregate of size j with the catalyzed birth rate kernel $K(k, j) = Kkj^{\upsilon}$, and a monomer death of an A species aggregate of size k occurs under the catalysis of a C species aggregate of size j with the catalyzed death rate kernel $L(k, j) = Lkj^{\upsilon}$, where υ is a parameter reflecting the dependence of the catalysis reaction rates of birth and death on the size of catalyst aggregate. The kinetic evolution behaviours of the three species are investigated by the rate equation approach based on the mean-field theory. The form of the aggregate size distribution of A species $a_k(t)$ is found to be dependent crucially on the competition between the catalyzed birth and death of A species, as well as the irreversible aggregation processes of the three species: (1) In the $\upsilon < 0$ case, the irreversible aggregation dominates the process, and $a_k(t)$ satisfies the conventional scaling form; (2) In the $\upsilon \geq 0$ case, the competition between the catalyzed birth and death dominates the process. When the catalyzed birth controls the process, $a_k(t)$ takes the conventional or generalized scaling form. While the catalyzed death controls the process, the scaling description of the aggregate size distribution breaks down completely.

Keywords: kinetic scaling behaviour, aggregation growth, catalyzed birth and death, rate equation
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1. Introduction

The aggregation growth phenomena are popular and important to a wide variety of fields in nature, such as physics, chemistry, biology *et al.* The characteristic aggregation growth phenomena include colliding growth, gelatination, island formation in epitaxial surface growth, stellar evolution and so on.[1−5] In the last few decades, many studies have been devoted to the investigation of the kinetics of these aggregation processes, and much understanding has been achieved. The most popular and originally studied mechanism for aggregate growth is the binary coalescence scheme, $A_k + A_j \rightarrow A_{k+j}$,[6−11] where, A_k denotes the aggregate consisting of i monomers, and two aggregates A_k and A_j can bond spontaneously to form a large aggregate A_{k+j}. This binary coalescence process was then generalized to multi-aggregate (or n-polymer) stochastic aggregation,[12−16] and then aggregation processes through aggregation-annihilation mechanisms were proposed.[17,18] Recently, much attention has been paid to the aggregation phenomena in sociology and economy to investigate the kinetic behaviours driven by some newly proposed mechanisms.[19−22] Isoplatov *et al*[23] introduced several different asset exchange models for the evolution of the wealth distribution in the economic interaction population. And Leyvraz and Redner[24] proposed a migration-driven aggregate growth model for the evolution of city population, where the migration direction depends on the relative sizes of the two aggregates. The results exhibit that the kinetics of such an exchange- or a migration-driven aggregation system obeys some scaling law different from those for conventional aggregation processes. More generally, the migration direction may not depend on the relative sizes of the two aggregates, Lin and Ke,[25] and Ke *et al*[26] proposed an exchange-driven aggregation model, where the migration of monomer goes from the larger to the smaller aggregates as well as from the

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smaller to the larger, and the aggregate size distribution is found to obey the conventional or generalized scaling law. Ke *et al*[27] also generalized the study to the kinetics of migration-driven aggregate growth on completely connected scale-free networks. The results show that the evolution behaviour of the aggregate size distribution is drastically different from those for the corresponding systems in normal space.

In an aggregate growth system of more than one species, there exist interactions between aggregates of unlike species. In recent years, the much interest has been aroused in a variety of catalysis-driven aggregation mechanisms.[28−31] Lin *et al*[32,33] introduced mutually catalyzed birth processes of population and assets in exchange-driven growth to model the interaction between aggregates of population and assets and study the competition between two aggregation mechanisms: catalyzed birth and exchange-driven growth. The results show that this competition between the two growth mechanisms gives rise to much more abundant kinetic behaviours. Here it is noticed that the catalyzed death process may also play the same important role as the catalyzed birth in the evolutions of city population, wealth distribution and other natural and social aggregation growths. More actually, the city population growth is driven by the merging of small population districts (which can be reasonably regarded as an irreversible aggregation process) as well as the competition between catalyzed birth and catalyzed death of the populations. For example, a city with more assets per person will attract more people to migrate into it, which can be regarded as the aggregation growth with asset-catalysis population birth. On the other hand, a city in war or plague will compel many people to die or emigrate to other cities, which can be regarded as catalyzed death of population in aggregation growth. Similarly, the merging of two separate companies to form a single large company can be reasonably regarded as an aggregation growth, and meanwhile, a company may correspondingly hire (or dismiss) employees due to some external factors (such as the market demand of its products), which can also be reasonably considered as catalyzed birth (or death) process in aggregation growth. Motivated by these situations, we propose an irreversible aggregation growth model of three different species A, B and C with the competition between catalyzed birth and catalyzed death. In this model, irreversible aggregation growth occurs between any two aggregates of the like species, and meanwhile, A species performs a monomer birth catalyzed by B species and a monomer death catalyzed by C species. Here B and C species perform conventional irreversible aggregation processes, whose kinetic evolution behaviours were well studied in former studies. The aggregates of A species perform catalyzed birth and death processes on the basis of their irreversible aggregation, so we focus our attention on the kinetic evolution behaviour of A species to study the competition between the catalyzed birth and death processes on the basis of irreversible aggregation and the competition between the effects of the catalyzed birth and death processes and that of irreversible aggregation. By discussing the evolution behaviours, we have found that the aggregate size distribution of A species obeys the conventional or generalized scaling law in some cases, while the scaling law may break down in other cases. It is believed that this model of irreversible aggregation growth with the competition between catalyzed birth and death may provide a fairly useful description of the dynamics of some physical (biological, chemical and social) systems.

The rest of the present paper is organized as follows. In Section 2, we introduce the model of the irreversible aggregation growth with the competition between catalyzed birth and catalyzed death, and describe the outline of the generalized Smoluchowski rate equation approach. In Section 3, we study the kinetics of the system in various cases with different dependences of catalyzed birth and death rates of A species on the catalyst size. Finally, a brief summary is given in Section 4.

2. The model of the competition between catalyzed birth and catalyzed death in irreversible aggregation growth

In our model, the irreversible aggregation growths of A, B and C species are described as $A_k + A_j \xrightarrow{I_1} A_{k+j}$, $B_k + B_j \xrightarrow{I_2} B_{k+j}$ and $C_k + C_j \xrightarrow{I_3} C_{k+j}$, respectively, where the self-coagulation rates I_1, I_2, and I_3 are assumed to be constants. The monomer birth of A species catalyzed by B species is $A_k + B_j \xrightarrow{K(k,j)} A_{k+1} + B_j$ with the catalysis rate kernel $K(k,j) = Kkj^{\upsilon}$ (K is a constant and υ is a pa-

rameter reflecting the dependence of the catalyzed birth rate on the catalyst aggregate size), and the monomer death of A species catalyzed by C species is $A_k + C_j \xrightarrow{L(k,j)} A_{k-1} + C_j$ with the catalysis rate kernel $L(k,j) = Lkj^v$ (L is also a constant).

We assume that the system has a spatial homogeneity, so that the fluctuations in densities of the reactants are ignored and the aggregates of species each are considered to be homogeneously distributed in the space throughout the process. Thus, the theoretical approach to the investigation of the kinetics of aggregate growth can be based on the mean-field theory, and the Smoluchowski rate equation can be used to study the aggregate size distributions of the three species.[6] Let $a_k(t)$ ($b_k(t)$, $c_k(t)$) be the concentration of the A_k (B_k, C_k) aggregates of size k at time t. By generalizing the rate equations of the self-coagulation model given by Smoluchowski[6] and the catalyzed birth model given by Lin *et al*,[32] we write the corresponding rate equations for our system as follows:

$$\begin{aligned}\frac{\mathrm{d}a_k}{\mathrm{d}t} &= \frac{1}{2}I_1\sum_{i+j=k}a_ia_j - I_1a_k\sum_{j=1}^{\infty}a_j \\ &+K(k-1)a_{k-1}\sum_{j=1}^{\infty}j^vb_j - Kka_k\sum_{j=1}^{\infty}j^vb_j \\ &+L(k+1)a_{k+1}\sum_{j=1}^{\infty}j^vc_j - Lka_k\sum_{j=1}^{\infty}j^vc_j, \quad (1)\end{aligned}$$

$$\frac{\mathrm{d}b_k}{\mathrm{d}t} = \frac{1}{2}I_2\sum_{i+j=k}b_ib_j - I_2b_k\sum_{j=1}^{\infty}b_j, \quad (2)$$

and

$$\frac{\mathrm{d}c_k}{\mathrm{d}t} = \frac{1}{2}I_3\sum_{i+j=k}c_ic_j - I_3c_k\sum_{j=1}^{\infty}c_j. \quad (3)$$

In Eq.(1) (as in Eqs.(2) and (3)), the first term accounts for the gain in $a_k(t)$ due to the aggregation $A_i + A_j \xrightarrow{I_1} A_k$ (for the case of $i + j = k$), and the second term accounts for the loss in $a_k(t)$ due to the aggregation $A_k + A_j \xrightarrow{I_1} A_{k+j}$. The third and fourth terms account respectively for the gain and the loss in $a_k(t)$ due to the monomer birth of A species catalyzed by B species. Similarly, The fifth and sixth terms account respectively for the gain and the loss in $a_k(t)$ due to the monomer death of A species catalyzed by C species.

Several methods of solving the rate equations in different aggregation processes have been developed. For a simple coagulation process, $A_i + A_j \xrightarrow{I(i,j)} A_{i+j}$, explicit solutions were obtained for $I(i,j) = 1, i + j$, and ij by introducing some suitable generating functions.[6−8] In the aggregation-annihilation process, the rate equations were solved with the help of a special ansatz $a_k(t) = A(t)[a(t)]^{k-1}$,[17,18] and by the generating function method as well.[17] And in the exchange-driven aggregation process, the rate equations were solved through making some scaling ansatzes directly[23,24,34] or with the help of the special ansatz $a_k(t) = A(t)[a(t)]^{k-1}$.[25,35] In the recent work, Lin *et al*[32] have given a thorough discussion on the applicability of this ansatz. Here we find that our current rate equations (1), (2) and (3) can be solved by using the ansatz

$$\begin{aligned}&a_k(t) = A(t)[a(t)]^{k-1}, \quad b_k(t) = B(t)[b(t)]^{k-1}, \\ &\text{and } c_k(t) = C(t)[c(t)]^{k-1}. \quad (4)\end{aligned}$$

The B and C species aggregates perform only the same irreversible aggregation processes, so their evolution kinetic behaviours are the same and relatively simple. The kinetics of simple irreversible aggregation process has been studied in the former work under the monodisperse initial condition $b_k(0) = B_0\delta_{k1}$ and $c_k(0) = C_0\delta_{k1}$ (B_0 and C_0 are the initial aggregate concentrations of B and C species, respectively).[36] The accurate solution to the aggregate size distribution of B species (similarly for C species) is

$$b_k(t) = 4B_0(2 + I_2B_0t)^{-2}\left(1 - \frac{2}{2 + I_2B_0t}\right)^{k-1}. \quad (5)$$

It satisfies the conventional scaling law in the long-time limit,[17]

$$b_k(t) \simeq 4B_0^{-1}I_2^{-2}t^{-2}\Phi(x), \quad (6)$$

where $x = k/S_B(t)$, and the scaling function $\Phi(x) = \exp(-x)$ with $S_B(t) = \frac{1}{2}I_2B_0t$ being the characteristic size of B aggregate, which plays a role similar to the correlation length in critical phenomenon. Moreover, the total number and the total mass of B aggregates are derived respectively as

$$\begin{aligned}&M_0^B(t) = 2B_0(2 + I_2B_0t)^{-1} \simeq 2I_2^{-1}t^{-1}, \\ &\text{and } \ M_1^B(t) = B_0, \quad (7)\end{aligned}$$

i.e. the total number decays linearly with time as t^{-1}, and the total mass is conserved.

We now focus our interest on the evolution behaviour of A species through solving rate equation

(1) under the monodisperse initial condition $a_k(0) = A_0\delta_{k1}$, where A_0 is the initial aggregate concentration of A species. Substituting ansatz (4) into rate equation (1), it can be transformed into the following differential equations:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{I_1 A}{2} + KM_\upsilon^B(1-a) + LM_\upsilon^C a^2 - LM_\upsilon^C a, \quad (8)$$

and

$$\frac{\mathrm{d}A}{\mathrm{d}t} = -\frac{I_1 A^2}{1-a} - KAM_\upsilon^B + LAM_\upsilon^C(2a-1), \quad (9)$$

where M_υ^B and M_υ^C are the υth moments of the aggregate size distributions $b_k(t)$ and $c_k(t)$, respectively. The υth moment of the aggregate size distribution $a_k(t)$ of A species (similarly for B and C species) can be expressed as $M_\upsilon^A(t) = \sum_{j=1}^{\infty} j^\upsilon a_j(t)$. Especially, the first two moments $M_0^A(t) = \sum_{j=1}^{\infty} a_j(t)$ and $M_1^A(t) = \sum_{j=1}^{\infty} j a_j(t)$ are the total number and the total mass of A species aggregates, respectively. Using ansatz (4), the first two moments of the aggregate size distribution $a_j(t)$ can be written as

$$M_0^A(t) = \sum_{j=1}^{\infty} a_j(t) = \frac{A}{1-a},$$
$$\text{and} \quad M_1^A(t) = \sum_{j=1}^{\infty} j a_j(t) = \frac{A}{(1-a)^2}. \quad (10)$$

On ansatz (4), The monodisperse initial condition $a_k(0) = A_0\delta_{k1}$ is transformed correspondingly as follows:

$$a(0) = 0, \text{ and } A(0) = A_0, \text{ at } t = 0. \quad (11)$$

To start with, we derive the relation between $A(t)$ and $a(t)$ from Eqs.(8) and (9) as

$$A = A_0(1-a)^2 E_B(t) E_C^{-1}(t), \quad (12)$$

where $E_B(t) = \exp\left[K\int_0^t M_\upsilon^B(t')\mathrm{d}t'\right]$ and $E_C(t) = \exp\left[L\int_0^t M_\upsilon^C(t')\mathrm{d}t'\right]$.

Substituting Eq.(12) into Eq.(8) yields

$$\frac{\mathrm{d}(1-a)}{\mathrm{d}t} = -\left[\frac{1}{2}I_1 A_0 E_B(t) E_C^{-1}(t) + LM_\upsilon^C\right](1-a)^2 + (LM_\upsilon^C - KM_\upsilon^B)(1-a). \quad (13)$$

This Bernoulli equation can be solved under initial condition (11) as follows:

$$a(t) = 1 - \frac{E_B^{-1}(t)E_C(t)}{1 + \frac{1}{2}I_1 A_0 t + \int_0^t E_B^{-1}(t')\mathrm{d}E_C(t')}. \quad (14)$$

In the next section we study the kinetics of the system in various cases with different values of catalyzed birth and catalyzed death rate kernel parameter υ, which reflects the dependence of the catalyzed birth and catalyzed death rates on the size of catalyst aggregate.

3. Kinetics of aggregation growth with the competition between catalyzed birth and catalyzed death

3.1. $\upsilon = 0$ case

We first study the case of $\upsilon = 0$, a case where the catalyzed birth and death rate kernels $K(k,j) = Kk$ and $L(k,j) = Lk$ are both independent of the size of catalyst aggregate.

In this case, we can obtain the asymptotical expression of $a(t)$ in the long-time limit. From expression (7), we have $M_0^B(t) = 2B_0(2 + I_2 B_0 t)^{-1}$ and $M_0^C(t) = 2C_0(2 + I_3 C_0 t)^{-1}$, and then we obtain $E_B(t) = (2 + I_2 B_0 t)^{2K/I_2}$ and $E_C(t) = (2 + I_3 C_0 t)^{2L/I_3}$. Substituting these expressions into expression (14), we derive the asymptotic expression of $a(t)$ in the long-time limit as

$$a \simeq \begin{cases} 1 - C_1^{-1} t^{-1-2(KI_2^{-1} - LI_3^{-1})}, & \text{for } 2(KI_2^{-1} - LI_3^{-1}) > -1, \\ 1 - (C_1 + C_2)^{-1}, & \text{for } 2(KI_2^{-1} - LI_3^{-1}) = -1, \\ KI_2^{-1}/LI_3^{-1} + C_1 C_2^{-2} I_1 A_0 t^{1+2(KI_2^{-1} - LI_3^{-1})}, & \text{for } 2(KI_2^{-1} - LI_3^{-1}) < -1, \end{cases} \quad (15)$$

where $C_1 = \frac{1}{2}I_1 A_0 (I_2 B_0)^{2K/I_2}(I_3 C_0)^{-2L/I_3}$ and $C_2 = LI_3^{-1}/(LI_3^{-1} - KI_2^{-1})$. And we further derive the asymptotic expression of $A(t)$ from expression (12) as

$$A \simeq \begin{cases} 2I_1^{-1}C_1^{-1}A_0^{-1}t^{-2-2(KI_2^{-1}-LI_3^{-1})}, & \text{for } 2(KI_2^{-1}-LI_3^{-1}) > -1, \\ 2I_1^{-1}C_1(C_1+C_2)^{-2}t^{-1}, & \text{for } 2(KI_2^{-1}-LI_3^{-1}) = -1, \\ 2I_1^{-1}C_1C_2^{-2}t^{2(KI_2^{-1}-LI_3^{-1})}, & \text{for } 2(KI_2^{-1}-LI_3^{-1}) < -1. \end{cases} \quad (16)$$

It is evident that the evolution behaviour of $a(t)$ is dependent on the value of $2(KI_2^{-1} - LI_3^{-1})$, which reflects the competition between the two opposite reactions of catalyzed birth and catalyzed death. Meanwhile, the effects of the irreversible aggregations of B and C species are to scale the B species-catalyzed birth and the C species-catalyzed death, respectively. This suggests that when both B and C species perform irreversible aggregation processes, the effective B species-catalyzed birth rate is $K_e = 2KI_2^{-1}$ and the effective C species-catalyzed death rate is $L_e = 2LI_3^{-1}$.

(i) When $2(KI_2^{-1} - LI_3^{-1}) > -1$, the aggregate size distribution of A species in the long-time limit is derived as

$$a_k(t) \simeq 2C_1^{-1}I_1^{-1}t^{-2-2(KI_2^{-1}-LI_3^{-1})}\exp(-x), \quad (17)$$

where $x = k/S_A(t)$, with the characteristic aggregate size $S_A(t) = C_1t^{1+2(KI_2^{-1}-LI_3^{-1})}$. It shows that the aggregate size distribution $a_k(t)$ satisfies the conventional scaling law,[18]

$$a_k(t) \simeq t^{-w}\Phi[k/S(t)], \quad (18)$$

with $S(t) \propto t^z$, and the scaling function $\Phi(x) = \exp(-x)$. The scaling exponents $w = 2 + 2(KI_2^{-1} - LI_3^{-1})$ and $z = 1 + 2(KI_2^{-1} - LI_3^{-1})$ are dependent on the competition between the effective B species catalyzed birth rate and the effective C species catalyzed death rate. Meanwhile, the scaling exponents w and z show that the kinetic evolution behaviour of A species is unexpectedly independent of the irreversible aggregation of A species itself.

The total number of A species aggregates is derived as

$$M_0^A = A(1-a)^{-1} \simeq 2I_1^{-1}t^{-1}, \quad (19)$$

which decays with time as t^{-1}. And the total mass of A species is

$$\begin{aligned} M_1^A &= A(1-a)^{-2} \\ &= A_0E_B(t)E_C^{-1}(t) \\ &\simeq 2I_1^{-1}C_1t^{2(KI_2^{-1}-LI_3^{-1})}. \end{aligned} \quad (20)$$

The expression of M_1^A is the same for all the subcases of $2(KI_2^{-1} - LI_3^{-1}) > -1$, $2(KI_2^{-1} - LI_3^{-1}) = -1$ and $2(KI_2^{-1} - LI_3^{-1}) < -1$, but its evolution behaviour is crucially dominated by the competition between the effective catalyzed birth and the effective catalyzed death. For the case of $K_e > L_e$, the catalyzed birth dominates the process and M_1^A increases with time, while in the case of $K_e < L_e$, the catalyzed death dominates the process and M_1^A decreases with time. And in the marginal case of $K_e = L_e$, the catalyzed birth and death processes play the same roles and counteract each other, and M_1^A keeps conserved.

(ii) When $2(KI_2^{-1} - LI_3^{-1}) = -1$, $a(t)$ approaches a constant value and the aggregate size distribution $a_k(t)$ is not be scaled,

$$\begin{aligned} a_k(t) \simeq\ & 2I_1^{-1}C_1(C_1+C_2)^{-2} \\ & \times[1-(C_1+C_2)^{-1}]^{k-1}t^{-1}. \end{aligned} \quad (21)$$

The total number M_0^A and the total mass M_1^A of species A are derived respectively as

$$\begin{aligned} M_0^A(t) &\simeq 2I_1^{-1}C_1(C_1+C_2)^{-1}t^{-1}, \\ \text{and}\quad M_1^A(t) &\simeq 2I_1^{-1}C_1t^{-1}. \end{aligned} \quad (22)$$

Both the total number and the total mass decrease linearly with time, and A species will die out in the end.

(iii) When $2(KI_2^{-1} - LI_3^{-1}) < -1$, the aggregate size distribution $a_k(t)$ is obtained as

$$\begin{aligned} a_k(t) \simeq\ & 2I_1^{-1}C_1C_2^{-2}\left(\frac{KI_2^{-1}}{LI_3^{-1}}\right)^k t^{-2(LI_3^{-1}-KI_2^{-1})} \\ & \times\exp[k/R(t)], \end{aligned} \quad (23)$$

where

$$R(t) = C_1^{-1}C_2^2(I_1A_0)^{-1}\left(\frac{KI_2^{-1}}{LI_3^{-1}}\right)t^{-1+2\left(LI_3^{-1}-KI_2^{-1}\right)}.$$

Obviously, the scaling description of the aggregate size distribution $a_k(t)$ breaks down completely in this case. The total number and the total mass of A species are derived respectively as

$$\begin{aligned} M_0^A(t) &\simeq 2I_1^{-1}C_1C_2^{-1}t^{-2(LI_3^{-1}-KI_2^{-1})}, \\ \text{and}\quad M_1^A(t) &\simeq 2I_1^{-1}C_1t^{-2(LI_3^{-1}-KI_2^{-1})}. \end{aligned} \quad (24)$$

The results imply that both the total number and the total mass of A species decrease with time in power law as $t^{-2(LI_3^{-1}-KI_2^{-1})}$ in the long-time limit. So A species will also die out finally in this case.

3.2. $\upsilon = 1$ case

We then discuss the case of $\upsilon = 1$, where the rate kernels of catalyzed birth and death $K(k,j) = Kkj$, $L(k,j) = Lkj$ are both proportional to the catalyst size.

Because the B and C species perform only the irreversible aggregation processes, the total masses of B and C species are both conserved, i.e. $M_1^B(t) = B_0$ and $M_1^C(t) = C_0$. The exact expressions of $E_B(t)$ and $E_C(t)$ can be directly derived as $E_B(t) = \exp(KB_0t)$ and $E_C(t) = \exp(LC_0t)$ respectively. The solution of $a(t)$ can be derived from expression (14) as

$$a = \begin{cases} 1 - \dfrac{(KB_0 - LC_0)\mathrm{e}^{-(KB_0-LC_0)}}{KB_0 - LC_0 + \dfrac{1}{2}I_1A_0(KB_0 - LC_0)t - LC_0\mathrm{e}^{-(KB_0-LC_0)}}, & \text{for } KB_0 \neq LC_0, \\ 1 - \left(\dfrac{1}{2}I_1A_0 + KB_0\right)^{-1} t^{-1}, & \text{for } KB_0 = LC_0. \end{cases} \tag{25}$$

In the long-time limit, it becomes

$$a \simeq \begin{cases} 1 - \left(\dfrac{1}{2}I_1A_0\right)^{-1} t^{-1}\mathrm{e}^{-(KB_0-LC_0)t}, & \text{for} \quad KB_0 > LC_0, \\ 1 - \left(\dfrac{1}{2}I_1A_0 + KB_0\right)^{-1} t^{-1}, & \text{for} \quad KB_0 = LC_0, \\ KB_0(LC_0)^{-1} + \dfrac{1}{2}I_1A_0[1 - KB_0(LC_0)^{-1}]^2 t\mathrm{e}^{-(LC_0-KB_0)t}, & \text{for} \quad KB_0 < LC_0. \end{cases} \tag{26}$$

And we further derive the asymptotic expression of $A(t)$ from expression (12) as

$$A \simeq \begin{cases} 4I_1^{-2}A_0^{-1}t^2\mathrm{e}^{-(KB_0-LC_0)t}, & \text{for} \quad KB_0 > LC_0, \\ A_0\left(\dfrac{1}{2}I_1A_0 + KB_0\right)^{-2} t^{-2}, & \text{for} \quad KB_0 = LC_0, \\ A_0[1 - KB_0(LC_0)^{-1}]^2\mathrm{e}^{-(LC_0-KB_0)t}, & \text{for} \quad KB_0 < LC_0. \end{cases} \tag{27}$$

(i) When $KB_0 > LC_0$, the aggregate size distribution of A species is obtained in the long-time limit, which satisfies a generalized scaling form as follows:

$$a_k(t) \simeq \frac{1}{2}I_1^{-2}A_0^{-1}t^{-2}\mathrm{e}^{-(KB_0-LC_0)t}\,\Phi(x), \tag{28}$$

where $x = k/S_A(t)$, and the scaling function $\Phi(x) = \exp(-x)$ with the characteristic size of aggregates $S_A(t) = \dfrac{1}{2}I_1A_0t\mathrm{e}^{(KB_0-LC_0)t}$. This generalized scaling expression of the aggregate size distribution can further be written in a more general form as follows:[32]

$$a_k(t) \simeq I_1^{-1}t^{-1}S_A^{-1}(t)\,\Phi(x). \tag{29}$$

where

$$x = k/S_A(t).$$

The total number and the total mass of A species can be derived respectively as

$$M_0^A(t) \simeq 2I_1^{-1}t^{-1}, \text{ and } M_1^A(t) \simeq A_0\mathrm{e}^{(KB_0-LC_0)t}. \tag{30}$$

The results imply that the total number of A species still decays as t^{-1}, while the total mass grows with time exponentially. This means that the catalyzed birth dominates the process.

(ii) For the case of $KB_0 = LC_0$, the aggregate size distribution of A species is obtained in the long-time limit, satisfying the conventional scaling form, as

$$a_k(t) \simeq A_0\left(\frac{1}{2}I_1A_0 + KB_0\right)^{-2} t^{-2}\,\Phi(x), \tag{31}$$

where $x = k/S_A(t)$, with the characteristic size

$S_A(t) = \left(\frac{1}{2}I_1A_0 + KB_0\right)t$, and the scaling exponents $w = 2$, $z = 1$.

Moreover, the total number and the total mass of A species can be obtained respectively as

$$M_0^A(t) \simeq A_0\left(\frac{1}{2}I_1A_0 + KB_0\right)^{-1}t^{-1},$$
$$\text{and } M_1^A(t) = A_0. \tag{32}$$

The results imply that the total number of A species still decays with time as t^{-1}, while the total number is conserved all along. It is just the result of the counteraction of catalyzed birth and catalyzed death.

(iii) When $KB_0 < LC_0$, the aggregate size distribution of A species is obtained in the long-time limit as

$$a_k(t) \simeq A_0[1 - KB_0(LC_0)^{-1}]^2 \times (KB_0/LC_0)^k \mathrm{e}^{-(LC_0-KB_0)t}\exp[k/R(t)], \tag{33}$$

with $R(t) = 2KB_0LC_0\ (LC_0 - KB_0)^{-2}\ (I_1A_0)^{-1}t^{-1}\ \mathrm{e}^{(LC_0-KB_0)t}$. It shows that in this case, the aggregate size distribution $a_k(t)$ is not scaled. Moreover, the total number and the total mass of A species can be obtained respectively as

$$M_0^A(t) \simeq A_0(LC_0 - KB_0)(LC_0)^{-1}\mathrm{e}^{-(LC_0-KB_0)t},$$
$$\text{and} \quad M_1^A(t) = A_0\mathrm{e}^{-(LC_0-KB_0)t}. \tag{34}$$

So, both the total number $M_0^A(t)$ and the total mass $M_1^A(t)$ decrease exponentially. That is, the effect of catalyzed death is greater than that of catalyzed birth, and A species will die out in the end.

Contrary to the former case of $\upsilon = 0$, the irreversible aggregation processes of B and C species play no role in the kinetic behaviour of A species in this case. Meanwhile, the initial concentrations of B and C species, B_0 and C_0, play an important role. The kinetic behaviour of A species is dominated by the competition between the effective B species catalyzed birth $K_\mathrm{e} = KB_0$ and the effective C species catalyzed death $L_\mathrm{e} = LC_0$.

3.3. The general υ case

In the general υ case, we can study the asymptotic kinetic behaviour of A species in the long-time limit.

The υth moment of the distribution $b_k(t)$ can be derived in the long-time limit from expression (5) as

$$M_\upsilon^B(t) = \sum_{j=1}^{\infty} j^\upsilon b_j \simeq \begin{cases} \Gamma(1+\upsilon)B_0\left(1+\frac{1}{2}I_2B_0t\right)^{\upsilon-1}, & \text{for} \quad \upsilon > -1, \\ \left(\frac{1}{2}I_2t\right)^{-1}\left(1+\frac{1}{2}I_2B_0t\right)^{-1}\ln\left(1+\frac{1}{2}I_2B_0t\right), & \text{for} \quad \upsilon = -1, \\ B_0\left(1+\frac{1}{2}I_2B_0t\right)^{-2}\zeta(-\upsilon), & \text{for} \quad \upsilon < -1, \end{cases} \tag{35}$$

where $\zeta(n) = \sum_{l=1}^{\infty} l^{-n} \quad (n > 1)$ is the Riemann zeta function. And the υth moment of the distribution $c_k(t)$ is

$$M_\upsilon^C(t) = \sum_{j=1}^{\infty} j^\upsilon c_j \simeq \begin{cases} \Gamma(1+\upsilon)C_0\left(1+\frac{1}{2}I_3C_0t\right)^{\upsilon-1}, & \text{for} \quad \upsilon > -1, \\ \left(\frac{1}{2}I_3t\right)^{-1}\left(1+\frac{1}{2}I_3C_0t\right)^{-1}\ln\left(1+\frac{1}{2}I_3C_0t\right), & \text{for} \quad \upsilon = -1, \\ C_0\left(1+\frac{1}{2}I_3C_0t\right)^{-2}\zeta(-\upsilon), & \text{for} \quad \upsilon < -1. \end{cases} \tag{36}$$

(i) In the case of $\upsilon > 0$, from expressions (35) and (36) we respectively derive

$$E_B(t) \simeq \exp(C_3t^\upsilon), \qquad \text{and} \qquad E_C(t) \simeq \exp(C_4t^\upsilon), \tag{37}$$

where $C_3 = K\Gamma(\upsilon)B_0^\upsilon\left(\frac{1}{2}I_2\right)^{\upsilon-1}$ and $C_4 = L\Gamma(\upsilon)C_0^\upsilon\left(\frac{1}{2}I_3\right)^{\upsilon-1}$.

Using these expressions we obtain $a(t)$ from expression (14) as

$$a \simeq \begin{cases} 1-\left(\dfrac{1}{2}I_1A_0\right)^{-1}t^{-1}\mathrm{e}^{-(C_3-C_4)t^{\upsilon}}, & \text{for} \quad C_3>C_4, \\ 1-\left(\dfrac{1}{2}I_1A_0t+C_4t^{\upsilon}\right)^{-1}, & \text{for} \quad C_3=C_4, \\ C_3C_4^{-1}+\dfrac{1}{2}I_1A_0(1-C_4C_3^{-1})^2t\mathrm{e}^{-(C_4-C_3)t^{\upsilon}}, & \text{for} \quad C_3<C_4. \end{cases} \tag{38}$$

And we further derive the asymptotic expression of $A(t)$ from expression (12) as

$$A \simeq \begin{cases} 4I_1^{-2}A_0^{-1}t^{-2}\mathrm{e}^{-(C_3-C_4)t^{\upsilon}}, & \text{for} \quad C_3>C_4, \\ A_0\left(\dfrac{1}{2}I_1A_0t+C_4t^{\upsilon}\right)^{-2}, & \text{for} \quad C_3=C_4, \\ A_0(1-C_3C_4^{-1})^2\mathrm{e}^{-(C_4-C_3)t^{\upsilon}}, & \text{for} \quad C_3<C_4. \end{cases} \tag{39}$$

It follows from expressions (38) and (39) that the kinetic behaviour of the aggregate size distribution $a_k(t)$ depends crucially on the relationship between C_3 and C_4, which reflects the competition between the catalyzed birth and catalyzed death reactions. So in this general case of $\upsilon > 0$, expressions (38) and (39) suggest that the effective rates of B species catalyzed birth and C species catalyzed death are $K_{\mathrm{e}} = C_3 = K\Gamma(\upsilon)B_0^{\upsilon}\left(\dfrac{1}{2}I_2\right)^{\upsilon-1}$ and $L_{\mathrm{e}} = C_4 = L\Gamma(\upsilon)C_0^{\upsilon}\left(\dfrac{1}{2}I_3\right)^{\upsilon-1}$, respectively.

For the case of $C_3 > C_4$, the case in which the catalyzed birth dominates the process, the aggregate size distribution of A species can be described in the generalized scaling form as follows:

$$a_k(t) = 2I_1^{-1}t^{-1}S_A^{-1}(t)\,\Phi(x), \tag{40}$$

where $x = k/S_A(t)$, with the characteristic size $S_A(t) = \dfrac{1}{2}I_1A_0t\mathrm{e}^{(C_3-C_4)t^{\upsilon}}$. The total number and the total mass of A species are obtained respectively as

$$M_0^A(t) \simeq 2I_1^{-1}t^{-1}, \text{ and } M_1^A(t) \simeq A_0\mathrm{e}^{(C_3-C_4)t^{\upsilon}}. \tag{41}$$

The results imply that the total number of A species still decreases as t^{-1}, while the total mass grows with time exponentially.

For the case of $C_3 = C_4$, the case in which the effects of catalyzed birth and catalyzed death are counteracted each other, the evolution behaviour of the aggregate size distribution $a_k(t)$ is dependent on the parameter υ.

If $0 < \upsilon < 1$, the aggregate size distribution of A species can be expressed in the conventional scaling form as follows:

$$a_k(t) \simeq 4I_1^{-2}A_0^{-1}t^{-2}\,\Phi(x), \tag{42}$$

where $x = k/S_A(t)$, with the characteristic size $S_A(t) = \dfrac{1}{2}I_1A_0t$. The scaling exponents are $w = 2$, and $z = 1$, which are independent of the value of the parameter υ. The total number and the total mass of A species are derived respectively as

$$M_0^A(t) \simeq 2I_1^{-1}t^{-1}, \quad \text{and} \quad M_1^A(t) = A_0. \tag{43}$$

The results indicate that the total number of A species decreases with time as t^{-1}, while the total mass is conserved.

On the other hand, if $\upsilon > 1$, the aggregate size distribution of A species also satisfies the conventional scaling form:

$$a_k(t) \simeq A_0C_4^{-2}t^{-2\upsilon}\,\Phi(x), \tag{44}$$

where $x = k/S_A(t)$, with the characteristic size $S_A(t) = C_4t^{\upsilon}$. But the scaling exponents $w = 2\upsilon$, $z = \upsilon$ are dependent on the value of the parameter υ. The total number and the total mass of A species are derived respectively as

$$M_0^A(t) \simeq A_0C_4^{-1}t^{-\upsilon}, \quad \text{and} \quad M_1^A(t) = A_0. \tag{45}$$

The results indicate that the total number of A species decreases with time as $t^{-\upsilon}$, while the total mass is conserved.

For the case of $C_3 < C_4$, which corresponds to the case in which the catalyzed death dominates the process, the aggregate size distribution of A species can be expressed as

$$a_k(t) \simeq A_0(1-C_3C_4^{-1})^2(C_3/C_4)^k\mathrm{e}^{-(C_4-C_3)t^{\upsilon}} \times \exp[k/R(t)], \tag{46}$$

where $R(t) = C_5(I_1A_0)^{-1}t^{-1}\mathrm{e}^{(C_4-C_3)t^{\upsilon}}$ with $C_5 = 2C_3C_4^{-1}(1-C_4C_3^{-1})^{-2}$. It is obvious that in this case the aggregate size distribution $a_k(t)$ is not scaled.

Moreover, both the total number $M_0^A(t)$ and the total mass $M_1^A(t)$ of A species decrease exponentially as $\exp[-(C_4-C_3)t^{\upsilon}]$. So, in this case, A species will also die out finally.

(ii) In the case of $\upsilon < 0$, from expressions (14), (35) and (36) one can easily obtain the asymptotic solution of $a(t)$ in the long-time limit as

$$a(t) \simeq 1 - C_6 t^{-1}, \tag{47}$$

where $C_6 = 2(I_1 A_0)^{-1} \exp[\int_0^{\infty}(LM_{\upsilon}^C - KM_{\upsilon}^B)\mathrm{d}t]$. And we further derive the asymptotic expression of $A(t)$ from expression (12) as

$$A \simeq 2I_1^{-1} C_6 t^{-2}. \tag{48}$$

The aggregate size distribution of A species is obtained to satisfy the conventional scaling form:

$$a_k(t) \simeq 2I_1^{-1} C_6 t^{-2} \Phi(x), \tag{49}$$

where $x = k/S_A(t)$, with the characteristic size $S_A(t) = C_6^{-1} t$. The scaling exponents are $w = 2$ and $z = 1$, which are independent of the value of the parameter υ and are the same as those in the pure coagulation aggregation process.[6,8] The total number $M_0^A(t)$ and the total mass $M_1^A(t)$ of A species can be derived respectively as

$$M_0^A(t) \simeq 2I_1^{-1} t^{-1}, \quad \text{and}$$
$$M_1^A(t) \simeq A_0 \exp\left[\int_0^{\infty} \left(KM_{\upsilon}^B - LM_{\upsilon}^C\right) \mathrm{d}t\right]. \tag{50}$$

From expression (50) we find that the total number of A species still decreases with time as t^{-1}, and the evolution of the total mass $M_1^A(t)$ is dominated by the competition between the catalyzed birth reaction and the catalyzed death reaction. When $\int_0^{\infty}(KM_{\upsilon}^B - LM_{\upsilon}^C)\mathrm{d}t > 0$, the total mass $M_1^A(t)$ grows with time, which reveals that the influence of catalyzed birth is greater than that of catalyzed death. While $\int_0^{\infty}(KM_{\upsilon}^B - LM_{\upsilon}^C)\mathrm{d}t < 0$, $M_1^A(t)$ decreases with time, which shows that the influence of catalyzed death is greater than that of catalyzed birth. And when $\int_0^{\infty}(KM_{\upsilon}^B - LM_{\upsilon}^C)\mathrm{d}t = 0$, $M_1^A(t)$ is conserved, which corresponds to the fact that the effects of catalyzed birth and catalyzed death counteract each other.

4. Summary

In summary, we have introduced a solvable aggregation growth model of three species A, B and C with the monomer birth and death of A species catalyzed by B and C species, respectively, to study the competition between the catalyzed birth and death in an irreversible aggregation process. The irreversible aggregation occurs between any two aggregates of the like species of A, B and C with the constant rate kernels I_1, I_2 and I_3, respectively. Meanwhile, the monomer birth and the monomer death of A species occur simultaneously under the catalysis of B and C species, respectively, with the catalyzed birth rate kernel $K(k,j) = Kkj^{\upsilon}$ and the catalyzed death rate kernel $L(k,j) = Lkj^{\upsilon}$, where υ is a parameter reflecting the dependence of the catalysis reaction rates of birth and death on the size of catalyst aggregate. In this model, the kinetic evolution behaviour of the A species aggregates is dominated by the two competitions of different aggregation mechanisms. One is the competition between the irreversible aggregations of aggregates of A, B and C species and the catalyzed birth and death of A species, and the other is the competition between the catalyzed birth and death of A species in the presence of the irreversible aggregations of A, B and C species. Those competitions between the different aggregation mechanisms bring A species to an abundant kinetic evolution behaviour. The kinetic evolution behaviour of the aggregate size distribution $a_k(t)$ of A species is illustrated in Table 1.

In Table 1, K_{e} and L_{e} are the effective rates of the catalyzed birth and death on the basis of the irreversible aggregation process, respectively. In the case of $\upsilon = 0$ or $\upsilon > 0$, $K_{\mathrm{e}} = K\Gamma(\upsilon)B_0^{\upsilon}\left(\frac{1}{2}I_2\right)^{\upsilon-1}$ and $L_{\mathrm{e}} = L\Gamma(\upsilon)C_0^{\upsilon}\left(\frac{1}{2}I_3\right)^{\upsilon-1}$.

From Table 1, we find that the kinetic behaviour of the aggregate size distribution $a_k(t)$ depends crucially on the value of υ, which reveals that the competition between the irreversible aggregations of aggregates of A, B and C species and the catalyzed birth and death of A species plays a prior dominating role in the kinetic scaling behaviour of A species to the competition between the catalyzed birth and death of A species. In the case of $\upsilon < 0$, where the effects of the catalyzed birth and death are rather weaker than in the case of $\upsilon = 0$ or $\upsilon > 0$, the aggregate size distribution $a_k(t)$ obeys the conventional scaling law. And the scaling exponents $w = 2$ and $z = 1$ are independent of the effects of the catalyzed birth and death, which is the same as in the irreversible aggregation

Table 1. The form of the aggregate size distribution of A species.

υ			the scaling behaviour of $a_k(t)$
$\upsilon < 0$			conventional scaling with $w = 2$ and $z = 1$
$\upsilon = 0$	$K_e - L_e > -1$		conventional scaling with $w = 2 + (K_e - L_e)$ and $z = 1 + (K_e - L_e)$
	$K_e - L_e \leq -1$		no scaling
$\upsilon > 0$	$K_e > L_e$		generalized scaling
	$K_e = L_e$	$0 < \upsilon \leq 1$	conventional scaling with $w = 2$ and $z = 1$
	$K_e = L_e$	$\upsilon > 1$	conventional scaling with $w = 2\upsilon$ and $z = \upsilon$
	$K_e < L_e$		no scaling behaviour

process.[6,8] So it is apparent that in this case the irreversible aggregation dominates the process and the catalyzed birth and death of A species play no role in the kinetic scaling behaviour of the aggregate size distribution $a_k(t)$.

As the value of υ increases to $\upsilon = 0$ or $\upsilon > 0$, the catalyzed birth rate kernel $K(k, j) = Kkj^{\upsilon}$ and the catalyzed death rate kernel $L(k, j) = Lkj^{\upsilon}$ become relatively large, and the effects of the catalyzed birth and death become stronger. From Table 1, we find that the effects of the catalyzed birth and death play an important role in the kinetic scaling behaviour of the aggregate size distribution $a_k(t)$ and the competition between the effective catalyzed birth and death dominates the kinetic behaviour. It is found that the effect of the catalyzed birth brings some kinetic scaling behaviours to the aggregate size distribution $a_k(t)$, while the effect of the catalyzed death causes the breakdown of the scaling behaviour. When the catalyzed birth dominates the process ($K_e - L_e > -1$ in the case of $\upsilon = 0$; $K_e \geq L_e$ in the case of $\upsilon > 0$), $a_k(t)$ obeys a conventional or generalized scaling law. On the other hand, when the catalyzed death dominates the process ($K_e - L_e \leq -1$ in the case of $\upsilon = 0$; $K_e < L_e$ in the case of $\upsilon > 0$), $a_k(t)$ is not scaled. And in the case of $\upsilon > 0$, there is a marginal case with $K_e = L_e$, where the effects of catalyzed birth and catalyzed death counteract each other and the irreversible aggregation dominates the process, and $a_k(t)$ obeys a conventional scaling law. But even in this case, the scaling exponents w and z are dependent on the catalysis reaction rate kernel parameter υ when $\upsilon > 1$.

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