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Analysis of performance and optimum configuration of two-stage semiconductor thermoelectric module*

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In this paper, the theoretical analysis and simulating calculation were conducted for a basic two-stage semiconductor thermoelectric module, which contains one thermocouple in the second stage and several thermocouples in the first stage. The study focused on the configuration of the two-stage semiconductor thermoelectric cooler, especially investigating the influences of some parameters, such as the current I_1 of the first stage, the area A_1 of every thermocouple and the number n of thermocouples in the first stage, on the cooling performance of the module. The obtained results of analysis indicate that changing the current I_1 of the first stage, the area A_1 of thermocouples and the number n of thermocouples in the first stage can improve the cooling performance of the module. These results can be used to optimize the configuration of the two-stage semiconductor thermoelectric module and provide guides for the design and application of thermoelectric cooler.

Keywords: thermocouples, coefficient of performance, cooling capacity
PACC: 6590

1. Introduction

As miniature cooling devices, the semiconductor thermoelectric refrigeration has the advantages of compact structure, high reliability, no vibration and direct electric energy conversion. It plays more and more important roles in many fields: ranging from superconductor applications, aerospace applications, electronic devices and food storage.[1] However, comparing with other refrigeration modes, we find that the poor coefficient of performance and the small cooling capacity are main obstacle for the semiconductor thermoelectric refrigeration. Recently, the research on improving the performance of semiconductor thermoelectric refrigeration has been developed continuously.

In semiconductor thermoelectric refrigeration, there are many factors influencing the performance of refrigeration, such as the thermoelectric materials[2−6] and size of thermocouples, the junction temperature difference and the electrical current etc. Therefore, a method of optimizing the performance of refrigeration by studying the relations between those factors has been used widely. Vikhor and Anatychuk[7] used optimal control theory to develop a precise technique for a computerized design of segmented thermoelectric coolers, and obtained the optimal parameter values of BiTe-based materials for thermoelectric coolers with single-, double- and three-segmented legs. Cheng and Shih[8] presented a method based on genetic algorithm to maximize the cooling capacity and the coefficient of performance for two-stage thermoelectric coolers. Throughout the calculation in genetic algorithm, the optimization was executed to generate the optimal parameters for a desired temperature difference of 90°C. Based on non-equilibrium thermodynamics, Chen *et al*[9] established the cycle models of a single-stage and a two-stage semiconductor thermoelectric refrigeration system. The internal structure parameter of the thermoelectric device is optimized, and the reasonable ranges of some parameters are determined. Luo *et al*[10] applied the theory of finite time thermodynamics to analyse and optimize the performance of a thermoelectric refrigerator, which is composed of multi-elements. The allocation of heat transfer surface area of a multi-element thermoelectric refrigerator was obtained for maximizing the cooling load and coefficient of performance in their study. Yu *et al*[11] developed a module of two-stage thermoelectric cooler

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and studied the influences of the allocation of the junction temperature difference in the module, the length of thermocouples and the number of thermocouples on the cooling performance of the module.

Two-stage semiconductor thermoelectric modules are often used for extending operating temperature range of the semiconductor thermoelectric refrigeration. The performance of the coolers depends not only on the physical properties of the thermocouples, but also on the configuration of two-stage thermoelectric coolers, such as the current of the first stage, the area of thermocouples and the number of thermocouples. However, the previous work[11] emphasized more on the effect of the length of thermocouples, the effect of the area of thermocouples was not considered deeply. In this paper, the optimum configuration of a basic two-stage semiconductor thermoelectric module was studied by investigating the current of the first stage, the area of thermocouples and the number of thermocouples. By analysing the effect of those parameters on the cooling performance, the optimum performance relations was obtained. The results may provide some guidance for the design and application of practical thermoelectric cooler.

We use the following symbols throughout this paper.

Table 1. Symbols used in this paper.

Symbol	Nomenclature
I	electrical current (A)
K	thermal conductance of a thermocouple (W/K)
A	cross-sectional area of every thermocouple (cm^2)
L	the length of thermocouples (cm)
R	electrical resistance of a thermocouple (Ω)
n	the number of thermocouples in the first stage
q_c	heat flow rate of a thermocouple at the cold junctions (W)
q_h	heat flow rate of a thermocouple at the hot junctions (W)
Q_c	cooling capacity of the semiconductor thermoelectric module (W)
Q_h	rejected heat flow of the thermoelectric module (W)
α	Seebeck coefficient (V/K)
ρ	electric resistivity (Ωcm)
λ	thermal conductivity (W/K cm)
ε	coefficient of performance
1	the first stage
2	the second stage
p	p-type semiconductor
n	n-type semiconductor

2. The two-stage semiconductor thermoelectric refrigeration

Currently, the one-stage semiconductor thermoelectric module is composed of a large number of thermocouples connected electrically in series and thermally in parallel. A thermocouple is made of a p- and a n-type semiconductor. A two-stage semiconductor thermoelectric module is stacked by one group of thermocouples on the top of the other, which is connected thermally in series. In an actual operation, the working electric currents for them are in series or in parallel. Two types of two-stage thermoelectric coolers are both used in semiconductor thermoelectric refrigeration, and the types with separate electrical currents may have the best performance.[12] Therefore, the type of two-stage thermoelectric cooler with separate working electrical currents was selected as the model in this study. For simplicity, there is only one thermocouple in the second stage, but several thermocouples in the first stage for this module, as shown in Fig.1. In addition, a piece of thermally conductive but electrically insulating ceramic plate is placed between the first stage and the second stage.

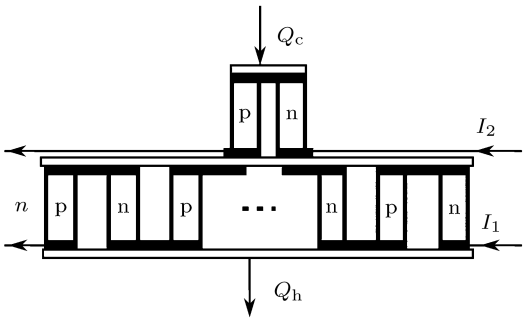


Fig.1. Schematic diagram of the semiconductor thermoelectric module.

For the first stage thermocouple, the pumped heat rates at the cold side and rejected heat rates at the hot side can be written respectively as

$$q_{c1} = \alpha_1 I_1 T_{c1} - K_1(T_{h1} - T_{c1}) - \frac{1}{2}R_1 I_1^2, \quad (1)$$

$$q_{h1} = \alpha_1 I_1 T_{h1} - K_1(T_{h1} - T_{c1}) + \frac{1}{2}R_1 I_1^2, \quad (2)$$

where

$$K_1 = \frac{2\lambda_1 A_1}{l_1}, \quad (3)$$

$$R_1 = \frac{2\rho_1 l_1}{A_1}. \quad (4)$$

Similarly, for one thermocouple in the second stage, it has

$$q_{c2} = \alpha_2 I_2 T_{c2} - K_2(T_{h2} - T_{c2}) - \frac{1}{2}R_2 I_2^2, \quad (5)$$

$$q_{h2} = \alpha_2 I_2 T_{h2} - K_2(T_{h2} - T_{c2}) + \frac{1}{2}R_2 I_2^2. \quad (6)$$

Where

$$K_2 = \frac{2\lambda_2 A_2}{l_2}, \quad (7)$$

$$R_1 = \frac{2\rho_2 l_2}{A_2}. \quad (8)$$

The conventional analysis method for thermoelectric module is also used in this study, which usually does not take the effect of the thermal and contact resistances of ceramic plate into consideration. In addition, we assume that a better thermal insulation is taken for the whole module. Therefore, the thermal resistance between two stages and heat loss for the whole two-stage thermoelectric module is neglected. There are following heat balance equations in the module:

$$T_{h2} = T_{c1}, \quad (9)$$

$$q_{h2} = n q_{c1}. \quad (10)$$

Based on the above heat balance equations, we find that the cooling capacity at the cold side and the rejected heat rate at the hot side of the whole two-stage thermoelectric cooler are respectively as

$$\begin{aligned} Q_c &= Q_{c2} \\ &= \alpha_2 I_2 T_{c2} - K_2(T_{h2} - T_{c2}) - \frac{1}{2}R_2 I_2^2, \end{aligned} \quad (11)$$

$$Q_h = n\left(\alpha_1 I_1 T_{h1} - K_1(T_{h1} - T_{c1}) + \frac{1}{2}R_1 I_1^2\right). \quad (12)$$

The coefficient of performance of two-stage thermoelectric module is thus defined as

$$\varepsilon = \frac{Q_c}{Q_h - Q_c} = \frac{1}{Q_h/Q_c - 1}. \quad (13)$$

It is well known that in the second stage the module exists an optimum electrical current corresponding to the maximum cooling capacity of the semiconductor thermoelectric module. The optimal parameters at the maximum cooling capacity can be derived from Eq.(11), which is written as

$$I_{2c\,max} = \frac{\alpha_2 T_{c2}}{R_2}. \quad (14)$$

3. Simulation results and discussion

In the calculation, the semiconductor thermoelectric properties of thermocouples of each stage are set as: Seebeck coefficient $\alpha = |\alpha_p| + |\alpha_n| = 2 \times 1.8 \times 10^{-4}\,\mathrm{V/K}$; thermal conductivity $\lambda_P = \lambda_N = 1.4 \times 10^{-2}\,\mathrm{W/Kcm}$; electrical resistivity $\rho_P = \rho_N = 7.7 \times 10^{-4}\,\Omega\mathrm{cm}$.

From the above module, we can see that the temperature, the junction temperature difference, the electrical current, and the number of thermocouples in the first stage and the size of thermocouples all influence the coefficient of performance. Therefore, in the following analysis, more emphasis is laid on the impacts caused by the current of the first stage, the area of every thermocouple and the number of thermocouples in the first stage. For this purpose, some parameters of semiconductor thermoelectric module are determined to be $T_{c2} = 248\,\mathrm{K}$, $T_{h1} = 308\,\mathrm{K}$, $A_2 = 0.1\,\mathrm{cm}^2$, $l_2 = 0.5\,\mathrm{cm}$. In addition, the electrical current I_2 of the second stage is applied to the optimal value $I_{2c\,max}$.

3.1. Effect of the current of the first stage

In two-stage semiconductor thermoelectric module, the current I_1 of the first stage is a main factor that affects the coefficient of performance when the current I_2 of the second stage is fixed.

Figure 2 shows the effect of I_1 and the number n of couples in the first stage on the coefficient of performance ε. In the figure, we can see that when n is a constant, the ε decreases with the increase of I_1. This results from the change of the rate $\eta = Q_{\rm h}/Q_{\rm c}$. Figure 3 shows the variation of η with I_1 when taking $n = 4$ as an example. Therefore, the ε can be improved by decreasing I_1 properly. However, the decrease of I_1 is limited. This is because when I_1 is very low the heat balance between two stages could not be met.

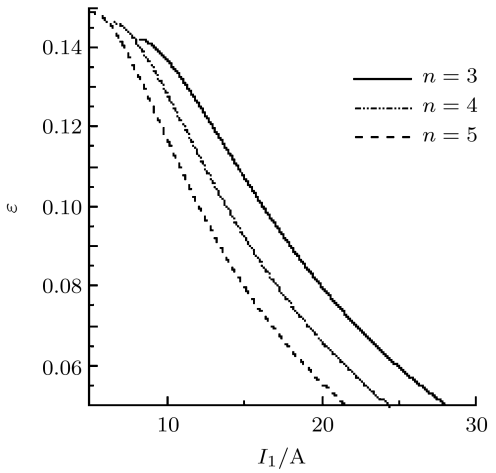


Fig.2. The coefficient of performance versus the current and the number of thermocouples in the first stage.

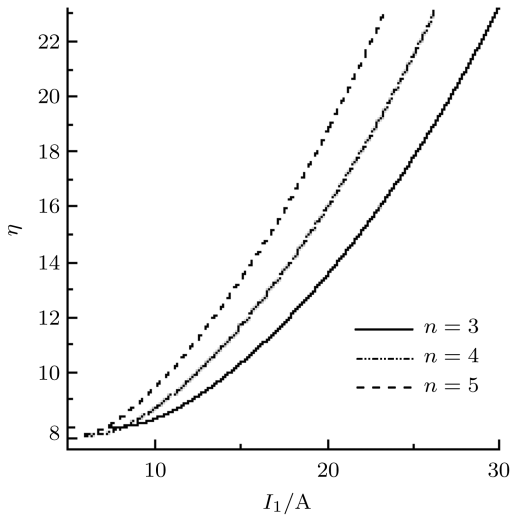


Fig.3. The rate η versus the current and the number of thermocouples in the first stage.

In addition, it is also shown in Fig.2 that increasing n will decrease ε. This results from that an increase in n causes the increase of η. This leads to the decrease of ε. The relationship among η, I_1 and n is shown in Fig.3. From Fig.3 we also get the result that the cooling capacity increases with the increase of the number n when I_1 keeps constant.

3.2. Effect of the area of every thermocouple

In semiconductor thermoelectric cooler, the area of every thermocouple not only influences the performance of refrigeration but also has a direct effect on cost. Especially for multi-stage refrigeration, the proper choice of the area of every thermocouple can effectively improve the performance of refrigeration. The original research on the area of every thermocouple was carried out in this paragraph. We assume that the area A_1 of every thermocouple is independent of the number n.

The following results are obtained when I_1 is $I_1 = 15\,\mathrm{A}$, L_1 is $L_1 = 0.4\,\mathrm{cm}$ and A_1 is variable. Figure 4 shows the effect of A_1 and n on ε. When n is given a value, we can see from Fig.4 that the coefficient of performance increases with the increase of A_1. This derives from the fact that the increase of A_1 causes the decrease of η, which is shown in Fig.5.

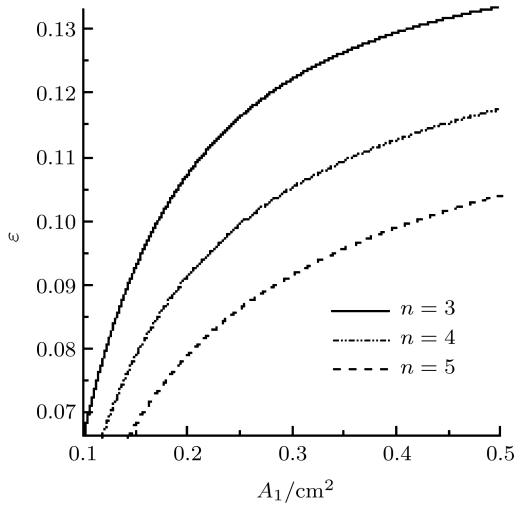


Fig.4. The coefficient of performance versus the area and the number of thermocouples in the first stage.

In addition we also get from Fig.4 that ε will decrease when n increases, because the increase of n

causes the increase of η (Fig.5). Meanwhile Fig.5 tells that the cooling capacity increases with the decrease of the area A_1 when the current I_1 is a constant.

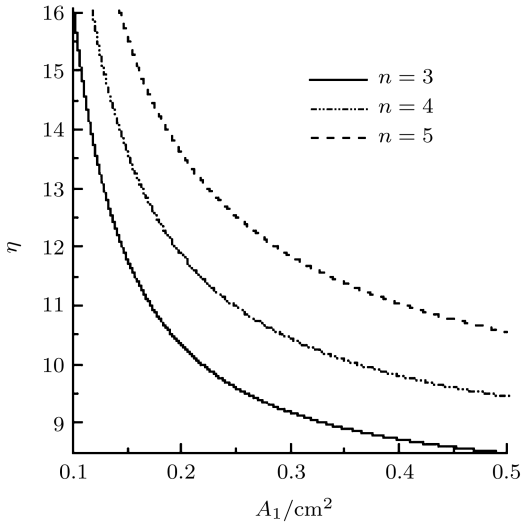


Fig.5. The rate η versus the area and the number of thermocouples in the first stage.

3.3. Optimum configuration

Based on the above analysis, we know that n, I_1 and A_1 all have influences on the ε, and those parameters are correlative. Therefore, the synthetical effect of n, I_1 and A_1 on ε should be considered for the optimum configuration of the two-stage thermoelectric cooler, which is shown in Fig.6. In Fig.6, the three curves of $A_1 = 0.2\,\mathrm{cm}^2$, $n = 3$; $A_1 = 0.3\,\mathrm{cm}^2$, $n = 3$, and $A_1 = 0.3\,\mathrm{cm}^2$, $n = 5$ are taken as examples for the analysis.

Comparing the curve of $A_1 = 0.2\,\mathrm{cm}^2$, $n = 3$ with the curve of $A_1 = 0.3\,\mathrm{cm}^2$, $n = 3$ in Fig.6, we can see clearly that the coefficient of performance of the former curve is smaller than that of the latter curve with a certain I_1 while keeping a steady cooling capacity. If keeping ε a constant, comparing the curve of $A_1 = 0.3\,\mathrm{cm}^2$, $n = 3$ with the curve of $A_1 = 0.3\,\mathrm{cm}^2$, $n = 5$, we can find that the corresponding I_1 of the former curve is bigger than that of the latter curve. This indicates that the cooling capacity of the former curve is larger than that of the latter curve on this occasion. Therefore, the increase of A_1 and the reduction of n can raise the coefficient of performance and decrease the cooling capacity.

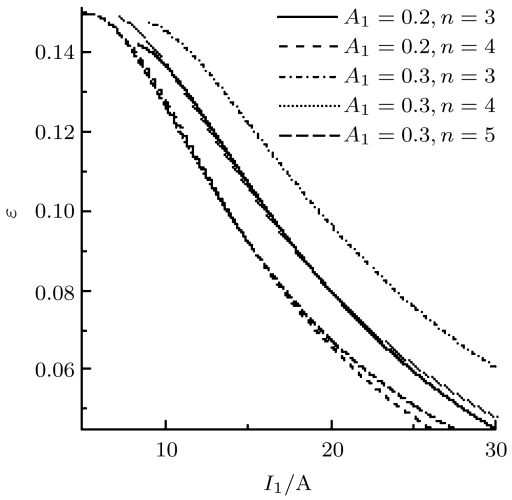


Fig.6. Variation of ε with I_1 for some given A_1 and n.

4. Conclusions

Based on theory of non-equilibrium thermodynamics, this paper established a two-stage semiconductor thermoelectric module, performed theoretical analysis and numerical simulation, and laid more emphasis on the influences of the current I_1 of the first stage, the area A_1 of every thermocouple and the number n of thermocouples in the first stage on the cooling performance of the cooler. We can get the conclusion: for a fixed current I_1 of the first stage, the increase of the area A_1 of every thermocouple and decrease of the number n of thermocouples in the first stage can improve the coefficient of performance and decrease the cooling capacity of the semiconductor thermoelectric cooler.

Therefore, those parameters such as the area A_1 and number n of the thermocouples should be considered synthetically in the optimum configuration of the two-stage semiconductor thermoelectric module. We should compute the optimal dimensions of the cooler to maximize its coefficient of performance, while fulfilling the cooling capacity requirements. Meanwhile it is expected that the results from the present study provide a guideline for the design and application of practical thermoelectric cooler and the development of thermoelectric materials.

References

[1] Riffat S B and Ma X L 2003 *Appl. Therm. Eng.* **23** 913

[2] Jiang J, Xu G J, Cui P and Chen L D 2006 *Acta Phys. Sin.* **55** 4849 (in Chinese)

[3] Li H, Tang X F, Liu T X, Song C and Zhang Q J 2005 *Acta Phys. Sin.* **54** 5481 (in Chinese)

[4] Luo P F, Tang X F, Xiong C and Zhang Q J 2005 *Acta Phys. Sin.* **54** 2403 (in Chinese)

[5] Tang X F, Chen L D, Goto T, Hirar T and Yang R Z 2002 *Acta Phys. Sin.* **51** 2823 (in Chinese)

[6] Tang X F, Chen L D, Goto T, Hiari T and Yuan R Z 2001 *Acta Phys. Sin.* **50** 1560 (in Chinese)

[7] Vikhor L N and Anatychuk L I 2006 *Appl. Therm. Eng.* **26** 1692

[8] Cheng Y H and Shih C K 2006 *Appl. Therm. Eng.* **26** 937

[9] Chen J C, Zhou Y H, Wang H J and Wang J T 2002 *Appl Energy* **73** 285

[10] Luo J, Chen L G, Sun F R and Wu C H 2003 *Energy Conserv. Manage.* **44** 3197

[11] Yu J L, Zhao H and Xie K S 2007 *Cryogenics* **47** 89

[12] Xuan X C 2002 *Semicond. Sci. Technol.* **17** 414