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Sideband manipulation of population inversion in a three-level Λ atomic system*

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Sideband manipulation of population inversion in a three-level Λ atomic configuration is investigated theoretically. Compared with the case of a nearly monochromatic field, a population inversion between an excited state and a ground state has been found in a wide sideband intensity range by increasing the difference in frequency between three components. Furthermore, the population inversion can be controlled by the sum of the relative phases of the sideband components of the trichromatic pump field with respective to the phase of the central component. Changing the sum phase from 0 to π, the population inversion between the excited state and the ground state can increase within nearly half of the sideband intensity range. At the same time, the sideband intensity range that corresponds to the system exhibiting inversion $\rho_{00} > \rho_{11}$ also becomes wider evidently.

Keywords: population inversion, sideband manipulation, relative phase
PACC: 4250, 4265

1. Introduction

Quantum interference can lead to many interesting optical phenomena such as lasing without population inversion,[1−9] enhancement of index,[10−14] electromagnetically induced transparency,[15−22] subluminal and superluminal light,[23−31] cancellation of spontaneous emission[32−38] and unexpected population inversion.[39−46] In Ref.[39], Meduri *et al* showed that for a V system, two coherent fields opened new channels of irreversible population transfer, and they found an unexpected population inversion on one of the optical transitions. In an early work, Whitley and Stroud[40] found that a ladder-type three-level atomic system driven by two coherent fields could also lead to an unexpected population inversion. Spontaneously generated coherence can also lead to an unexpected population inversion. Malakyan and Unanyan[41] showed that the population inversion could be greatly enhanced on one of the optical transitions due to the spontaneous-emission-induced coherence in a ladder-type three-level atomic system. Gong *et al*[42] found that the unexpected population inversions could be achieved on both of the optical transitions due to spontaneously generated coherent effects in a closed Λ-type atomic system. In addition, by changing the rates of spontaneous emission from an excited state to two metastable states in a coherently driven Λ system, Hu and Peng[43] achieved a maximum population inversion of 0.4. Especially, Ficek *et al*[44] analysed the steady-state population in a two-level atom driven by three laser fields of unequal frequencies. They found that the population inversion was dependent on initial relative phases between the driving lasers.

Enlightened by the above work, in this present paper, we study the steady-state population inversion in a three-level Λ atomic system, in which a trichromatic field and a monochromatic field are applied respectively to transitions from an excited state to two lower states. By appropriately tuning the frequency difference of the trichromatic pump field, a population inversion between an excited state and a ground state can be achieved in a wide sideband intensity range compared with the case of a monochromatic field which corresponds to detunings between three components being close to 0. Moreover, we also consider phase dependence in this paper. We find that the

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population inversion crucially depends on the sum of the relative phases of the sideband components of the trichromatic pump field with respect to the phase of the central component. With the sum phase changing from 0 to π, the population inversion between the excited state and the ground state can be increased within nearly half of the sideband intensity range. At the same time, the sideband intensity range that corresponds to the system exhibiting inversion $\rho_{00} > \rho_{11}$ also becomes wider evidently. If the sum phase is fixed, the change of the relative phases has no influence on the population inversion, which is different from the point proposed in Ref.[44].

2. Model and equations

The system under consideration is a Λ-type system as shown in Fig.1. It consists of an excited state and two lower states, denoted by $|0\rangle$, $|1\rangle$ and $|2\rangle$ respectively. A trichromatic field $(E_0 + E_1 e^{i\delta t} + E_2 e^{-i\delta t})e^{-i\omega_0 t}$ + c.c. is applied to the $|0\rangle - |2\rangle$ transition, and a monochromatic field $E_3 e^{-i\omega_3 t}$ + c.c. is coupled to $|0\rangle - |1\rangle$ transition, where, $E_i (i = 0 - 3)$ are the amplitudes of components of the trichromatic and monochromatic fields, ω_0 is the central frequency of the trichromatic field, ω_3 is the frequency of the monochromatic field, and δ is the difference in frequency between the three components of the trichromatic field. $\Delta_1 = \omega_{01} - \omega_p$ and $\Delta_2 = \omega_{02} - \omega_0$ are the detunings of the respective fields from the corresponding transitions. $2\gamma_1$ and $2\gamma_2$ are the atomic decay rates. $\Omega_i = \dfrac{\boldsymbol{\mu}_{20} \cdot E_i}{\hbar} (i = 0, 1, 2)$ and $\Omega_3 = \dfrac{\boldsymbol{\mu}_{10} \cdot E_3}{\hbar}$ denote Rabi frequencies associated with the respective fields, in which $\boldsymbol{\mu}_{20}$ and $\boldsymbol{\mu}_{10}$ are the atomic transition electric dipole moments. Generally, we assume that the Rabi frequencies Ω_0 and Ω_3 and the dipole moments (μ_{10} and μ_{20}) each are real. At the same time we take $\Omega_j = \mid \Omega_j \mid e^{-i\phi_j}$ $(j = 1, 2)$, where $e^{-i\phi_j}$ $(j = 1, 2)$, where $\phi_{1,2}$ are the relative phases of the sideband components $E_{1,2}$ of the trichromatic pump field with respect to that of the central component E_0, respectively. We also define the sum of the relative phases of the sideband components with respect to that of the central component as $\Phi = \phi_1 + \phi_2$. Under the rotating-wave approximation, the following equations are derived for the density matrix elements:

$$
\begin{aligned}
\dot{\rho}_{00} &= -2(\gamma_1 + \gamma_2)\rho_{00} + i\Omega_3\rho_{10} - i\Omega_3^*\rho_{01} \\
&\quad + iG\rho_{20} - iG^*\rho_{02}, \\
\dot{\rho}_{11} &= 2\gamma_1\rho_{00} - i\Omega_3\rho_{10} + i\Omega_3^*\rho_{01}, \\
\dot{\rho}_{22} &= 2\gamma_2\rho_{00} - iG\rho_{20} + iG^*\rho_{02}, \\
\dot{\rho_{01}} &= -(\gamma_1 + \gamma_2 + i\Delta_1)\rho_{01} - i\Omega_3\rho_{00} \\
&\quad + i\Omega_3\rho_{11} + iG\rho_{21}, \\
\dot{\rho_{02}} &= -(\gamma_1 + \gamma_2 + i\Delta_2)\rho_{02} + i\Omega_3\rho_{12} \\
&\quad + iG(1 - \rho_{11} - 2\rho_{00}),
\end{aligned}
$$

and

$$
\dot{\rho_{21}} = -i(\Delta_2 - \Delta_1)\rho_{21} - i\Omega_3\rho_{20} + iG^*\rho_{01}. \quad (1)
$$

The above equations are constrained by $\rho_{00} + \rho_{11} + \rho_{22} = 1$ and $\rho_{ij}^* = \rho_{ji}$. Here we have set $G = (\Omega_0 + \Omega_1 e^{i\delta t} + \Omega_2 e^{-i\delta t})$ and $G^* = (\Omega_0^* + \Omega_1^* e^{-i\delta t} + \Omega_2^* e^{i\delta t})$.

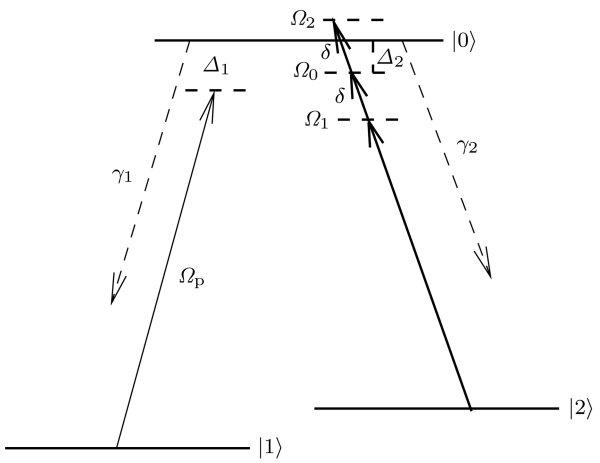


Fig.1. Schematic diagram of energy levels for a Λ-system, which consists of an excited state and two lower states, denoted by $|0\rangle$, $|1\rangle$ and $|2\rangle$ respectively. The two lower states are respectively coupled to the excited state via a trichromatic field and a monochromatic field.

In order to solve these equations, the density matrix elements are expanded in a Fourier series as

$$
\rho_{jk} = \sum_{n=-\infty}^{+\infty} \rho_{jk}^{(n)} e^{in\delta t} (j, k = 0, 1, 2), \quad (2)
$$

i.e.

$$
\rho_{01} = \sum_{n=-\infty}^{+\infty} \rho_{01}^{(n)} e^{in\delta t}, \ \rho_{10} = \sum_{n=-\infty}^{+\infty} \rho_{10}^{(n)} e^{in\delta t},
$$

$$
\rho_{02} = \sum_{n=-\infty}^{+\infty} \rho_{02}^{(n)} e^{in\delta t}, \ \rho_{20} = \sum_{n=-\infty}^{+\infty} \rho_{20}^{(n)} e^{in\delta t},
$$

$$
\rho_{12} = \sum_{n=-\infty}^{+\infty} \rho_{12}^{(n)} e^{in\delta t}, \ \rho_{21} = \sum_{n=-\infty}^{+\infty} \rho_{21}^{(n)} e^{in\delta t},
$$

$$\rho_{00}=\sum_{n=-\infty}^{+\infty}\rho_{00}^{(n)}\mathrm{e}^{\mathrm{i}n\delta t},\quad\text{and}\quad\rho_{11}=\sum_{n=-\infty}^{+\infty}\rho_{11}^{(n)}\mathrm{e}^{\mathrm{i}n\delta t}. \tag{3}$$

Substituting expression (3) into Eq.(1) and applying the method described in Ref.[47], the steady solution for the vector $\boldsymbol{X}=\left(\rho_{01}^{(-N)},\cdots,\rho_{11}^{(-N)};\cdots;\rho_{01}^{(0)},\cdots,\rho_{11}^{(0)};\cdots;\rho_{01}^{(N)},\cdots,\rho_{11}^{(N)}\right)^{\mathrm{T}}$ can be obtained.

3. Analyses and results

It is easy to see from Eq.(1) and expressions (2) and (3) that the steady-state population inversions $\rho_{00}-\rho_{11}$ and $\rho_{00}-\rho_{22}$ are determined by the atomic decay rates (γ_1, γ_2), Rabi frequencies (Ω_0, Ω_1, Ω_2, Ω_3), and frequency detunings (δ, Δ_1, Δ_2). In what follows we investigate the trichromatic manipulation of population inversion through the numerical simulation. For simplicity, all parameters are reduced to dimensionless ones by being scaled with γ_1.

Firstly, we focus on the behaviour of population inversion due to multi-colour interference effect in the case in which the sum of the initial relative phases of the sideband components of the trichromatic field Φ is equal to 0. Setting the central Rabi frequencies $\Omega_0=\Omega_3=36.0$, the atomic decay rates $\gamma_2=3.0$, and frequency detunings $\Delta_1=-\Delta_2=20.0$, we plot in Fig.2 the evolutions of population inversion $\rho_{00}-\rho_{11}$ versus the sideband Rabi frequency $|\Omega_1|=|\Omega_2|$ with different values of the difference in frequency of the trichromatic field, in which curves $(a-d)$ respectively correspond to the cases of $\delta=0.0$, 5.0, 20.0 and 30.0. From Fig.2, we see that, in a sideband intensity range of $|\Omega_1|=|\Omega_2|=(0,26.7]$, the steady-state population inversion $\rho_{00}-\rho_{11}$ rises obviously with the frequency difference increasing from 0.0 to 5.0 and 20.0. For example, at the point of $|\Omega_1|=|\Omega_2|=21.1$, the steady-state population difference $\rho_{00}-\rho_{11}$ is 0.000 for $\delta=0.0$, 0.035 for $\delta=5.0$, and 0.058 for $\delta=20.0$. As is well known, if δ=0.0, three components of the trichromatic field are nearly in resonance. As a result, the trichromatic field returns to the monochromatic case which is described as curve (a). Gradually, increasing the sideband intensity $|\Omega_1|=|\Omega_2|$, the steady-state population inversion $\rho_{00}-\rho_{11}$ rapidly decreases to less than zero. The sideband intensity range corresponding to the steady-state population inversion $\rho_{00}-\rho_{11}>0$ is only $|\Omega_1|=|\Omega_2|=[0,21.1]$. While, when the values of the difference frequency between three components of the trichromatic field increase to $\delta=5.0$, and 20.0, the cases described as curves $(b,\ c)$ are quite different. For example, in the case of $\delta=5.0$ (curve (b)), the sideband intensity range corresponding to the steady-state population inversion $\rho_{00}-\rho_{11}>0$ is $|\Omega_1|=|\Omega_2|=[0,34.1]$. Notably, the atomic system can exhibit inversion $\rho_{00}>\rho_{11}$ in a wide sideband intensity range compared with the case of $\delta=0.0$ (curve (a)). These are the results of multi-colour interference.

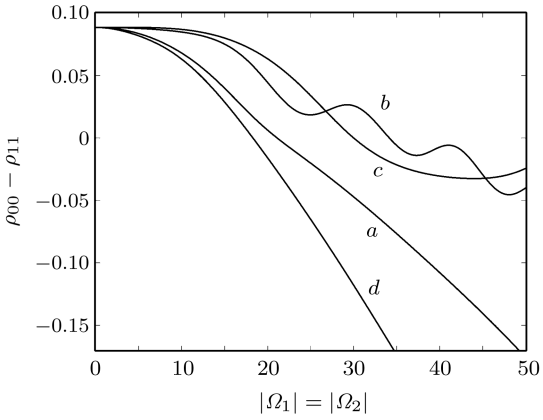


Fig.2. Variations of population inversion versus the sideband Rabi frequency $|\Omega_1|=|\Omega_2|$ with different values of the difference in frequency of the trichromatic field. Curves $(a-d)$ respectively correspond to $\delta=0.0$, 5.0, 20.0 and 30.0. Other parameters are $\Phi=0$, $\Omega_0=|\Omega_3|=36.0$, $\gamma_2=3.0$, $\Delta_1=-\Delta_2=20.0$.

We have also numerically calculated the population inversion when the frequency difference δ increases to a value as large as $\delta=30.0$. In this case, we can see from curve (d) that by gradually increasing the sideband intensity $|\Omega_1|=|\Omega_2|$, the steady-state population inversion $\rho_{00}-\rho_{11}$ decreases more rapidly than in the case of $\delta=0.0$ (curve (a)). This result can be understood as the following. In Ref.[43], it was proposed that the large inversions occurred when the frequency detunings of two arms had the opposite signs. For the mode considered in the present paper, three components of the trichromatic field coupled to the $|0\rangle-|2\rangle$ transition and the monochromatic field applied to the $|0\rangle-|1\rangle$ transition form tree pairs of arms. We fix the frequency detunings $\Delta_1=20.0$ and $\Delta_2=-20.0$. When $\delta=5.0$ and 20.0, the frequency detunings of three pairs of arms all have opposite signs. So in the sideband intensity range considered here, population inversions are larger than that in the

case of $\delta = 0.0$ due to the positive effects of three pairs of arms. But when $\delta = 30.0$, the frequency detunings of one pair of arms corresponding to Ω_1 evidently have the same sign. This will destroy the increase of population inversion, resulting in more rapid decrease than that in the case of $\delta = 0.0$.

It is well known that the multi-level system coupled by the multi-colour fields exhibits rich phase-dependent spectrum features associated with the light absorption, the fluorescence emission and coherent population trapping.[48−52] We now turn to considering the phase dependence of the population inversion. In Fig.3, we plot the evolution of population inversion $\rho_{00} - \rho_{11}$ versus the sideband Rabi frequency $|\Omega_1| = |\Omega_2|$, with the sum phase changing from $\Phi = 0$ (solid line) to $\Phi = \pi$ (dashed line) in the case of $\delta = 10.0$. Other parameters are the same as those in Fig.2. Comparing the solid line with the dashed line

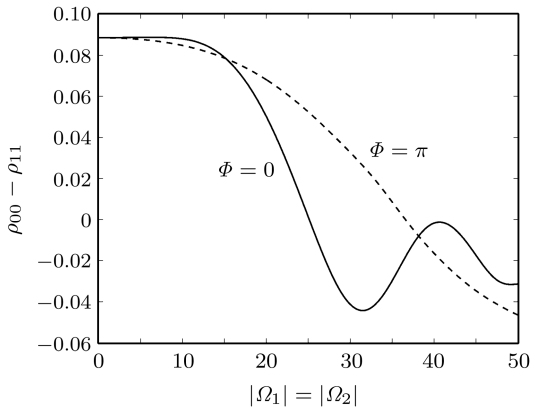


Fig.3. Variations of population inversion versus the sideband Rabi frequency $|\Omega_1| = |\Omega_2|$ with different values of the sum phase Φ in the case of $\delta = 10.0$. Solid line $\Phi = 0$ and dashed line to $\Phi = \pi$. Other parameters are the same as those in Fig.2.

in Fig.3, we find that the population inversion depends strongly on the sum phase. Changing the sum phase from 0 to π, the population inversion between an excited state and a ground state can increase within nearly half of the sideband intensity range corresponding to $\rho_{00} > \rho_{11}$. At the same time, the sideband intensity range that corresponds to population inversion $\rho_{00} > \rho_{11}$ also becomes wider evidently. Noticeably, it is the sum phase but not the respective relative phase that modifies the population inversion. When the sum phase is fixed, the changes in respective relative phase have no influence on the population inversion. As a result, we can achieve the increased population inversion in the corresponding sideband intensity region through controlling the sum of the relative phases of the sideband components of the trichromatic pump field with respect to the central component.

The above dependence of the sum phase can be analysed by using the dressed atomic states.[53] It is well known that the phase dependence occurs in a closed loop system in which all transitions are in a loop configuration. In the case of trichromatic pump field, the sideband pump components lead to the two-step transitions. Two different pathways in the closed loop give rise to the same phase $(\phi_1 + \phi_2)$.[42] Subsequently, the population inversion which is related to ρ_{00} and ρ_{11} is determined by the sum phase Φ, not by the respective relative phases ϕ_1 and ϕ_2.

4. Conclusions

We have studied the sideband manipulation of the steady-state population inversion in a three-level Λ atomic configuration. The numerical calculation results clearly show that this system can exhibit a population inversion between an excited and a ground state in a wide sideband intensity range due to the multi-colour interference compared with the case of a nearly monochromatic field, which means that detunings between three components are close to 0. The sum of the relative phases of the sideband components of the trichromatic pump field with respect to the phase of the central component also plays a crucial role in the population inversion. Changing the sum phase from 0 to π, the increased population inversion between the excited state and ground state can be obtained within nearly half of the sideband intensity range corresponding to $\rho_{00} > \rho_{11}$ in the case of $\Phi = 0$.

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