Vol 17 No 4, April 2008
1674-1056/2008/17(04)/1254-09

Chinese Physics B

© 2008 Chin. Phys. Soc.
and IOP Publishing Ltd

# Wigner functions and tomograms of the photon-depleted even and odd coherent states*

Wang Ji-Suo(王继锁)† and Meng Xiang-Guo(孟祥国)

*Department of Physics, Liaocheng University, Liaocheng* 252059, *China*

(Received 9 July 2007; revised manuscript received 2 August 2007)

Using the coherent state representation of Wigner operator and the technique of integration within an ordered product (IWOP) of operators, this paper derives the Wigner functions for the photon-depleted even and odd coherent states (PDEOCSs). Moreover, in terms of the Wigner functions with respect to the complex parameter $\alpha$ the nonclassical properties of the PDEOCSs are discussed. The results show that the nonclassicality for the state $|\beta,m\rangle_{\rm o}$ (or $|\beta,m\rangle_{\rm e}$) is more pronounced when $m$ is even (or odd). According to the marginal distributions of the Wigner functions, the physical meaning of the Wigner functions is given. Further, the tomograms of the PDEOCSs are calculated with the aid of newly introduced intermediate coordinate-momentum representation in quantum optics.

**Keywords:** photon-depleted even and odd coherent state, Wigner function, intermediate coordinate-momentum representation, tomogram

**PACC:** 4250, 0530

## 1. Introduction

The Wigner function[1,2] in phase space is an important statistical method for studying fields in quantum optics. Since its two marginal distributions lead to measuring probability density in coordinate space and momentum space respectively, the Wigner function is most popularly used. Recently, various photon states have been generated by the micromaser[3] and Wigner functions of some cavity fields can be measured in the whole phase space by a scheme based on interaction between cavity fields and atoms.[4] Therefore, it is an important topic to construct and measure the Wigner functions of quantum states by using some theoretical schemes. In this paper, we shall obtain the Wigner functions for the photon-depleted even and odd coherent states (PDEOCSs). It is of physical interest in experiment because it is generated in the process of the two-level excited atoms through a cavity.[5]

The PDEOCSs are defined as[5,6]

$$|\beta,m\rangle_{\rm o}=C_{{\rm o},m}\left[\left(a^{+}\right)^{-1}\right]^{m}|\beta\rangle_{\rm o}=\frac{C_{{\rm o},m}}{\sqrt{\sinh|\beta|^{2}}}\sum_{n=[m/2]}^{\infty}\frac{\beta^{2n+1}\sqrt{(2n+1-m)!}}{(2n+1)!}\,|2n+1-m\rangle\,, \tag{1}$$

$$|\beta,m\rangle_{\rm e}=C_{{\rm e},m}\left[\left(a^{+}\right)^{-1}\right]^{m}|\beta\rangle_{\rm e}=\frac{C_{{\rm e},m}}{\sqrt{\cosh|\beta|^{2}}}\sum_{n=[(m+1)/2]}^{\infty}\frac{\beta^{2n}\sqrt{(2n-m)!}}{(2n)!}\,|2n-m\rangle\,, \tag{2}$$

where $\beta$ is a complex number. $C_{{\rm o},m}$ and $C_{{\rm e},m}$ are the normalization constants,

$$|C_{{\rm o},m}|^{2}=\frac{\sinh|\beta|^{2}}{\sum\limits_{n=[m/2]}^{\infty}\dfrac{|\beta|^{2(2n+1)}}{\left[(2n+1)!\right]^{2}}(2n+1-m)!},\quad |C_{{\rm e},m}|^{2}=\frac{\cosh|\beta|^{2}}{\sum\limits_{n=[(m+1)/2]}^{\infty}\dfrac{|\beta|^{4n}}{\left[(2n)!\right]^{2}}(2n-m)!}. \tag{3}$$

*Project supported by the National Natural Science Foundation of China (Grant No 10574060) and the Natural Science Foundation of Shandong Province of China (Grant No Y2004A09).

†E-mail: jswang@lcu.edu.cn

Using the following relations[5]

$$a^k\left(a^{-1}\right)^k=\left[\left(a^+\right)^{-1}\right]^k a^{+k}=I,\quad \left(a^{-1}\right)^k a^k=a^{+k}\left[\left(a^+\right)^{-1}\right]^k=I-\sum_{i=0}^{k-1}|i\rangle\langle i|, \tag{4}$$

we can easily prove that the states $|\beta,m\rangle_{\mathrm{o,e}}$ are the eigenvectors of the operator $\left[\left(a^+\right)^{-1}\right]^m a^2a^{+m}$, i.e.,

$$\left[\left(a^+\right)^{-1}\right]^m a^2a^{+m}|\beta,m\rangle_{\mathrm{o,e}}=\beta^2|\beta,m\rangle_{\mathrm{o,e}}. \tag{5}$$

Therefore, the states $|\beta,m\rangle_{\mathrm{o,e}}$ are called as the PDEOCSs.

## 2. The Wigner functions for the PDEOCSs

In the coherent state representation, the Wigner operator $\Delta(\alpha,\alpha^*)$ is[7]

$$\Delta(\alpha,\alpha^*)=\int\frac{\mathrm{d}^2z}{\pi}|\alpha+z\rangle\langle\alpha-z|\,e^{\alpha z^*-\alpha^*z}, \tag{6}$$

where $|\alpha+z\rangle$ is the coherent state[8] and $\alpha$ is a complex number. Using the technique of integration within an ordered product (IWOP) of operators,[7] we can simplify the Wigner operator $\Delta(\alpha,\alpha^*)$ as

$$\Delta(\alpha,\alpha^*)=\frac{2}{\pi}D(2\alpha)(-)^N, \tag{7}$$

where $N=a^+a$, $D(2\alpha)=\exp\left[2\left(\alpha a^+-\alpha^*a\right)\right]$ is the displaced operator.

For an expression of the pure state $|\varphi\rangle$ in terms of the number states $|i\rangle$

$$|\varphi\rangle=\sum_{i=0}^{\infty}f_i|i\rangle, \tag{8}$$

its Wigner function is

$$W(\alpha,\alpha^*)=\langle\varphi|\Delta(\alpha,\alpha^*)|\varphi\rangle=\begin{cases}\sum\limits_{j,i=0}^{\infty}\dfrac{2}{\pi}f_if_j^*(-)^i\sqrt{\dfrac{i!}{j!}}\mathrm{e}^{-|\alpha|^2/2}\alpha^{j-i}L_i^{j-i}\left(|\alpha|^2\right), & j\geq i,\\ \sum\limits_{j,i=0}^{\infty}\dfrac{2}{\pi}f_if_j^*(-)^i\sqrt{\dfrac{j!}{i!}}\mathrm{e}^{-|\alpha|^2/2}\left(-\alpha^*\right)^{i-j}L_j^{i-j}\left(|\alpha|^2\right), & j<i,\end{cases} \tag{9}$$

where $L_n^\rho(x)$ is an associated Laguerre polynomials[9]

$$L_n^\rho(x)=\sum_{l=0}^{n}\binom{n+\rho}{n-l}\frac{(-x)^l}{l!}. \tag{10}$$

By means of simple calculations, we find that the Wigner function $W_{\mathrm{o},m}(\alpha,\alpha^*)$ of the states $|\beta,m\rangle_{\mathrm{o}}$ can be written as a sum of the mixture part $W_{\mathrm{o},m}^{\mathrm{M}}(\alpha,\alpha^*)$ and the quantum interference part $W_{\mathrm{o},m}^{\mathrm{I}}(\alpha,\alpha^*)$:

$$W_{\mathrm{o},m}(\alpha,\alpha^*)=W_{\mathrm{o},m}^{\mathrm{M}}(\alpha,\alpha^*)+W_{\mathrm{o},m}^{\mathrm{I}}(\alpha,\alpha^*), \tag{11}$$

where the mixture part $W_{\mathrm{o},m}^{\mathrm{M}}(\alpha,\alpha^*)$ has the following form

$$W_{\mathrm{o},m}^{\mathrm{M}}(\alpha,\alpha^*)=\frac{2}{\pi}\sum_{n=[m/2]}^{\infty}\frac{C_{\mathrm{o},m}^2(-)^{2n+1-m}(2n+1-m)!|\beta|^{4n+2}}{\left[(2n+1)!\right]^2\sinh|\beta|^2}L_{2n+1-m}^{(0)}\left(4|\alpha|^2\right)\exp\left(-2|\alpha|^2\right), \tag{12}$$

and the interference part $W_{\mathrm{o},m}^{\mathrm{I}}(\alpha,\alpha^*)$ is

$$\begin{aligned}W_{\mathrm{o},m}^{\mathrm{I}}(\alpha,\alpha^*)=\frac{2}{\pi}\sum_{\substack{n,k=[m/2]\\ n\neq k}}&\frac{C_{\mathrm{o},m}^2(-)^{2n+1-m}\beta^{2n+1}\beta^{*2k+1}}{(2n+1)!\,(2k+1)!\sinh|\beta|^2}\exp\left(-2|\alpha|^2\right)\times\Big[(2n+1-m)!\,(2\alpha)^{2(k-n)}\\ &\times L_{2n+1-m}^{2(k-n)}\left(4|\alpha|^2\right)+(2k+1-m)!\left(-2\alpha^*\right)^{2(n-k)}L_{2k+1-m}^{2(n-k)}\left(4|\alpha|^2\right)\Big],\end{aligned} \tag{13}$$

where the constant $C_{\mathrm{o},m}$ is given by Eq.(3). Similarly, using Eqs.(2), (3) and (6)–(10) we also obtain the Wigner functions for the states $|\beta,m\rangle_{\mathrm{e}}$. The mixture part $W_{\mathrm{e},m}^{\mathrm{M}}(\alpha,\alpha^*)$ and the interference part $W_{\mathrm{e},m}^{\mathrm{I}}(\alpha,\alpha^*)$ are given by, respectively

$$W_{\mathrm{e},m}^{\mathrm{M}}(\alpha,\alpha^*) = \frac{2}{\pi}\sum_{n=[(m+1)/2]}^{\infty}\frac{(-)^{2n-m}C_{\mathrm{e},m}^2(2n-m)!|\beta|^{4n}}{[(2n)!]^2\cosh|\beta|^2}L_{2n-m}^{(0)}\left(4|\alpha|^2\right)\exp\left(-2|\alpha|^2\right), \tag{14}$$

$$W_{\mathrm{e},m}^{\mathrm{I}}(\alpha,\alpha^*) = \frac{2}{\pi}\sum_{\substack{n,k=[(m+1)/2]\\ n\neq k}}\frac{(-)^{2n-m}C_{\mathrm{e},m}^2\beta^{2n}\beta^{*2k}}{(2n)!\,(2k)!\cosh|\beta|^2}\exp\left(-2|\alpha|^2\right)\times\Big[(2n-m)!\,(2\alpha)^{2(k-n)}$$
$$\times L_{2n-m}^{2(k-n)}\left(4|\alpha|^2\right)+(2k-m)!\,(-2\alpha^*)^{2(n-k)}L_{2k-m}^{2(n-k)}\left(4|\alpha|^2\right)\Big]. \tag{15}$$

By virtue of the method of numerical computation we find that, for small values of $\beta$ (for example, $\beta = 0.1$), the shape of the Wigner function distribution is the same as those of the Fock states $|0\rangle$ and $|1\rangle$, which have an almost Gaussian shape. In fact, it is clear that the coefficients in the Fock representation of the state $|\beta,m\rangle_{\mathrm{o}}$ are rapidly decreasing for small values of $\beta$, so the dominance of the effect of the vacuum and single-photon state over the effect of the higher excitations. As $\beta$ increases, it means that the vacuum and single-photon state shall begin to lose its higher probability in the whole number state expansion.

In Figs.1 and 2, we plot the Wigner functions $W_{\mathrm{o},m}(\alpha,\alpha^*)$ of the states $|\beta,m\rangle_{\mathrm{o}}$ against $\alpha = \mathrm{Re}\alpha + \mathrm{iIm}\alpha$ when we take different values of $m$ and $\beta$ ($m = 2,3;\beta = 1.8,3,4.5$) for the six three-dimensional figures. From Figs.1(a) and 2(a) it is clearly that, at the central position, there exists a downward peak when $m$ is even, however there exists an upward peak when $m$ is odd. If $\beta$ is taken smaller values, the results are in general valid for the state $|\beta,m\rangle_{\mathrm{o}}$ while for all values of $m$. The feature indicates that the two types of the states are orthogonal. Moreover, we can also see that the negative parts of $W_{\mathrm{o},m}(\alpha,\alpha^*)$ rely on the parameters $m$ and $\beta$. Since the negativity of the Wigner function signifies nonclassical effects of the corresponding state, the nonclassicality is more pronounced when $m$ is even for the same value of $\beta$. But with the increase of $m$ and $\beta$, the negative parts of $W_{\mathrm{o},m}(\alpha,\alpha^*)$ become larger for the state $|\beta,m\rangle_{\mathrm{o}}$. This shows that the nonclassicality is more pronounced. On the other hand, as $\beta$ increases, the Wigner functions $W_{\mathrm{o},m}(\alpha,\alpha^*)$ in the phase space are compressed along the Im$\alpha$ (or $p$) direction at the expense of an increase along the Re$\alpha$ (or $x$). The greater the value of $\beta$ is, the further the peak of the Wigner function is compressed. So $\beta$ is

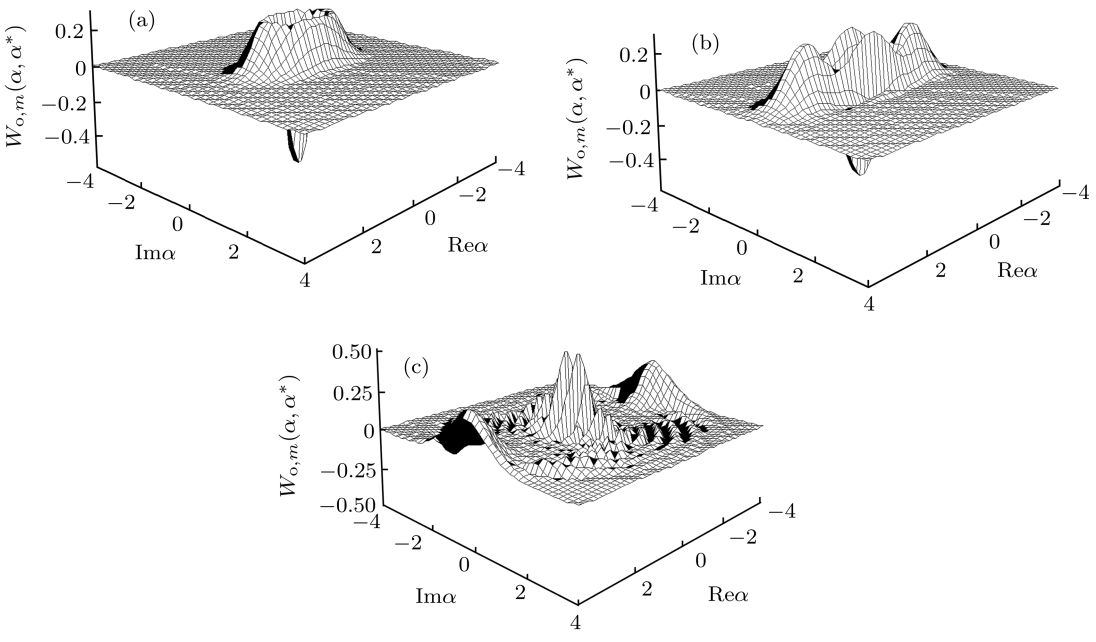



**Fig.1.** The Wigner function of the state $|\beta,m\rangle_{\mathrm{o}}$ with $\alpha$ when (a) $m = 2,\beta = 1.8$; (b) $m = 2,\beta = 3$; (c) $m = 2,\beta = 4.5$.

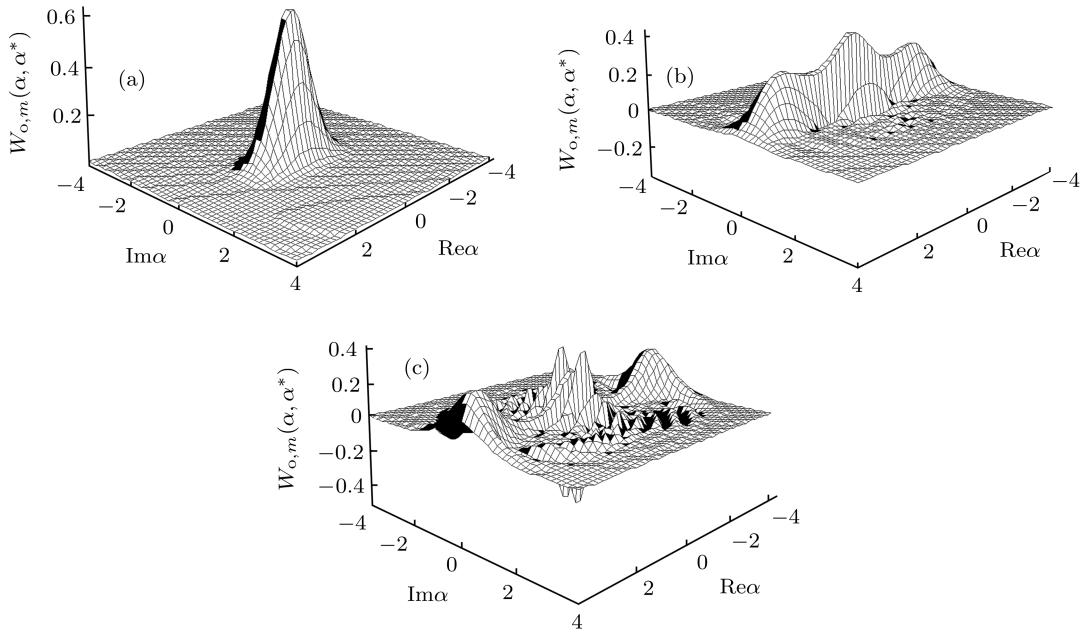



**Fig.2.** The Wigner function of the state $|\beta,m\rangle_{o}$ with $\alpha$ when (a) $m=3,\beta=1.8$; (b) $m=3,\beta=3$; (c) $m=3,\beta=4.5$.

equivalent to the squeezed parameter. Thus the phenomenon of squeezing can be well interpreted from the phase space point of view. When the parameter $\beta$ is enough large, the single peak begin to split several peaks and form the multi-peak structure. And the peak values increase and the peaks are further compressed along the direction $p$ as can be seen in Figs.1 and 2. The multi-peak structure causes by the quantum interference part $W^{I}_{o,m}(\alpha,\alpha^*)$ for the state $|\beta,m\rangle_{o}$. In conclusion, the behaviour of the Wigner functions $W_{o,m}(\alpha,\alpha^*)$ is in a good agreement with the quantum features of the state $|\beta,m\rangle_{o}$.

On the other hand, by virtue of Figs.3 and 4 we can find that, for the state $|\beta,m\rangle_{e}$, there exists an

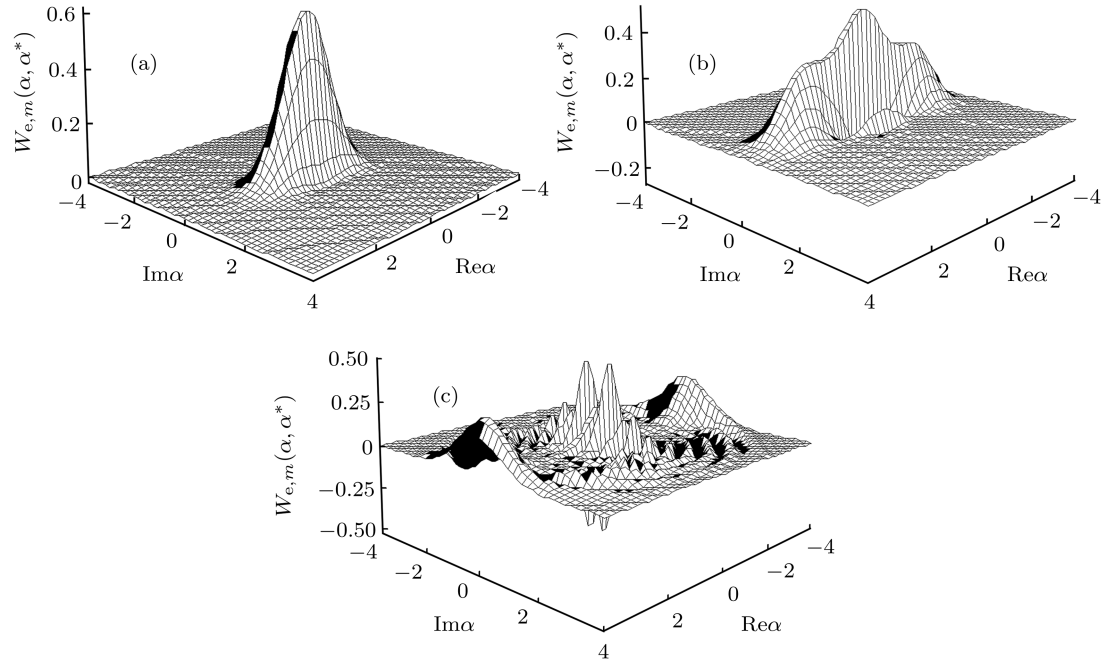



**Fig.3.** The Wigner function of the state $|\beta,m\rangle_{e}$ with $\alpha$ when (a) $m=2,\beta=1.8$; (b) $m=2,\beta=3$; (c) $m=2,\beta=4.5$.

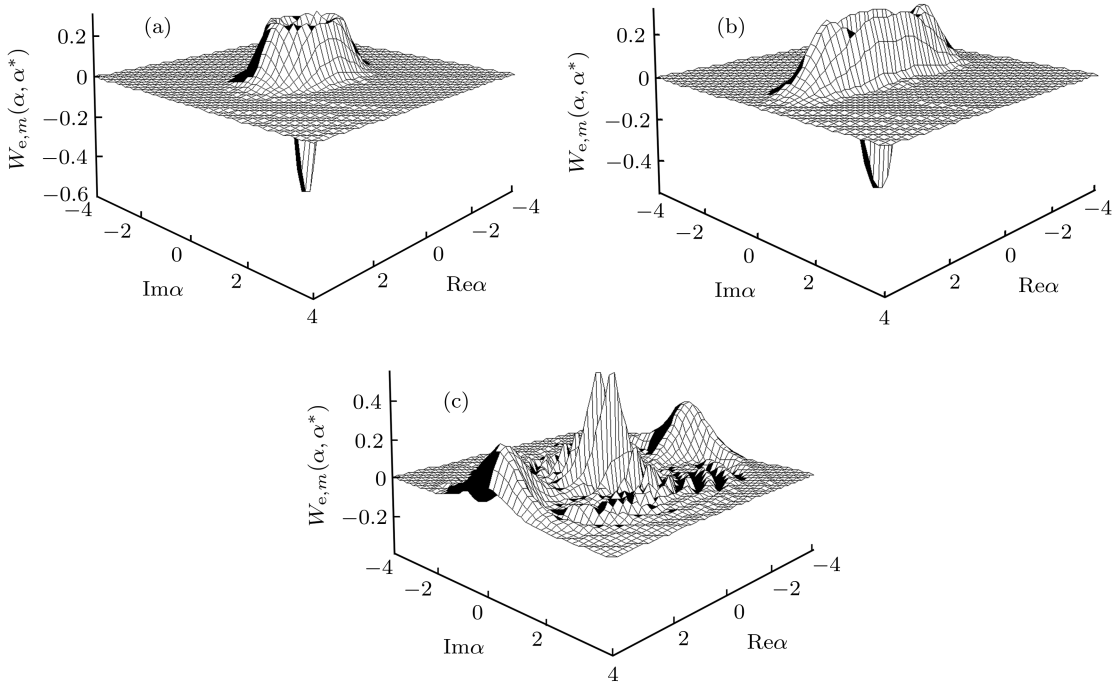



**Fig.4.** The Wigner function of the state $|\beta,m\rangle_{\rm e}$ with $\alpha$ when (a) $m=3,\beta=1.8$; (b) $m=3,\beta=3$; (c) $m=3,\beta=4.5$.

upward peak when $m$ is even, however there exists a downward peak when $m$ is odd. This is just contrary to the features of the states $|\beta,m\rangle_{\rm o}$. But it also shows that the two types of the states are orthogonal. And when $m$ is even (or odd) , the states $|\beta,m\rangle_{\rm e}$ are also orthogonal. Moreover, the nonclassicality is more pronounced when $m$ is odd for the states $|\beta,m\rangle_{\rm e}$. Comparison of Figs.3 and 4 shows that, as $\beta$ increases, the negativity parts increase for $m$ taking even numbers and decreases for $m$ taking odd numbers. Namely, with the increase of $\beta$ the nonclassical effects increase when $m$ is even and decreases when $m$ is odd. This is completely different from the variations of the Wigner functions for the states $|\beta,m\rangle_{\rm o}$ with the parameter $\beta$. Otherwise, the other variations of the Wigner functions of the states $|\beta,m\rangle_{\rm e}$ with respect to $m$ and $\beta$ are the same as those of the states $|\beta,m\rangle_{\rm o}$.

In the following section we shall obtain the marginal distributions of the PDEOCSs. For given matrix element $\rho$, the marginal distributions of its Wigner function are[1]

$$P(q)=\frac{1}{2\pi}\int {\rm d}pW(p,q)=\langle q|\,\rho\,|q\rangle\,, \tag{16}$$

$$P(p)=\frac{1}{2\pi}\int {\rm d}qW(p,q)=\langle p|\,\rho\,|p\rangle\,, \tag{17}$$

where $|q\rangle$ and $|p\rangle$ are the standard eigenvectors of the coordinate operator $Q$ and the momentum operator $P$, respectively. Eqs.(16) and (17) are the marginal distributions of its Wigner function in the $q$ and $p$ directions, respectively.

In Eqs.(12) and (13), if setting $\alpha=(q+{\rm i}p)/\sqrt{2}$, then using Eq.(16) we can obtain the marginal distribution of the Wigner function of the state $|\beta,m\rangle_{\rm o}$ in the $q$ direction,

$$P_{{\rm o},m}(q)=\frac{1}{2\pi}\int {\rm d}pW_{{\rm o},m}(p,q)=|\,\langle q|\,\beta,m\rangle_{\rm o}\,|^2. \tag{18}$$

Since the amplitude of $P_{{\rm o},m}(q)$

$$\langle q|\,\beta,m\rangle_{\rm o}=\frac{C_{{\rm o},m}}{\sqrt{\sinh|\beta|^2}}\sum_{n=[m/2]}^{\infty}\frac{\beta^{2n+1}\sqrt{(2n+1-m)!}}{(2n+1)!}\langle q|\,2n+1-m\rangle$$

$$
\begin{aligned}
&= \frac{C_{\mathrm{o},m}}{\sqrt{\sinh|\beta|^2}} \sum_{n=[m/2]}^{\infty} \frac{\beta^{2n+1}}{(2n+1)!} \int \mathrm{d}q' \langle q| a^{+(2n+1-m)} |q'\rangle \langle q'| 0\rangle \\
&= \frac{C_{\mathrm{o},m}}{\sqrt{\sinh|\beta|^2}} \sum_{n=[m/2]}^{\infty} \frac{\beta^{2n+1}}{(2n+1)!\sqrt{2^{2n+1-m}}} \int \mathrm{d}q' \left(q - \frac{\mathrm{d}}{\mathrm{d}q}\right)^{2n+1-m} \delta(q-q') \langle q'| 0\rangle \\
&= \frac{C_{\mathrm{o},m}}{\sqrt{\sinh|\beta|^2}} \sum_{n=[m/2]}^{\infty} \frac{\beta^{2n+1}}{(2n+1)!\sqrt{2^{2n+1-m}\sqrt{\pi}}} \left(q - \frac{\mathrm{d}}{\mathrm{d}q}\right)^{2n+1-m} \mathrm{e}^{-q^2/2} \\
&= \frac{C_{\mathrm{o},m}}{\sqrt{\sinh|\beta|^2}} \sum_{n=[m/2]}^{\infty} \frac{\beta^{2n+1}}{(2n+1)!\sqrt{2^{2n+1-m}\sqrt{\pi}}} \mathrm{e}^{-q^2/2} H_{2n+1-m}(q), \qquad (19)
\end{aligned}
$$

where we have used the formula for Hermite polynomials[9]

$$
H_n(q) = \mathrm{e}^{q^2/2} \left(q - \frac{\mathrm{d}}{\mathrm{d}q}\right)^n \mathrm{e}^{-q^2/2}. \qquad (20)
$$

Thus

$$
\begin{aligned}
P_{\mathrm{o},m}(q) &= \frac{1}{2\pi} \int \mathrm{d}p W_{\mathrm{o},m}(p,q) = |\langle q|\, \beta, m\rangle_{\mathrm{o}}|^2 \\
&= \frac{C_{\mathrm{o},m}^2 \mathrm{e}^{-q^2}}{\pi^{1/2}\sinh|\beta|^2} \left| \sum_{n=[m/2]}^{\infty} \frac{\beta^{2n+1}}{(2n+1)!2^{(2n+1-m)/2}} H_{2n+1-m}(q) \right|^2 . \qquad (21)
\end{aligned}
$$

Using Eqs.(1) and (16), we obtain another marginal distribution of the Wigner function in the $p$ direction,

$$
\begin{aligned}
P_{\mathrm{o},m}(p) &= \frac{1}{2\pi} \int \mathrm{d}q W_{\mathrm{o},m}(p,q) = |\langle p|\, \beta, m\rangle_{\mathrm{o}}|^2 \\
&= \frac{C_{\mathrm{o},m}^2 \mathrm{e}^{-p^2}}{\pi^{1/2}\sinh|\beta|^2} \left| \sum_{n=[m/2]}^{\infty} \frac{(-\mathrm{i})^{2n+1-m}\beta^{2n+1}}{(2n+1)!2^{(2n+1-m)/2}} H_{2n+1-m}(p) \right|^2 . \qquad (22)
\end{aligned}
$$

From Eqs.(21) and (22), we find that $P_{\mathrm{o},m}(q)$ (or $P_{\mathrm{o},m}(p)$) is the probability of finding the particles with given value $q$ of the position (or the value $p$ of the momentum) in the state $|\beta, m\rangle_{\mathrm{o}}$. Similarly, using Eqs.(2) and (14)–(17) we have

$$
\begin{aligned}
P_{\mathrm{e},m}(q) &= \frac{1}{2\pi} \int \mathrm{d}p W_{\mathrm{e},m}(p,q) = |\langle q|\, \beta, m\rangle_{\mathrm{e}}|^2 \\
&= \frac{C_{\mathrm{e},m}^2 \mathrm{e}^{-q^2}}{\pi^{1/2}\cosh|\beta|^2} \left| \sum_{n=[(m+1)/2]}^{\infty} \frac{\beta^{2n}}{(2n)!2^{(2n-m)/2}} H_{2n-m}(q) \right|^2 , \qquad (23) \\
P_{\mathrm{e},m}(p) &= \frac{1}{2\pi} \int \mathrm{d}q W_{\mathrm{e},m}(p,q) = |\langle p|\, \beta, m\rangle_{\mathrm{e}}|^2 \\
&= \frac{C_{\mathrm{e},m}^2 \mathrm{e}^{-p^2}}{\pi^{1/2}\cosh|\beta|^2} \left| \sum_{n=[(m+1)/2]}^{\infty} \frac{(-\mathrm{i})^{2n-m}\beta^{2n}}{(2n)!2^{(2n-m)/2}} H_{2n-m}(p) \right|^2 , \qquad (24)
\end{aligned}
$$

which are the marginal distributions of the Wigner functions of the states $|\beta, m\rangle_{\mathrm{e}}$. Similarly, $P_{\mathrm{e},m}(q)$ (or $P_{\mathrm{e},m}(p)$) is the probability of finding the particles with given value $q$ of the position (or the value $p$ of the momentum) in the state $|\beta, m\rangle_{\mathrm{e}}$. Therefore, these marginal distributions give the probability of finding the particles with given position (or the momentum) in the state $|\beta, m\rangle_{\mathrm{o,e}}$. This is also the physical meaning of the Wigner functions of the PDEOCSs.

## 3. The tomograms of the PDEOCSs

The use of tomograms in quantum mechanics and quantum optics provides the possibility of describing a quantum state with a positive probability distribution. A direct description of quantum states by means of quantum tomograms for the system observables is interesting from both the theoretical and experimental points of view. Therefore, in recent years, tomogram approach has brought much interest of physicists. In this section we want to derive the tomogram functions of $|\beta,m\rangle_{\rm o,e}$. The tomogram function of the state $|\beta,m\rangle_{\rm o,e}$ is defined as

$$\begin{aligned} &T\left(q,f,g\right)\\ &=\iint \mathrm{d}p'\mathrm{d}q'\delta\left(q-fq'-gp'\right)W\left(q',p'\right), \end{aligned} \tag{25}$$

where $W\left(q',p'\right)$ given by Eqs.(12)–(15) are the Wigner functions for the states $|\beta,m\rangle_{\rm o,e}$. However, it will usually be very difficult to evaluate the integration if we directly substitute Eqs.(12)–(15) into Eq.(25), fortunately we use the following relation between the Wigner operator and the projector of intermediate coordinate-momentum state[10]

$$\begin{aligned} &|q\rangle_{f,g\,f,g}\langle q|\\ &=\iint \mathrm{d}p'\mathrm{d}q'\delta\left(q-fq'-gp'\right)\Delta\left(q',p'\right), \end{aligned} \tag{26}$$

where the state $|q\rangle_{f,g}$ is expressed as

$$\begin{aligned} |q\rangle_{f,g} =\ & \frac{1}{\left[\pi\left(f^2+g^2\right)\right]^{1/4}}\exp\left[-\frac{q^2}{2\left(f^2+g^2\right)}\right.\\ &\left.+\sqrt{2}a^{+}\frac{q}{f-\mathrm{i}g}-\frac{f+\mathrm{i}g}{2\left(f-\mathrm{i}g\right)}a^{+2}\right]|0\rangle, \end{aligned} \tag{27}$$

which is the eigenvector of the operator $(fQ+gP)$ corresponding to the eigenvalue $q$, $f$ and $g$ are two real parameters. Thus the tomograms of the states $|\beta,m\rangle_{\rm o,e}$ are

$$\begin{aligned} &T\left(q,f,g\right)\\ &=\iint \mathrm{d}p'\mathrm{d}q'\delta\left(q-fq'-gp'\right){}_{\rm o,e}\langle\beta,m|\\ &\quad\times\langle\beta,m|\,\Delta\left(q',p'\right)|\beta,m\rangle_{\rm o,e}\\ &=|{}_{f,g}\langle q\,|\beta,m\rangle_{\rm o,e}|^2, \end{aligned} \tag{28}$$

which show that for tomographic approach there exists the intermediate coordinate-momentum state $|q\rangle_{f,g}$, and the Radon transforms of the Wigner operator are just the pure-state density matrices $|q\rangle_{f,g\,f,g}\langle q|$. As a result, the tomogram functions for quantum states can be considered as the module-square of the states' wave function in this representation. This is a new way to derive the tomogram functions for quantum states. Using the completeness relations of the coherent states and Eqs.(1) and (27), we can obtain the tomogram amplitude of the state $|\beta,m\rangle_{\rm o,e}$

$$\begin{aligned} &{}_{f,g}\langle q|\,\beta,m\rangle_{\rm o}\\ &={}_{f,g}\langle q|\int\frac{\mathrm{d}^2z}{\pi}|z\rangle\langle z|\,\beta,m\rangle_{\rm o}\\ &=\frac{\left[\pi\left(f^2+g^2\right)\right]^{-1/4}C_{\rm o,m}}{\sqrt{\sinh|\beta|^2}}\sum_{n=[m/2]}^{\infty}\frac{\beta^{2n+1}}{(2n+1)!}\int\frac{\mathrm{d}^2z}{\pi}z^{*2n+1-m}\\ &\quad\times\exp\left[-|z|^2-\frac{q^2}{2\left(f^2+g^2\right)}+\frac{\sqrt{2}q}{f+\mathrm{i}g}z-\frac{f-\mathrm{i}g}{2\left(f+\mathrm{i}g\right)}z^2\right]\\ &=\frac{\left[\pi\left(f^2+g^2\right)\right]^{-1/4}C_{\rm o,m}}{\sqrt{\sinh|\beta|^2}}\sum_{n=[m/2]}^{\infty}\frac{\beta^{2n+1}}{(2n+1)!}\frac{\partial^{2n+1-m}}{\partial\lambda^{2n+1-m}}\int\frac{\mathrm{d}^2z}{\pi}\\ &\quad\times\exp\left[-|z|^2-\frac{q^2}{2\left(f^2+g^2\right)}+\frac{\sqrt{2}q}{f+\mathrm{i}g}z-\frac{f-\mathrm{i}g}{2\left(f+\mathrm{i}g\right)}z^2+\lambda z^*\right]\Bigg|_{\lambda=0} \end{aligned}$$

$$= \frac{\left[\pi\left(f^2+g^2\right)\right]^{-1/4} C_{\mathrm{o},m}}{\sqrt{\sinh|\beta|^2}} \sum_{n=[m/2]}^{\infty} \frac{\beta^{2n+1}}{(2n+1)!} \frac{\partial^{2n+1-m}}{\partial\lambda^{2n+1-m}} \exp\left[-\frac{q^2}{2\left(f^2+g^2\right)} + \frac{\sqrt{2}q}{f+\mathrm{i}g}\lambda - \frac{f-\mathrm{i}g}{2\left(f+\mathrm{i}g\right)}\lambda^2\right]\Bigg|_{\lambda=0}$$

$$= \frac{\left[\pi\left(f^2+g^2\right)\right]^{-1/4} C_{\mathrm{o},m}}{\sqrt{\sinh|\beta|^2}} \sum_{n=[m/2]}^{\infty} \left(\sqrt{\frac{f-\mathrm{i}g}{2\left(f+ig\right)}}\right)^{2n+1-m} \frac{\beta^{2n+1}}{(2n+1)!} H_{2n+1-m}\left(q/\sqrt{f^2+g^2}\right) \times \exp\left[-\frac{q^2}{2\left(f^2+g^2\right)}\right], \tag{29}$$

where we have used the following integral formula[10]

$$\int \frac{\mathrm{d}^2 z}{\pi} \exp\left(\zeta|z|^2 + \xi z + \eta z^* + f z^2 + g z^{*2}\right) = \frac{1}{\sqrt{\zeta^2-4fg}} \exp\left[\frac{-\zeta\xi\eta + \xi^2 g + \eta^2 f}{\zeta^2-4fg}\right], \tag{30}$$

with

$$\mathrm{Re}\left(\zeta \pm f \pm g\right) < 0, \quad \mathrm{Re}\left(\frac{\zeta^2-4fg}{\zeta \pm f \pm g}\right) < 0, \tag{31}$$

and the generating function of $H_n(x)$[9]

$$H_n(q) = \frac{\mathrm{d}^n}{\mathrm{d}q^n}\, \mathrm{e}^{2qt-q^2}\Big|_{t=0}. \tag{32}$$

Then using Eq.(12) we get the tomogram of $|\beta, m\rangle_{\mathrm{o}}$

$$T_{\mathrm{o},m}(q,f,g) = \frac{C_{\mathrm{o},m}^2}{\left[\pi\left(f^2+g^2\right)\right]^{1/2}\sinh|\beta|^2} \left|\sum_{n=[m/2]}^{\infty} \left(\sqrt{\frac{f-\mathrm{i}g}{2\left(f+\mathrm{i}g\right)}}\right)^{2n+1-m} \frac{\beta^{2n+1} H_{2n+1-m}\left(q/\sqrt{f^2+g^2}\right)}{(2n+1)!}\right|^2 \times \exp\left[-q^2/\left(f^2+g^2\right)\right]. \tag{33}$$

In the same way, we can derive the tomogram functions of the states $|\beta, m\rangle_{\mathrm{e}}$,

$$T_{\mathrm{e},m}(q,f,g) = \iint \frac{\mathrm{d}p'\mathrm{d}q'}{2\pi h} \delta\left(q - fq' - gp'\right)_{\mathrm{e}} \langle\beta, m|\, \Delta\left(p', q'\right)|\beta, m\rangle_{\mathrm{e}}$$

$$= \frac{C_{\mathrm{e},m}^2}{\left[\pi\left(f^2+g^2\right)\right]^{1/2}\cosh|\beta|^2} \left|\sum_{n=[(m+1)/2]}^{\infty} \left(\sqrt{\frac{f-\mathrm{i}g}{2\left(f+\mathrm{i}g\right)}}\right)^{2n-m} \frac{\beta^{2n} H_{2n-m}\left(q/\sqrt{f^2+g^2}\right)}{(2n)!}\right|^2 \times \exp\left[-q^2/\left(f^2+g^2\right)\right]. \tag{34}$$

It is a good function of describing a quantum state with a positive probability distribution function like Husimi Q-function. Therefore, in experiment if only we can measure $|_{f,g}\langle q\,|\beta, m\rangle_{\mathrm{o,e}}|^2$ in the intermediate coordinate-momentum representation, the tomogram functions for the states $|\beta, m\rangle_{\mathrm{o,e}}$ shall be obtained and further the states $|\beta, m\rangle_{\mathrm{o,e}}$ shall be measured.

In summary, we have used the coherent state representation of Wigner operator and the intermediate coordinate-momentum representation to derive the Wigner functions and tomograms of the PDEOCSs. This is a new way and it is suitable to obtain the Wigner functions and tomograms of other quantum states. Moreover, the results in this paper may be useful for experiments as reference.

## References


[1] Wigner E P 1932 *Phys. Rev.* **40** 749
O'Connell R F and Wigner E P 1981 *Phys. Lett.* A **83** 145
Vogel K and Risken H 1989 *Phys. Rev.* A **40** 2847

[2] Dariano G M, Macchiavello C and Paris M G A 1994 *Phys. Rev.* A **50** 4298
Agarwal G S and Wolf E 1972 *Phys. Rev.* D **2** 2161

[3] Vracoe B T H, Brattke S, Weidinger M and Walther H 2000 *Nature* **403** 743
Brattke S, Vracoe B T H and Walther H 2001 *Phys. Rev. Lett.* **86** 3534
[4] Zhang Z M 2003 *Chin. Phys. Lett.* **20** 227
Zhang Z M 2004 *Chin. Phys. Lett.* **21** 5
[5] Yang Q Y, Wei L F and Ding L E 2003 *Acta Phys. Sin.* **52** 1390 (in Chinese)
[6] Yang Q Y, Sun J W, Wei L F and Ding L E 2005 *Acta Phys. Sin.* **54** 2704 (in Chinese)
[7] Fan H Y and Ruan T N 1984 *Commun. Theor. Phys.* **3** 345
Fan H Y and Ruan T N 1985 *Commun. Theor. Phys.* **4** 181
Fan H Y 1987 *Phys. Lett.* A **124** 303
[8] Klauder J R 1963 *J. Math. Phys.* **4** 1005
Glauber R J 1963 *Phys. Rev.* **130** 2529
[9] Wang Z X and Guo D R 1965 *General Theory of Special Functions* (Beijing: Science Press) p359 (in Chinese)
[10] Fan H Y 2001 *Entangled State Representations in Quantum Mechanics and Their Application* (Shanghai: Shanghai Jiao Tong University Press) p7 (in Chinese)