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Fidelity of quantum information for V-type three-level atom interacting with a number state light field in Kerr medium

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In this paper the evolution characteristics of the fidelity of quantum information for the V-type three-level atom interacting with number state light field in Kerr medium are investigated. It shows that the periodicity of the evolutions of fidelity of quantum information is influenced by the Kerr coefficient, the photon number of the initial field and intensity of light. The evolutions of the fidelity of quantum information are modulated by the initial number state field. The Rabi oscillation frequency and the modulation frequency of fidelity for the field and the system vary with the value of the Kerr coefficient. The evolutions of fidelity of quantum information obviously show the quantum collapse and revival behaviours in the system of atom interacting with light field.

Keywords: quantum optics, fidelity, number state, Kerr medium
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1. Introduction

Fidelity, the degree of closeness to the original state of information in the transmission, is of fundamental importance in information science as well as in the quantum optics. It has been adopted broadly as an important physical parameter in quantum communication and quantum calculations.[1−4] Fidelity can measure the performance of quantum teleportation and describe the similarity between the input state and the teleported state. In order to obtain better output results, The effort of increasing the fidelity of quantum teleportation in continuous-variable systems has greatly been made.[5−7]

Manipulation and extraction of quantum information have attracted much attention[1−19] for more than ten years. One of the crucial problems in the rapidly developing field of quantum communication is how to send a quantum state from one place to another. The carrier of the encoded information in quantum communication, quantum computer, and quantum cryptography is quantum state, so its fidelity must be taken into account in the transmission. The quantum information is not wholly reflected by its entropy for the quantum system. However, the density operator interval, equivalent to the fidelity in describing the connection and the entanglement of information, provides more related information about quantum state.[8]

This paper studies the evolution characteristics of fidelity of quantum information for V-type three-level atom interacting with number state light field. It is shown that the periodicity of the evolutions of fidelity of quantum information is remarkably influenced by Kerr coefficient, the photon number of the initial field, and intensity of light. The evolutions of the fidelity of quantum information are modulated by the initial number state field. The evolutions of fidelity of quantum information obviously show the quantum collapse and revival behaviours in the system of atom interacting with light field.

2. Model and fidelity of quantum information

Suppose that a V-type three-level atom has an excited state $|1\rangle$, an intermediate state $|2\rangle$ and a ground state $|3\rangle$ interacting non-resonantly with the number field in Kerr medium. In both the dipole and the rotating-wave approximations, the Hamiltonian of

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system is given by

$$\hat{H}_{\mathrm{I}} = \mu\hat{a}^{+}\hat{a}\hat{a}^{+}\hat{a} + \lambda_1(\hat{\sigma}_{12}^{-}\hat{a}^{+} + \hat{\sigma}_{21}^{+}\hat{a}) + \lambda_2(\hat{\sigma}_{23}^{+}\hat{a} + \hat{\sigma}_{32}^{-}\hat{a}^{+}), \quad (1)$$

where μ is the Kerr coefficient, relating to the three-order nonlinear polarization of medium. $\hat{a}^{+}$ and $\hat{a}$ are respectively the standard creation and annihilation operators of monochromatic photons with frequency ω. λ_i $(i = 1, 2)$ is the atom–field coupling constant which may be treated as real and positive. $\hat{\sigma}_{ij}^{+}$ and $\hat{\sigma}_{ij}^{-}$ $(i, j = 1, 2, 3, i \neq j)$ are the ladder operators. $\eta = 1$ and energy of the excited atomic state to be equal to zero are set in the equation

If the atom is initially in the excited state $|1\rangle$ and the light field is in an arbitrary superposition of the number state at the time $t = 0$, the state vector for the atom-field system can be written as[14,16]

$$|\Psi(0)\rangle = \sum_m q_m |1, m\rangle, \quad (2)$$

where q_m is the number state expansion coefficient of the initial field.

At a time $t > 0$, the state vector in the interaction picture is found from Eq.(1) to be

$$|\Psi_{\mathrm{s}}(t)\rangle = \sum_{m=0}^{\infty} q_m[A_m(t)|1, m\rangle + B_{m+1}(t)|2, m+1\rangle + C_m(t)|3, m\rangle], \quad (3)$$

where

$$A_m(t) = \frac{\mathrm{e}^{-\frac{1}{2}\mathrm{i}t(k_2 - \mathrm{i}k_1)}}{-4k_1^2}\left\{8\left(1 + \mathrm{e}^{tk_1} + 2\mathrm{e}^{\frac{1}{2}t(\mathrm{i}k_3 + k_1)}\right) + k_4 + k_3\left[\left(1 + \mathrm{e}^{tk_1} + 2\mathrm{e}^{\frac{1}{2}t(\mathrm{i}k_3 + k_1)}\right)k_3 - \mathrm{i}(-1 + \mathrm{e}^{tk_1})k_1\right]\right\}, \quad (4a)$$

$$B_{m+1}(t) = \frac{-\left(\mathrm{i}\mathrm{e}^{-\frac{1}{2}t(\mathrm{i}k_2 + k_1)}\left(-1 + \mathrm{e}^{tk_1}\right)k_5\right)}{k_1}, \quad (4b)$$

$$C_m(t) = \frac{\mathrm{e}^{-\frac{1}{2}\mathrm{i}t(k_2 - \mathrm{i}k_1)}}{-4k_1^2}\left\{8\left(1 + \mathrm{e}^{tk_1} - 2\mathrm{e}^{\frac{1}{2}t(\mathrm{i}k_3 + k_1)}\right) + k_4 + k_3\left[\left(1 + \mathrm{e}^{tk_1} - 2\mathrm{e}^{\frac{1}{2}t(\mathrm{i}k_3 + k_1)}\right)k_3 - \mathrm{i}(-1 + \mathrm{e}^{tk_1})k_1\right]\right\}, \quad (4c)$$

$$k_1 = \sqrt{-8(m+1)\lambda^2 - (\mu + 2m\mu)^2},\ k_2 = \mu + 2m\mu + 2m^2\mu,\ k_3 = (2m+1)\mu,\ k_4 = (m+1)\lambda^2,\ k_5 = \sqrt{m+1}\lambda. \quad (4d)$$

The resulting density operators of the whole system $\rho_{\mathrm{s}}(t)$, the atom $\rho_{\mathrm{a}}(t)$, and the field $\rho_{\mathrm{f}}(t)$ are given by[9,10]

$$\rho_{\mathrm{s}}(t) = |\Psi_{\mathrm{s}}(t)\rangle\langle\Psi_{\mathrm{s}}(t)|, \quad (5)$$

$$\rho_{\mathrm{f}}(t) = \mathrm{Tr}_{\mathrm{a}}\rho_{\mathrm{s}}(t), \quad (6)$$

$$\rho_{\mathrm{a}}(t) = \mathrm{Tr}_{\mathrm{f}}\rho_{\mathrm{s}}(t), \quad (7)$$

where $\mathrm{Tr}_{\mathrm{a}}(\mathrm{Tr}_{\mathrm{f}})$ means the trace over atomic (field) states.

The fidelity is defined by Schumacher[1,11] as

$$F(\rho_1, \rho_2) = \left[\mathrm{Tr}(\sqrt{\rho_1}\rho_2\sqrt{\rho_1})^{1/2}\right]^2, \quad (8)$$

where ρ_1 and ρ_2 are the density operators respectively corresponding to initial and final states. This is a more satisfactory definition in general. It ranges obviously between 0 and 1. If the initial state and the final state are orthogonal each other, then $F(\rho_1, \rho_2) = 0$, indicating that the quantum information (i.e. quantum state) is totally distorted in the transmission. If the initial state and the final state are coincided, then $F(\rho_1, \rho_2) = 1$, indicating that two density matrices equal to each other. In this case, an ideal transmitting process takes place. If $0 < F(\rho_1, \rho_2) < 1$, indicating that certain distortion exists in the transmission process of information.

Based on Eqs.(3)–(7) as well as the initial condition of system, the F_{a}, F_{f} and F_{s} can be respectively expressed as

$$F_{\mathrm{a}}(t) = \left|\sum_{m=0}^{\infty} q_m A_m(t)\right|^2 = \sum_{m=0}^{\infty} P_m |A_m(t)|^2, \quad (9)$$

$$F_{\mathrm{f}}(t) = \left|\sum_{m=0}^{\infty} q_m^2 A_m^*(t)\right|^2 + \left|\sum_{m=1}^{\infty} q_{m+1}^2 B_{m+1}^*(t)\right|^2 + \left|\sum_{m=0}^{\infty} q_m^2 C_m^*(t)\right|^2, \quad (10)$$

$$F_{\rm s}(t) = \left|\sum_{m=0}^{\infty} q_m^2 A_m^*(t)\right|^2 = \left|\sum_{m=0}^{\infty} P_m A_m^*(t)\right|^2 . \quad (11)$$

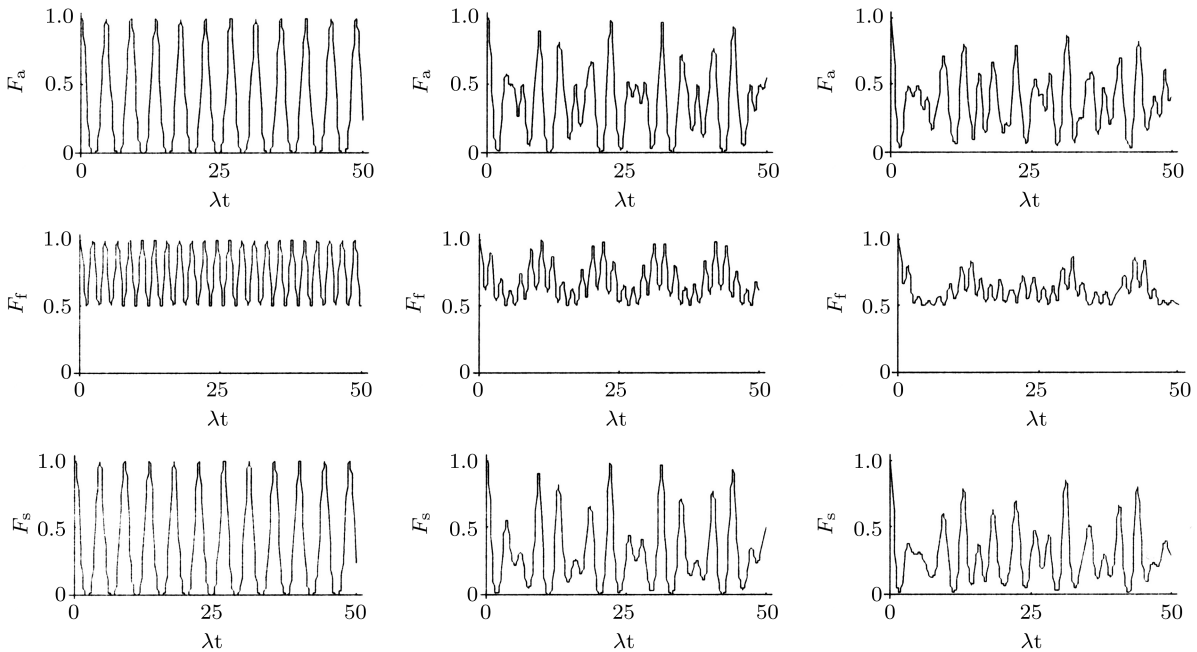
It is obvious that the fidelity of quantum state for the atom, the field, and the system is related to the coupling constant λ of atom-field, Kerr coefficient μ, the photon number of the initial field, and the initial field intensity. Since it is a kinetic system, the fidelity also depends on the time t. Substituting Eq.(4) into Eqs.(9)–(11), one can get the resulting expressions of fidelity for the atoms, the field, and the system. If $t = 0$, the fidelity of quantum state for the atom, the field and the system is certainly equal to 1. In the following, the effects of Kerr coefficient, the photon number of the initial field, and intensity of light on the fidelity are discussed by using numerical results.

3. Numerical results and discussions

3.1. Initial number state light field

Based on Eqs.(11)–(13), we find that the fidelity of a general system is a weighted fidelity average of the number states. The weight factor is the distribution function of photon number, i.e. $P_m = |q_m|^2$.[4,14,16] The numerical fidelities $F_{\rm a}$, $F_{\rm f}$, and $F_{\rm s}$ for the final state have been calculated by using different sets of the initial state. The evolutions of the fidelities are plotted as functions of scaled time λt following.

Figure 1 presents $F_{\rm a}$, $F_{\rm f}$ and $F_{\rm s}$ for three different values of the photon number m of a number state field, where the Kerr coefficient is fixed to be zero. It is obviously shown from the figure that the evolutions of fidelity of quantum information are influenced by the photon number of the initial field. If $m = 0$, the evolutions of the fidelities exhibit equi-amplitude oscillations with a perfect periodicity. The evolution of the fidelity for the atom is similar to that for the system with the amplitudes equal to 1, while the oscillation period of them is roughly equal to 4 (i.e. $\lambda t = 4$). The evolution of the fidelity for the field has a higher fidelity with the values of field fidelity larger than 0.5. The amplitude of the fidelity for the field is equal to 0.5, and its oscillation period is roughly equal to 2 (i.e. $\lambda t = 2$). With the increase of m, the evolution of the fidelities shows the irregular oscillations, and amplitude of them shows more or less decrease, implying that the quantum state fidelity is modulated by the initial field of number state. However, the nature that the evolutions of fidelity for the fields are better than those for the atom and the system is conserved well.



(a) $m = 0$ (b) $m = 2$ (c) $m = 5$

Fig.1. The evolutions of fidelity for three-level atom interacting with different number state light fields for $\mu = 0$.

Figure 2 plots the F_a, F_f and F_s for the different values of Kerr coefficient with $m = 1$. It can be seen from the figure that the time evolutions of F_a, F_f and F_s are more complicated. If Kerr medium is contained in a cavity, the fidelities of quantum state of the system, the atom, and the field decrease with the increase of μ on the whole, while the evolutions of the fidelity show a more irregular oscillation. If $\mu = 1$, the evolutions of fidelity exhibit oscillations with equi-amplitude and perfect periodicity with the oscillation period roughly equal to 6 (i.e. $\lambda t = 6$) and the amplitude equal to 1. With increasing Kerr coefficient μ, the evolution of the fidelity for the atom shows more and more irregular oscillations, while the evolutions of fidelities for the field and the system exhibit the oscillations with more and more high-frequency. Since F_s is the product of F_a and F_f, the fidelity of quantum state for the system consequently should be the superpositions of the fidelities of quantum state for the light field and the atom, exhibiting interference of the complex coefficients of system. This result is different from that of the Refs.[4] and [9]. In addition, Fig.2 shows that the Rabi oscillation frequency and the modulation frequency of fidelity for the field and the system vary with the value of the Kerr coefficient.

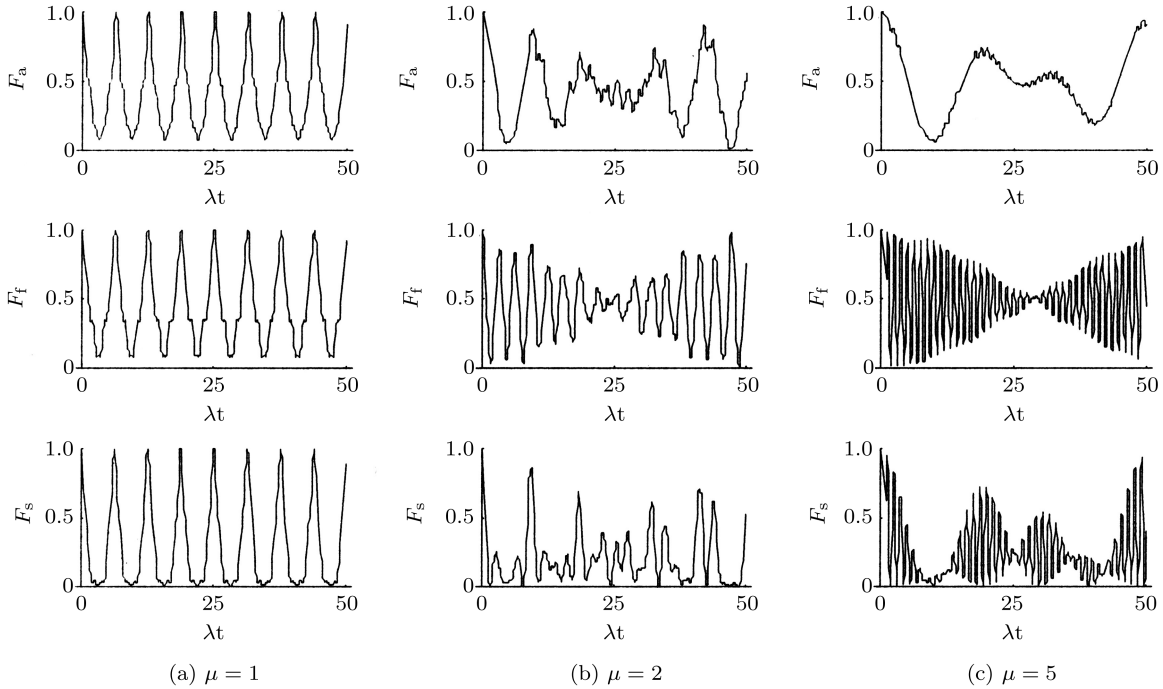


Fig.2. The evolutions of fidelities for three-level atom interacting with the number state light field for $m = 1$.

3.2. Initial coherent state light field

The evolutions of the fidelity for the atom, the field and the system with different initial field intensity $\bar{n}$ and the same $\mu = 0$ are plotted in Fig.3. The coherent state can be described as superposition of photon number states. The Rabi oscillation with a different frequency is produced by the different photon number states interacted with atom. The high-frequency oscillation is made up of the superposition on the time evolution of atomic population. In general, the relationship between Rabi oscillation frequency and the photon number m is not linear. The Rabi oscillation is modulated by the photon number and results in the high-frequency oscillation of F_a on the fidelity of quantum information for the atom, which is different from the evolutions of the fidelities with the number state light fields. If $\bar{n} = 0$, the evolutions of fidelity exhibit oscillations with equi-amplitude and perfect periodicity. With increasing the intensity of initial light field, the time evolutions of fidelity of quantum information obviously show the quantum collapse and revival in the system, and the fidelity decreases gradually. It means that the distortion increases gradually. The probability of finding m photons in the coherent state is given by a Poisson distribution. With the increase

of light intensity, the width of the Poisson distribution and the oscillation frequency of F_a rises, the interaction between the light field and the atom increases, and the fidelity of the system decreases. If $\bar{n} = 65$, the values of fidelity all close to zero with little oscillations, implying that the strong initial coherent light field greatly weakens the fidelities of quantum information.

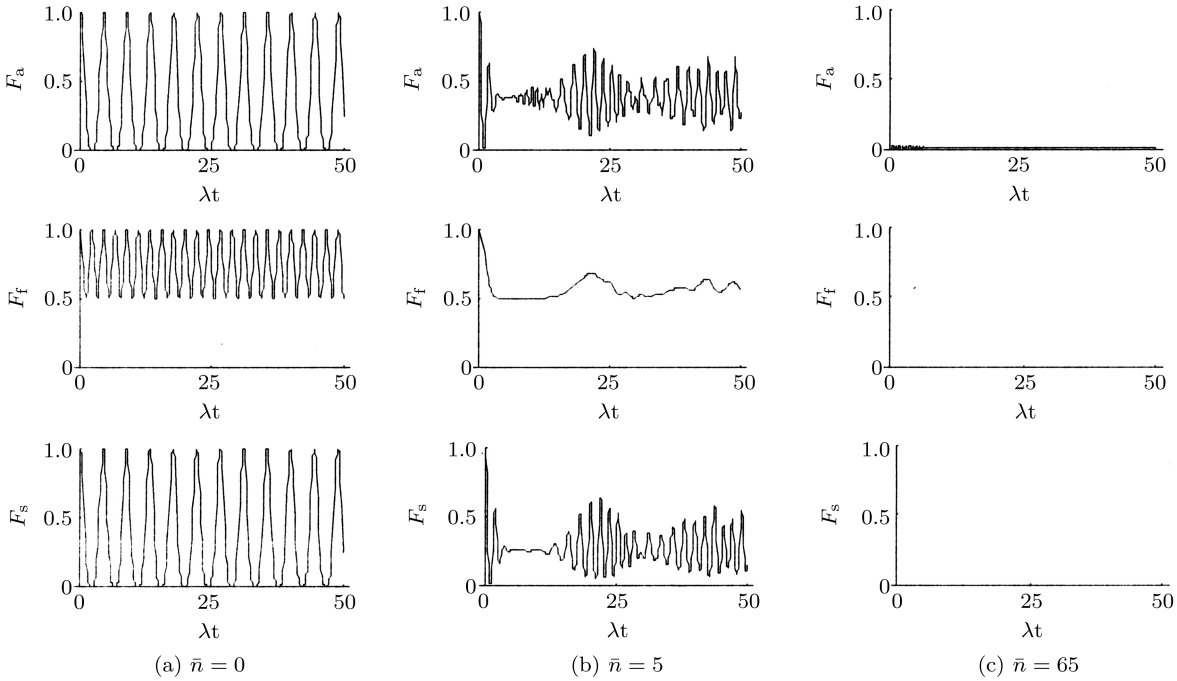


Fig.3. The evolutions of fidelity for three-level atom interacting with the coherent state light field for $\mu = 0$.

The F_a, F_f and F_s with the different values of Kerr coefficient and the same $\bar{n} = 2$ are plotted in Fig.4. It is obviously shown from Fig.4 that the periodicity of the evolutions of fidelities of quantum information in the weak initial light field is influenced by Kerr medium. With increasing the Kerr coefficient,

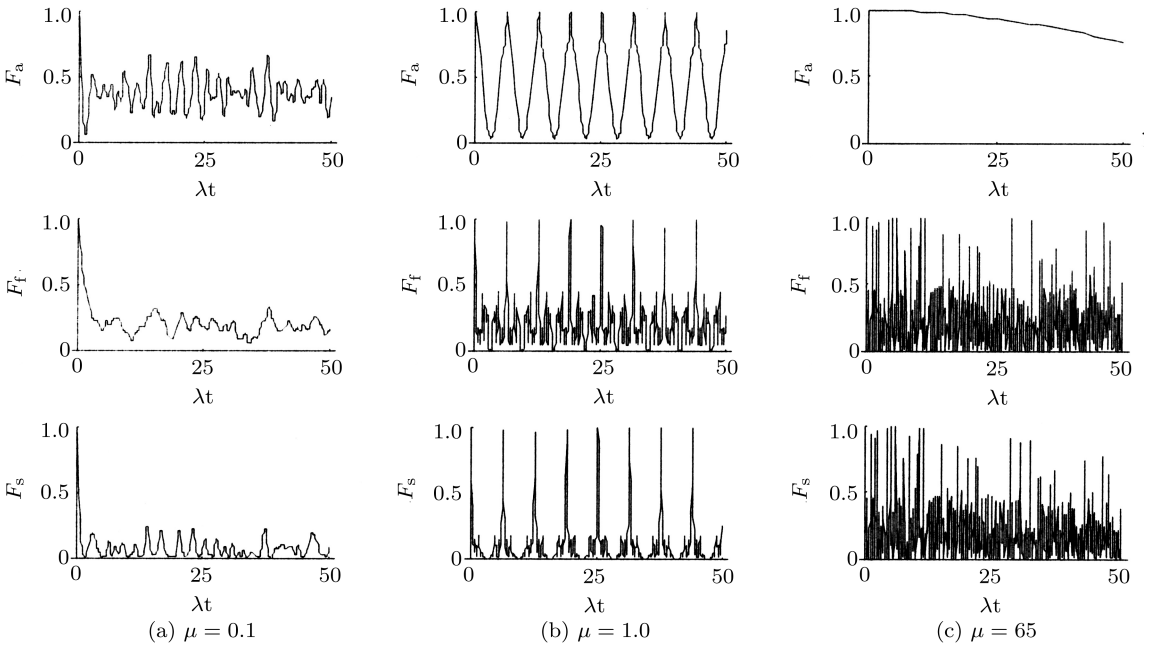


Fig.4. The evolutions of fidelity for three-level atom interacting with the coherent state light field for $\bar{n} = 2$.

the oscillation frequencies of the fidelities $F_{\rm f}$ and $F_{\rm s}$ increase, while that of $F_{\rm a}$ decreases obviously, indicating Kerr medium relatively weakens the relationship between the light field and the atom. If $\mu = 1.0$, the evolutions of the fidelities of quantum information exhibit the oscillations with equi-amplitude and perfect periodicity (i.e. $\lambda t = 6$). When $\mu = 65$, the fidelity of atom is close to 1, and the evolutions of fidelities for both the field and the system show the revival oscillation, while the fidelities of the quantum states for the field and the system remain the high-frequency oscillation.

4. Conclusions

The evolution characteristics of fidelity of quantum information for V-type three-level atom interacting with light field are investigated by using the numerical method. The general properties of the whole system, the atomic subsystem, and the field subsystem have been discussed. We come to some conclusions on the fidelity of quantum state:

1) The evolutions of the fidelity of quantum information are modulated by the initial number state field. The evolution of the fidelity of field shows the high-oscillation, and the fidelity of the field is better than those of the system and the atom.

2) The frequency of the evolution of the fidelity for quantum information is influenced by Kerr coefficient. The evolutions of the fidelities for the field and the system exhibit the oscillations with high-frequency. The Rabi oscillation frequency and the modulation frequency of fidelity for the field and the system vary with the value of the Kerr coefficient.

3) The evolutions of fidelity of quantum information obviously show the quantum collapse and revival behaviours in the system of atom interacting with light field. The light field is the key factor influencing the fidelity of quantum information. The stronger the initial light field, the lower the fidelities of quantum information. The change of initial light field intensity varies the Rabi oscillation frequency and the modulation frequency for the fidelity of quantum state of atom, field and system.

Our results show that the fidelity can provide more detailed information about the system behaviour than some global quantities such as the field entropy, so it could be of interest to perform similar calculations for other systems, such as the Lambda-type J-C model and the ladder-type J-C model etc.

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