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Controlling beam halo-chaos via backstepping design*

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A backstepping control method is proposed for controlling beam halo-chaos in the periodic focusing channels (PFCs) of high-current ion accelerator. The analysis and numerical results show that the method, via adjusting an exterior magnetic field, is effective to control beam halo chaos with five types of initial distribution ion beams, all statistical quantities of the beam halo-chaos are largely reduced, and the uniformity of ion beam is improved. This control method has an important value of application, for the exterior magnetic field can be easily adjusted in the periodical magnetic focusing channels in experiment.

Keywords: high-current ion beam, beam halo-chaos, chaos control, backstepping control
PACC: 0545, 4265J, 2990

1. Introduction

The high-current and high-power-duty ion linac is high-tech, and it has become a focal point of intensive investigation in a variety of nuclear physics and atomic energy development projects. Some possible significant applications of the high-current ion beam include accelerator driven radioactive clean nuclear power systems which are safer, cheaper and cleaner,[1] the transmutation of radioactive wastes, accelerator production of radioactive isotopes for medical use, high-power free electron lasers, heavy ion fusion and so on.

In the research and development of high-intensity accelerators, one of the most important problems is beam halo-chaos formation, which not only causes beam losses, restricts the beam-current strength, and damages accelerator components, but also leads to serious excessive radioactivity of accelerator and great harm to surroundings and human health. Therefore, the beam halo-chaos significantly restricts the application of high-intensity ion linacs. Today, it is a key issue to explore the mechanism of halo-chaos formation and the methods of controlling or suppressing it. Since the 1990s, researchers have not only been hunting for the formation mechanism,[2−8] but have also developed some effective methods of controlling the beam halo-chaos.[9−15]

In fact, the beam halo-chaos is essentially a complex spatiotemporal chaos. It is nonlinear in intensity and highly sensitive to small external perturbations, which are shown by some good examples including the LAMPF linac in Los Alamos.[3] How to suppress and control the beam halo-chaos is a very important and challenging subject in the sense of theory and practice. Fang and Chen[9] proposed a set of nonlinear feedback control methods based on the mechanism of control chaos, and the measurability to variables of complex system such as the wavelet-based function feedback,[10] the sample function feedback,[11] variables delayed feedback and variable structure method.[12,13] It will be valuable and realizable in engineering application if some external parameters (for example the exterior field) can be used to control the beam halo-chaos in a high-current accelerator.[14] In this paper, the formation of beam halo-chaos in the PFCs with a raised cosine magnetic field is considered, and a magnetic field backstepping design for controlling the beam halo-chaos in the PFCs is proposed.

The organization of this paper is as follows. Introduced in Section 2 is the envelope equation with the Kapchinsdkij–Vladimiskij (K–V) initial ion beam distribution and the motion of single ion. The halo-chaos formation in the case of raised cosine focusing field channel is considered. In Section 3 a control equation of exterior magnetic field via backstepping design is deduced. In Section 4, control results of the beam envelope equation and the orbit of single ion are given. In Section 5, the control method, in which the

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root-mean-square radius ($r_{\rm rms}$) of beam is selected as a feedback variable, is applied to the multi-particle model for initial ion beam distributions of five types. The particle-in-cell (PIC) simulation results illustrate that this control method is very effective to suppress beam halo-chaos. Conclusions drawn from the present study are given in Section 6.

2. Basic theories

2.1. Envelope equation and single ion motion of K–V beam

The intense charged-particle beam with the K–V equilibrium distribution is assumed to be a cylindrical beam. Ion beam dimensionless envelope equation in periodic solenoidal focusing field channels, based on the theories of breathing-mode round beams in high-current ion linacs, is described by the following differential equation[2,3]

$$\frac{{\rm d}^2 r_{\rm b}}{{\rm d}s^2} + \kappa(s) r_{\rm b} - \frac{K}{r_{\rm b}} - \frac{1}{r_{\rm b}^3} = 0, \tag{1}$$

where $r_{\rm b}$ is the beam envelope radius, the periodic function $\kappa(s) = \kappa(s+S)$ is the strength of the focusing magnetic field, S is the periodicity length for PFCs, the normalized beam perveance K is a measure of the beam self–field intensity, and $s = z/\beta_{\rm b} ct = z/S$ is the axial coordinate, $\beta_{\rm b} c$ is the average axial speed of the beam particles, and c is the light speed. Let ${\rm d}r_{\rm b}/{\rm d}s = \rho$, then Eq.(1) can be transformed into the following states equation:

$$\begin{aligned} &\frac{{\rm d}r_{\rm b}}{{\rm d}s} = \rho, \\ &\frac{d\rho}{{\rm d}s} = -\kappa(s) r_{\rm b} + \frac{K}{r_{\rm b}} + \frac{1}{r_{\rm b}^3}. \end{aligned} \tag{2}$$

When the beam envelope matches with the PFC system, the beam current is in a static (stationary) state, so $r_{\rm b} = r_{\rm b0} = {\rm consts.}$, where $r_{\rm b0}$ is an ion beam matched radius. That is the case corresponding to ${\rm d}r_{\rm b}/{\rm d}s = 0$ and ${\rm d}\rho/{\rm d}s = 0$. Note that the matched radius $r_{\rm b0}$ of beam is the aim that is to be realized. $r_{\rm b0}$ is worked out approximately from the following expression:

$$r_{\rm b0} = \frac{1}{\sqrt{2}} \sqrt{\frac{K}{\overline{\kappa(s)}} + \sqrt{\frac{K^2}{\overline{\kappa(s)^2}} + \frac{4}{\overline{\kappa(s)}}}}, \tag{3}$$

in which $\overline{\kappa(s)}$ is an average of the periodic magnetic field over one period.

Assuming that the core has a K–V distribution and test ions have no angular momentum, the transverse motion of an individual ion is expressed in terms of the dimensionless variables as[7,8]

$$\begin{aligned} &\frac{{\rm d}^2 r}{{\rm d}s^2} + \kappa(s) r - \frac{K}{r_{\rm b}^2} r = 0, \qquad |r| \le r_{\rm b}, \\ &\frac{{\rm d}^2 r}{{\rm d}s^2} + \kappa(s) r - \frac{K}{r} = 0, \qquad |r| > r_{\rm b}, \end{aligned} \tag{4}$$

where $|r|$ is the radial distance of individual particle from the core axis. Equation (4) shows the relation between the ion and the beam core, and it can be transformed into one order form

$$\begin{aligned} &\frac{{\rm d}r}{{\rm d}s} = v, \\ &\frac{{\rm d}v}{{\rm d}s} + \kappa(s) r - \frac{K}{r_{\rm b}^2} r = 0, \quad |r| \le r_{\rm b}, \\ &\frac{{\rm d}v}{{\rm d}s} + \kappa(s) r - \frac{K}{r} = 0, \quad |r| > r_{\rm b}, \end{aligned} \tag{5}$$

where $\kappa(s)$ has been selected as a periodic square form with a filling factor in most of reports.[2–4] It is clear that $\kappa(s) = \kappa(s+S)$, where S is the periodicity length of focusing field. Figure 1 is a sketch showing a periodic square focusing magnetic field with a amplitude κ_0 and a filling factor Γ. Some parameters in Ref.[14] are chosen as follows: turn-depression $\eta = 0.8$, the leading phase in vacuum $\sigma_0 = 115^\circ$, and the filling factor $\Gamma = 0.4$. With these parameters, we can work out the other parameters matched radius $r_{\rm b0} = 0.7891642$, perveance $K = 0.9032079$ and the strength of focusing field $\kappa_0 = 10.07$. Equation (2) behaves as chaotic motion under the condition of the above parameters and the mismatched initial state $\rho(0) = 1.0$, and $r_{\rm b}(0) = 1.578$. The single scroll chaotic attractor of Eq.(2) is displayed in Fig.2.

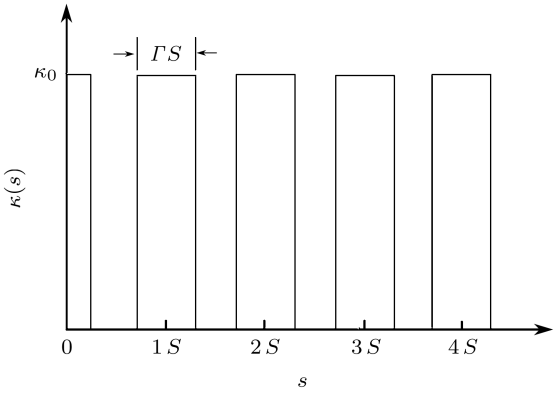


Fig.1. Periodic square focusing field.

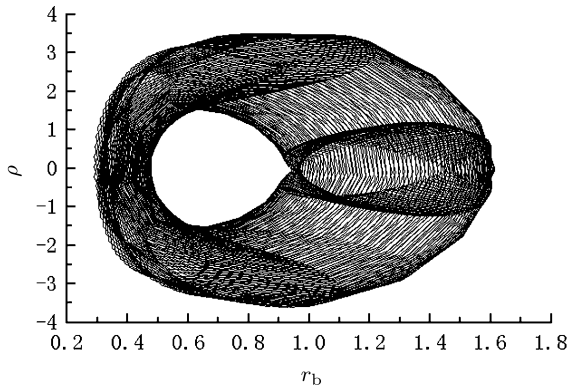


Fig.2. The single scroll chaotic attractor of envelope.

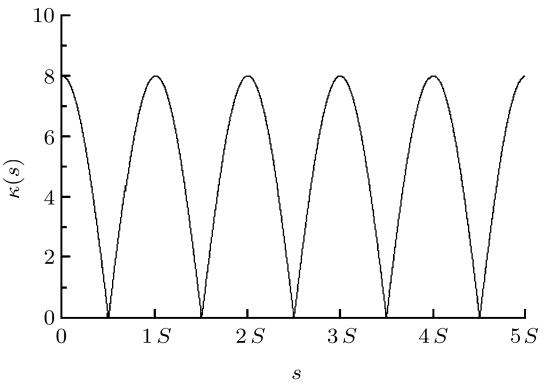


Fig.3. Raised cosine curve.

2.2. Focusing magnetic field with the raised cosine form

If the actual edge effect of $\kappa(s)$ with a square form is considered, $\kappa(s)$ will be similar to the magnetic field in a periodic spiral tube.[16] Reference [15] presents a raised cosine function $\kappa(s)$ as the following expression to approximately depict the practical focusing field:[15]

$$\kappa(s) = \kappa_0 \left|\cos 2\pi\, f\, s\right|, \tag{6}$$

where κ_0 is the amplitude of magnetic field intensity. The period of magnetic field is $S = 0.5/f$. Figure 3 displays the curve of $\kappa(s)$ with $\kappa_0 = 8.0$ and $f = 0.5\,\mathrm{Hz}$. The curve is close to some traits of actual periodic spiral tube field distribution.

The average magnetic field of the PFCs over one period, $\overline{\kappa(s)}$, obtained from expression (6), is equal to $2\kappa_0/\pi$. Let $K = 0.9032$, and select $\kappa_0 = 10.07$, and then we can obtain $r_{b0} = 0.6867$ from expression (3). Substituting expression (6) into Eqs.(2) and (5), and choosing the integration size $\Delta\, s = 0.02$, the mismatched initial conditions $r_b(0) = 1.4$, $\rho(0)$ =0.5, $r(0) = 0.4$, and $v(0) = 0.2$, we can use the fourth-order Runge–Kutta algorithm to calculate Eqs.(2) and (5) numerically with double-precision. In order to ensure the stability of the evolution of system, 20000 steps are taken in computation.

Figure 4 shows the single scroll chaotic attractor of envelope. From Fig.4, it is evident that the evolution of envelope shows a chaotic state under the mismatched initial conditions. Figure 5 reveals that the transverse radius evolution of individual ion motion is also chaotic. Figure 6 displays a single scroll chaotic attractor of transverse motion of ion. Those chaotic behaviours of ion and the beam core are responsible for the formation of beam-halo chaos.

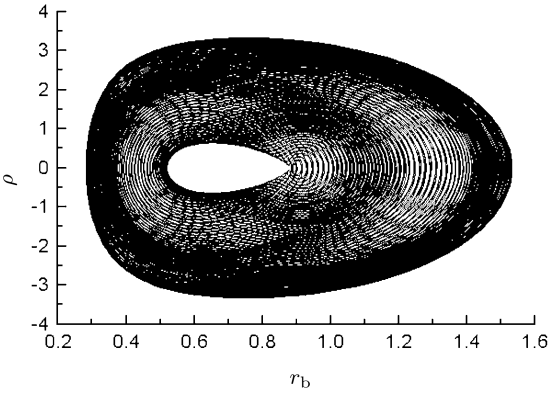


Fig.4. The single scroll chaotic attractor of envelope.

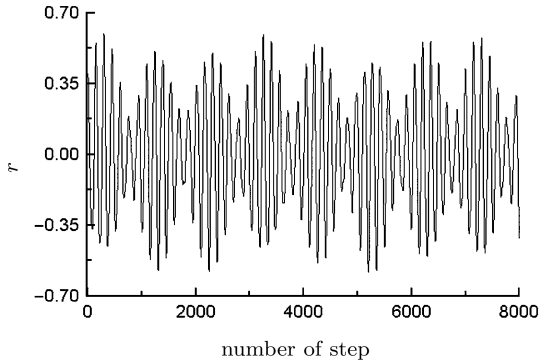


Fig.5. Evolution of transverse motion radius of ion.

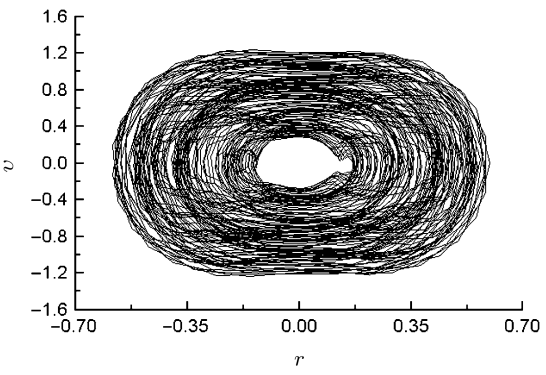


Fig.6. The single scroll chaotic attractor of ion.

3. Control method

Backstepping design has been commonly used in the field of nonlinear system control in recent years.[17,18] In this paper, we adopt this method to solve the problem of controlling high current beam halo-chaos in the PFCs. The backstepping control acts on the exterior magnetic field $\kappa(s)$ of the PFCs directly for realizing the control in engineering. Under the control condition, $\kappa(s)$ is adjusted as $\kappa'(s)$, so Eqs.(2) and (5) can be modified into the following forms respectively:

$$\begin{aligned} &\frac{\mathrm{d}r_\mathrm{b}}{\mathrm{d}s} = \rho, \\ &\frac{\mathrm{d}\rho}{\mathrm{d}s} = \dot{\rho} = -\kappa'(s) r_\mathrm{b} + \frac{K}{r_\mathrm{b}} + \frac{1}{r_\mathrm{b}^3}, \end{aligned} \tag{7}$$

and

$$\begin{aligned} &\frac{\mathrm{d}r}{\mathrm{d}s} = v, \\ &\frac{\mathrm{d}v}{\mathrm{d}s} + \kappa'(s) r - \frac{K}{r_\mathrm{b}^2} r = 0, \quad |r| \le r_\mathrm{b}, \\ &\frac{\mathrm{d}v}{\mathrm{d}s} + \kappa'(s) r - \frac{K}{r} = 0, \quad |r| > r_\mathrm{b}. \end{aligned} \tag{8}$$

In order to obtain the backstepping controller, z_1 is introduced to describe the error of the beam envelope radius, and it is expressed as

$$z_1 = r_\mathrm{b} - r_\mathrm{b0}, \tag{9}$$

so the axial coordinate derivative of z_1 can be written as

$$\dot{z}_1 = \frac{\mathrm{d}z_1}{\mathrm{d}s} = \frac{\mathrm{d}r_\mathrm{b}}{\mathrm{d}s} = \rho. \tag{10}$$

A virtual variable α_1 is defined as

$$\alpha_1 = -c_1 z_1, \tag{11}$$

in which $c_1 > 0$, and c_1 is a control factor. Variable z_2 and the Lyapunov function V are defined as

$$z_2 = \rho - \alpha_1, \tag{12}$$

$$V = \frac{1}{2}(z_1^2 + z_2^2), \tag{13}$$

respectively. Using the equations as mentioned above yields the time derivative of V as follows:

$$\begin{aligned} \dot{V} &= z_1 \dot{z}_1 + z_2 \dot{z}_2 = z_1 \rho + z_2(\dot{\rho} - \dot{\alpha}_1) \\ &= z_1(z_2 + \alpha_1) + z_2(\dot{\rho} - \dot{\alpha}_1) \\ &= -c_1 z_1^2 + z_1 z_2 + z_2(\dot{\rho} - \dot{\alpha}_1), \end{aligned} \tag{14}$$

where $-c_1 z_1^2 \le 0$. If $\kappa(s)$ can be controlled into $\kappa'(s)$, and let

$$-z_1 = \dot{\rho} - \dot{\alpha}_1, \tag{15}$$

inequality $\dot{V} \le 0$ will hold true, thus the aim of $r_\mathrm{b} \to r_\mathrm{b0}$ is realized. Using Eq.(7) and expression (15), the control equation of magnetic field $\kappa'(s)$ of PFCs is deduced as

$$\kappa'(s) = \frac{1}{r_\mathrm{b}} \left(\frac{K}{r_\mathrm{b}} + \frac{1}{r_\mathrm{b}^3} + r_\mathrm{b} - r_\mathrm{b0} + c_1 \rho \right). \tag{16}$$

4. Numerical results of equation under control

In this section, the validity of control method is verified by applying it to the ion beam envelope and the individual ion motion The choice of system parameters corresponds to the perveance $K = 0.9032$, the magnitude of magnetic field $\kappa_0 = 10.07$, and the frequency $f = 0.5\,\mathrm{Hz}$. We select the same mismatched initial conditions of system as presented in the second section, and control factor $c_1 = 2$. The backstepping control Eq.(16) starts after free evolution of 2000 iteration steps for Eqs.(7) and (8). Figure 7 shows the evolution of the beam radius r_b after control. From Fig.7, it is clear that r_b can be controlled to the matched radius 0.6867, which shows that the aim is realized. Figure 8 shows the evolution of the transverse motion radius of ion. It can be seen from Fig.8 in which the chaotic status of particle is almost suppressed effectively to one period orbit after control.

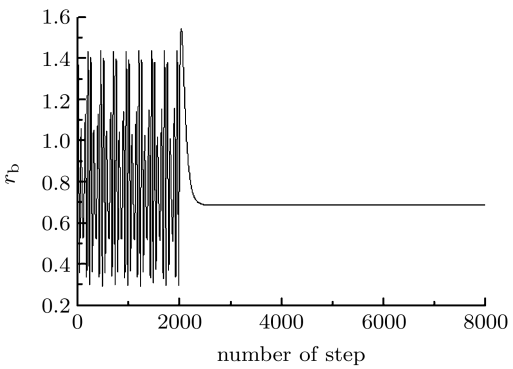


Fig.7. Evolution of the beam radius.

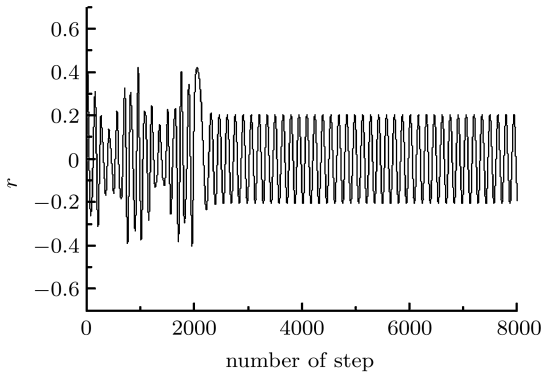


Fig.8. Evolution of the transverse motion radius of ion.

5. PIC simulation results with comparative analysis

5.1. Control strategy and parameters used in PIC simulation

The PIC simulation is used for studying the multi-ion motion in the PFCs. In order to improve the feasibility in engineering, we select the amplitude of magnetic field κ_0 as an adjustable parameter and keep the frequency f unchanged in expression (6). Based on the relationship between the $\kappa(s)$ and the κ_0 in expression (6), controller equation (16) can be modified into

$$\kappa_0' = \frac{1}{r_\mathrm{b}}\left(\frac{K}{r_\mathrm{b}} + \frac{1}{r_\mathrm{b}^3} + r_\mathrm{b} - r_{\mathrm{b}0} + c_1\rho\right). \tag{17}$$

Therefore expression (6) can be rewritten as

$$\kappa'(s) = \kappa_0' \left|\cos 2\pi f s\right|. \tag{18}$$

To avoid overshoot, a limiter of κ_0' is introduced, and κ_0' is adjusted as

$$\kappa_0' = \begin{cases} \dfrac{1}{r_\mathrm{b}}\left(\dfrac{K}{r_\mathrm{b}} + \dfrac{1}{r_\mathrm{b}^3} + r_\mathrm{b} - r_{b0} + c_1\rho\right), & \kappa_0' \le 200, \\ 200, & \kappa_0' > 200. \end{cases} \tag{19}$$

The main parameters are selected as follows: the radial meshes over the length $r_\mathrm{b}(0)$ are 100, the total number of ions is 50000, mismatched factor $M = 2.0$, perveance $K = 0.9032079$, amplitude of focusing field $\kappa_0 = 10.07$ and frequency $f = 0.5\,\mathrm{Hz}$. Using the Eq.(3), the corresponding matched radius of K–V beam $r_{\mathrm{b}0} = 0.686755$ may be worked out, we select the integration size $\Delta s = 0.02$ and 50000 ions in PIC program. The root-mean-square radius of beam, r_rms, as a measurable feedback variable, replaces the envelope radius r_b in expression (19). Choosing a statistical physical quantity as a feedback variable has been reported in References [10,11]. Considering the actual time-lag between the observed input and the feedback response, and reducing the control cost, the control strategy is carried in each half of the period. In order to ensure the stability of the control results, 2000 periods of PFCs are adopted.

5.2. Simulation results with comparative analysis

In order to characterize the effects of halo-chaos under control, we adopt some statistical physical quantities as follows:[10,13] the halo-chaos strength factor H, which is defined as the ratio of the number of particles whose positions are larger than $1.75\,r_{\mathrm{b}0}$ to the number of total particles; the statistical average value of beam halo-chaos strength factor in each period, H_av; the maximum value of the average relative emittance $\varepsilon_{\mathrm{av\ max}}$ corresponding to transverse direction; the transverse mean-square momentum P_av^2; the maximal value of the ion beam radius r_max. Table 1 summarizes the main results of controlling the beam halo-chaos with five types of initial ion distributions.

Table 1. Comparison among properties of beam halo-chaos with five types of initial ion distributions before and after applying the exterior magnetic field adaptive switching control (A/B stands for after/ before).

Initial Ion Distribution	K–V A/B Control	Water-bag A/B Control	Parabola A/B Control	3-δ Gauss A/B Control	Full Gauss A/B Control
H_max	0.0000/0.1437	0.0000/0 .0959	0.0000/0.1418	0.0000/ 0.1196	0.0003/0.1337
$r_\mathrm{max}/r_{\mathrm{b}0}$	1.2149/4.3809	1.6305/3.9998	1.2605/ 4.5346	1.6346/4.2887	2.3036/5.3532
$\varepsilon_{\mathrm{av\ max}}$	1.7617/4 .1876	2.3731/4.1466	1.6897/4.2193	2.1134/4.1780	2.0118/4.1932

The following conclusions are drawn from Table 1:

(1) The maximum value of the halo-chaos strength factor $H_{\max} \to 0$ except full Gauss beam after control, namely, the halo-chaos strength factor $H \to 0$ and the regeneration of the halo-chaos are completely eliminated after control.

(2) The transverse emittance of ion beam is also significantly reduced. It is decreased by about 1–3 times after control, which means the beam is more compact after control.

(3) After the direct control over the strength of the focusing magnetic field, almost all statistical properties of the beam halo-chaos with five types of initial distributions are in good condition. Because of observability for the feedback variable $r_{\rm rms}$, for instance, $r_{\rm rms}$ is considered approximately as $r_{\rm b}/\sqrt{2}$ for K–V beam,[2] thus this control method is easy to realize in experiment.

Figure 9 shows the phase diagram of beam halo-chaos with an initial parabolic distribution in the transection space (x, y) at period = 1600 section before and after applying the control. Figure 9 shows that before control, not only is the ion beam distribution lacking in the uniformity and compactness, but also the beam halo-chaos is evident. After control, the transection area is largely reduced, nearly contracted by about 20 times, also the halo-chaos is eliminated, and the uniformity of ion beam turns better.

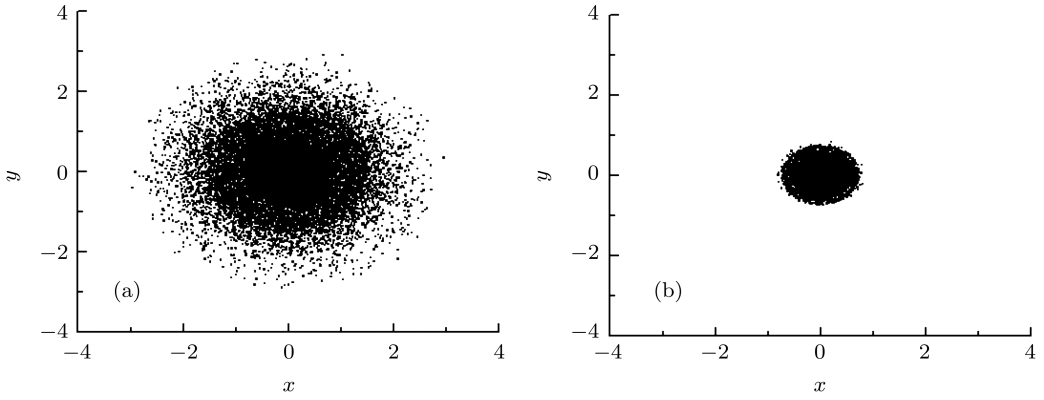


Fig.9. Ion distribution diagram in the (x, y) space periodic section (period = 1600). (a) before control; (b) after control.

6. Summary and conclusions

In this paper, the formation of beam halo-chaos in the PFCs with a raised cosine magnetic field is considered, and the backstepping design for controlling the beam halo-chaos in the PFCs is presented. Simulation results demonstrate that the chaotic envelope can be well controlled by this method, and the radius of beam converges stably at the matched radius. This method is also applied to the multi-particle model in the PFCs. The initial ion beams with five types of distributions including K–V beam are selected as the typical examples. The PIC simulation results show that the halo-chaos and its regeneration can be eliminated effectively, and that both the compactness and the uniformity of ion beam are improved evidently. The method possesses some advantages as below: the feedback variable $r_{\rm rms}$ is measurable, a direct relation between the measurable feedback variable and exterior magnetic field in the PFCs can be founded, and it is easy to realize in engineering. Therefore, these research results give a valuable reference to the application to high-current accelerator.

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