Vol 17 No 4, April 2008
1674-1056/2008/17(04)/1189-07

Chinese Physics B

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Effect of interaction strength on gap solitons of Bose–Einstein condensates in optical lattices

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(Received 8 August 2007; revised manuscript received 29 October 2007)

We have developed a systematic analytical approach to the study on the dynamic properties of the linear and the nonlinear excitations for quasi-one-dimensional Bose–Einstein condensate trapped in optical lattices. A novel linear dispersion relation and an algebraic soliton solution of the condensate are derived analytically under consideration of Bose–Einstein condensate with a periodic potential. By analysing the soliton solution, we find that the interatomic interaction strength has an important effect on soliton dynamic properties of Bose–Einstein condensate.

Keywords: Bose–Einstein condensates, solitons, optical lattices
PACC: 0530J, 0290, 1110L

1. Introduction

Realization of Bose–Einstein condensation in dilute atomic gases[1−3] has provided physicists with a new fertile ground for exploring many aspects of this fascinating phenomenon, including stationary state characters,[4−6] dynamic properties,[7−13] and time-dependent behaviours.[14,15] With the development of the experimental technique, neutral atoms confined in an array of magnetic or optical traps offer a scalable system for the application in quantum computation and quantum information processing. Recently, Bose–Einstein condensates (BECs) have been successfully loaded in optical lattices.[16−19] In these experiments, the optical lattice is created by the interference of laser beams. The properties of the atoms are characterized by the depth and the period of this optically induced potential. The intensity of the lattice potential can be modulated from very weak to very strong.[16] Many efforts have been made to study the lattice effect and the relevant phenomena, such as Landau-Zener tunnelling.[20] Josephson effect,[18] Bloch oscillations,[21] superfluid and dissipative dynamics,[22,23] phase diagram,[24] and nonlinear dynamics of a dipolar[25] or spinor BEC.[26] Theoretically, these entities are possible for solitons to exist, for the nonlinearity compensates for the inter-atomic dispersion caused by intersite tunnelling. For example, Louis *et al*[27] demonstrated gap solitons occurring in the band gaps of the matter-wave spectrum and analysed their stability in different band gaps for repulsive and attractive inter-atomic interactions. This made it possible to employ the perturbation method. By means of a multiple-scale expansion, Konotop and Salerno[28] explained that bright solitons could come into being in a BEC with a positive scattering length and dark solitary pulses could be stable with a negative scattering length.

One of the most attractive features for cold atomic gases is how the interatomic interaction affects the nonlinear dynamic properties of the condensates. Especially, in the process of Feshbach resonances,[29,30] the scattering length can be tuned from highly repulsive to strongly attractive by changing the applied magnetic field. This indicates that the interaction strength can be controlled by using different experimental devices. In this paper, we study how the interatomic interaction affects the nonlinear dynamic stability of dilute cold atoms in optical lattices. It is found that the stability of the soliton is determined by both the strength of interatomic interaction and the depth of optical lattices. The outline of this paper is as follows. In Section 2, we present our model and derive the dimensionless form of amplitude and phase of the wavefunction of condensate by means of the multiple-scale method. Subsequently, the linear dispersion relation that arises from the ground-state and the group speed of this system is obtained analytically in Section 3. In Section 4, by developing a systematic analytical approach to the investigation on the dynamic properties of solitons for quasi-one-

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dimensional (1D) BEC in optical lattices based on a generalized method of multiple scales, we obtain algebraic soliton solutions and discuss their stability. Some conclusions drawn from the present study are given in Section 5.

2. Model

The evolution of the mean-field theory of a BEC confined in an elongated optical lattice is described by the time-dependent three-dimensional GP equation[27,31]

$$\mathrm{i}\hbar\frac{\partial\Psi}{\partial T}=\left[-\frac{\hbar^2}{2m}\nabla^2+V(X,Y,Z)+g\left|\Psi\right|^2\right]\Psi, \quad (1)$$

where $\Psi(X,Y,Z,T)$ is the wavefunction of cigar-shape condensate; (X,Y) are the direction of strong transverse confinement and Z is the direction of lattice; $\int \mathrm{d}r\left|\Psi\right|^2=N$ is the number of atoms in the condensate; $g=4\pi\hbar^2a_\mathrm{s}/m$ is the interaction strength with the atomic mass m and the s-wave scattering length a_s ($a_\mathrm{s}>0$ for a repulsive interaction). The combined potential $V(X,Y,Z)$ of the optical lattice and magnetic trap can be given by

$$V(R^2,Z)=E_0\sin^2\left(\frac{\pi Z}{d}\right)+\frac{1}{2}m\left(\omega_\perp^2R^2+\omega_Z^2Z^2\right), \quad \omega_Z\ll\omega_\perp, \quad (2)$$

where $R^2=X^2+Y^2$, E_0 is the well depth of the optical lattice, d is the characteristic lattice constant, and ω_Z and $\omega_\perp$ are the frequencies of the magnetic trap in the axial (Z) and the transverse (X and Y) directions, respectively.

For convenience, we introduce the dimensionless variables $(r,z)=a_0^{-1}(R,Z)$, $t=\omega_\perp T$ with $a_0=\left(\hbar/m\omega_\perp\right)^{1/2}$ (transverse harmonic-oscillator length), and $\psi=\sqrt{a_0^3/N}\,\Psi$. So the dimensionless GP equation is written as

$$\mathrm{i}\frac{\partial\psi}{\partial t}=-\frac{1}{2}\nabla^2\psi+\left[V_0\sin^2\left(\frac{\pi z}{D}\right)+\frac{1}{2}\left(r^2+\Omega^2z^2\right)\right]\psi+Q\left|\psi\right|^2\psi, \quad (3)$$

where $V_0=E_0/\left(\hbar\omega_\perp\right)$, $D=a_0^{-1}d$, $\Omega=\omega_Z/\omega_\perp\ll1$, and $Q=4\pi Na_\mathrm{s}/a_0$. Using the expression of $\psi=\sqrt{n}\exp(\mathrm{i}\Phi)$, we obtain a set of coupled equations

$$\frac{\partial n}{\partial t}+\nabla\cdot(n\nabla\Phi)=0, \quad (4)$$

$$\frac{\partial\Phi}{\partial t}+V_0\sin^2\left(\frac{\pi z}{D}\right)+\frac{1}{2}\left(r^2+\Omega^2z^2\right)+\frac{1}{2}\left(\nabla\Phi\right)^2-\frac{1}{2\sqrt{n}}\nabla^2\sqrt{n}+Qn=0. \quad (5)$$

The ratio Ω between the confinement strength in the axial (Z) direction and that in the transverse (X and Y) direction varies from 0.1[17,18] to $1/\sqrt{2}$.[32,33] Here, we suppose a cigar-shape condensate in a strongly elongated trap ($\Omega\sim0.1$) which is regarded as a quasi-1D optical lattice in the direction of a weak confinement. Then, the wavefunction may be expressed as $\psi(r,z,t)=G_0(r)\,\phi(z,t)$, with $\phi(z,t)=A(z,t)\exp\left[-\mathrm{i}\mu t+\mathrm{i}\varphi(z,t)\right]$. Its corresponding modulus and phase, respectively, are

$$\sqrt{n}=G_0(r)\,A(z,t), \quad (6)$$

$$\Phi=-\mu t+\varphi(z,t). \quad (7)$$

According to the tight confinement of the condensate in the transverse direction, we assume that the spatial structure of the function $G_0(r)$ can be well described by a solution of the two-dimensional radially symmetric quantum harmonic-oscillator problem, $\nabla_\perp^2G_0+2G_0-r^2G_0=0$. Taking the ground-state solution, $G_0(r)=C\exp\left(-r^2/2\right)$, where $C^2=1/\pi$ is found from the normalization condition, $\int_{-\infty}^{+\infty}\left|G_0\right|^2r\mathrm{d}r=1$, and inserting them into Eqs.(4) and (5), we can obtain

$$\frac{\partial A}{\partial t}+\frac{\partial A}{\partial z}\frac{\partial\varphi}{\partial z}+\frac{1}{2}A\frac{\partial^2\varphi}{\partial z^2}=0, \quad (8)$$

$$-\frac{1}{2}\frac{\partial^2A}{\partial z^2}+\left[\frac{1}{2}\left(\frac{\partial\varphi}{\partial z}\right)^2-\mu+1+\frac{\partial\varphi}{\partial t}+V_0\sin^2\left(\frac{\pi z}{D}\right)+\frac{1}{2}\Omega^2z^2\right]A+Q'\left|A\right|^2A=0, \quad (9)$$

where $Q'=Q/2\pi=2Na_\mathrm{s}/a_0$. In order to obtain an effective 1D GP equation in the dimensionless variables, we have multiplied Eq.(9) by G_0^* and then integrated the resulting equation once with respect to the transverse coordinates to eliminate the dependence on transverse plane. An approach similar to this one has been widely used for quasi-1D (cigar-shape) BEC problems.[34,35]

3. Linear Bloch modes

Let $A=u_0(z)+\alpha(z,t)$ (without loss of generality, we assume that $u_0(z)$, which characterizes the

condensate background, is positive) with $\alpha(z,t) = \alpha_0 \exp(\mathrm{i}\beta)$, $\varphi = \varphi_0 \exp(\mathrm{i}\beta)$, and $\beta = kz - \omega t$. Considering α_0 and φ_0 to be small constants, we obtain

$$-\mathrm{i}\omega\alpha_0 + \mathrm{i}k\varphi_0 \frac{\partial u_0}{\partial z} - \frac{1}{2}k^2 u_0 \varphi_0 = 0, \tag{10}$$

and

$$(\mu - 1)\, u_0 = -\frac{1}{2}\frac{\partial^2}{\partial z^2} u_0 + V_0 \sin^2\left(\frac{\pi z}{D}\right) u_0 + \frac{1}{2}\Omega^2 z^2 u_0 + Q' |u_0|^2 u_0, \tag{11}$$

$$\frac{1}{2}k^2\alpha_0 - (\mu - 1)\,\alpha_0 - \mathrm{i}\omega\varphi_0\mu_0 + V_0 \sin^2\left(\frac{\pi z}{D}\right)\alpha_0 + \frac{1}{2}\Omega^2 z^2 \alpha_0 + 3Q' |u_0|^2 \alpha_0 = 0, \tag{12}$$

from linearized Eqs.(8) and (9), respectively. Assuming $Q' = 0$, which corresponds to linear conditions, Eq.(11) can be written as the Mathieu equation[36]

$$\frac{\mathrm{d}^2}{\mathrm{d}y^2} u_0 - [2q\cos(2y) - p]\, u_0 = 0, \tag{13}$$

where $y = \pi z/D$, $q = -V_0 D^2/(2\pi^2)$, and $p = q + 2D^2(\mu - 1)/\pi^2$. By using the Floquet–Bloch theorem, the solutions of Eq.(13) can be presented as a superposition of Bloch waves. By employing the plane-wave method, a typical band structure corresponding to Eq.(13) for $V_0 = 10$ and $d = 1\,\mu m$ is obtained as shown in Fig.1 of Ref.[31].

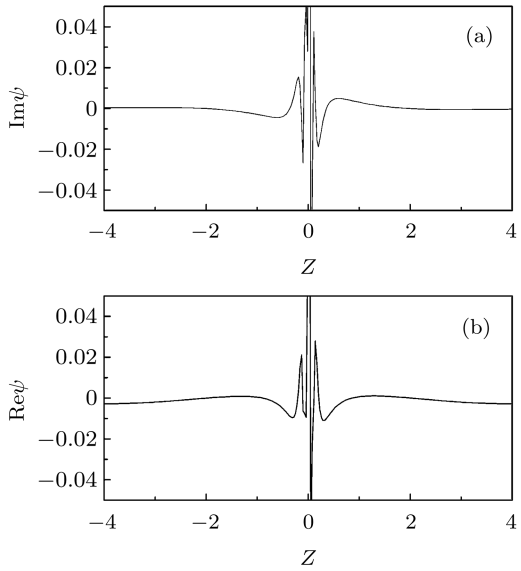


Fig.1. The imaginary (a) and the real (b) parts of wave-function of the condensates each as a function of z for $V_0 = 0.06$ (in units of $\hbar\omega_\perp$), $Q' = 0.03$ (in units of $2Na_0^{-1}a_s$).

In the case of $Q' \neq 0$, which corresponds to weakly interacting condensates, Eq.(13) possesses periodic solutions. According to the Bloch's theorem, the Floquet–Bloch modes are characterized by lattice vector k. The Bloch wave is determined by

$$u_0(z) = u_{0k} \exp(\mathrm{i}kz), \tag{14}$$

where u_{0k} is a periodic function with a period of lattice. Substituting expression (14) into Eqs.(10)–(12), we obtain the linear dispersion relation

$$\omega = \pm\sqrt{3Q'}\, k u_{0k}, \tag{15}$$

where the positive and the negative signs correspond respectively to the wave propagating to the right and the left. Here, the real part of ω is chosen as that in Ref.[27]. The reason is that if ω has a nonvanishing imaginary part, the solution is not oscillatory (decay or ascending exponentially with z). From expression (15), we have a group speed of the system $V_\mathrm{g} = \mathrm{d}\omega/\mathrm{d}k = \pm\sqrt{3Q'}\, u_{0k}$. For an homogeneous system [i.e. for the case of $V(X,Y,Z) = 0$ in Eq.(1)] a corresponding group speed is $V_{\mathrm{g}0} = \pm\sqrt{3Q}\, u_{0k}$ in our notation. Therefore, we have $V_\mathrm{g}/V_{\mathrm{g}0} = 1/\sqrt{2\pi}$. The factor $1/\sqrt{2\pi}$ is due to the transverse confinement of the system. It is obvious that the linear dispersion relation and the group speed of the system possess a periodicity, which arises from the periodicity of the optical lattices.

4. Nonlinear Bloch modes

Using the asymptotic expansion, we look for an approximate solution in the following form:

$$A = u_0(z) + \varepsilon\left[a^{(0)} + \varepsilon a^{(1)} + \varepsilon^2 a^{(2)} + \cdots\right], \tag{16}$$

$$\varphi = \varepsilon\left[\varphi^{(0)} + \varepsilon\varphi^{(1)} + \varepsilon^2\varphi^{(2)} + \cdots\right]. \tag{17}$$

Assume that $u_0(z)$ and optical lattices each are a function of a fast variable $z_0 = z$ characterizing the variation of ground-state of BEC, $a^{(i)}$ and $\varphi^{(i)}$ $(i = 0, 1, \cdots)$ each are a function of multiple-scale slow variables

$\xi = \varepsilon(z - V_g t)$ and $\tau = \varepsilon^2 t$, where ε is a smallness parameter characterizing the relative amplitude of the excitation. Substituting them into Eqs.(8) and (9), respectively, we have

$$V_g \frac{\partial a^{(0)}}{\partial \xi} - \frac{\partial u_0}{\partial z_0}\frac{\partial \varphi^{(0)}}{\partial \xi} = 0, \tag{18}$$

$$V_g \frac{\partial a^{(1)}}{\partial \xi} - \frac{\partial u_0}{\partial z_0}\frac{\partial \varphi^{(1)}}{\partial \xi} = \frac{\partial a^{(0)}}{\partial \tau} + \frac{1}{2}u_0 \frac{\partial^2 \varphi^{(0)}}{\partial \xi^2}, \tag{19}$$

and

$$(\mu - 1)u_0 = -\frac{1}{2}\frac{\partial^2}{\partial z^2}u_0 + V_0 \sin^2\left(\frac{\pi z}{D}\right)u_0 + Q'|u_0|^2 u_0, \tag{20}$$

$$-(\mu - 1)a^{(0)} + V_0 \sin^2\left(\frac{\pi z}{D}\right)a^{(0)} + 3Q'|u_0|^2 a^{(0)} = 0, \tag{21}$$

$$-(\mu - 1)a^{(1)} + V_0 \sin^2\left(\frac{\pi z}{D}\right)a^{(1)} + 3Q'|u_0|^2 a^{(1)} = V_g u_0 \frac{\partial \varphi^{(0)}}{\partial \xi} - 3Q' u_0 \left(a^{(0)}\right)^2. \tag{22}$$

As is known, solitons can travel a long distance without attenuation and change in shape, which results from the balance between dispersion and nonlinearity. By comparing Eq.(20) with Eq.(21), we have $-\frac{1}{2}\frac{\partial^2}{\partial z^2}u_0 = 2Q'|u_0|^2 u_0$, in which the left-hand side describes the dispersion properties of the condensate background and the right-hand side depicts its nonlinearity. This may be used to theoretically explain the mechanism for the formation of solitons in BEC confined in optical lattices.

In view of the fact that solitons of lattices exist only in the gap between two adjacent bands,[27,31] we study the solution of condensate inside the band gap. In general, inside the band gap, the lattice vector is complex, $k = \pi/D + \mathrm{i}k_i$, where k_i is the decay rate of the ith mode among these modes.[31] Equation (20) has an exact solution of Bloch wave on the Brillouin zone edge[22,23,37]

$$\begin{aligned} u_0 &= \frac{\sqrt{2Q'+V_0}+\sqrt{2Q'-V_0}}{\sqrt{8Q'}}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{D}+\mathrm{i}k_i\right)z} + \frac{\sqrt{2Q'+V_0}-\sqrt{2Q'-V_0}}{\sqrt{8Q'}}\mathrm{e}^{-\mathrm{i}\left(\frac{\pi}{D}+\mathrm{i}k_i\right)z} \\ &= \sqrt{\frac{2Q'+V_0}{2Q'}}\cosh\left(k_i - \mathrm{i}\frac{\pi}{D}\right)z - \sqrt{\frac{2Q'-V_0}{2Q'}}\sinh\left(k_i - \mathrm{i}\frac{\pi}{D}\right)z, \end{aligned} \tag{23}$$

with $\mu = 1 + Q' + (1/2)\left[(\pi/D)^2 + V_0\right]$. From Eqs.(18), (21), and (22), we have

$$\begin{aligned} \varepsilon a^{(0)} &= -\frac{V_g^2}{3Q' u_{0z}(z - V_g t)} \\ &= \frac{\sqrt{2}V_g^2\left[3(k_i - \mathrm{i}\pi/D)(z - V_g t)\sqrt{Q'}\right]^{-1}}{\left[\sqrt{2Q'-V_0}\cosh(k_i - \mathrm{i}\pi/D)z - \sqrt{2Q'+V_0}\sinh(k_i - \mathrm{i}\pi/D)z\right]}, \end{aligned} \tag{24}$$

$$\begin{aligned} \varepsilon \varphi^{(0)} &= -\frac{V_g^3}{3Q' u_{0z}^2(z - V_g t)} \\ &= \frac{-2V_g^3\left[3(k_i - \mathrm{i}\pi/D)^2(z - V_g t)\right]^{-1}}{\left[2Q'\cosh 2(k_i - \mathrm{i}\pi/D)z - V_0 - \sqrt{4(Q')^2 - V_0^2}\sinh 2(k_i - \mathrm{i}\pi/D)z\right]}, \end{aligned} \tag{25}$$

where u_{0z} is the derivative with respect to z. To obtain expressions (24) and (25), we have used $\xi = \varepsilon(z - V_g t)$. Thus, we obtain the soliton solution of Eq.(3) in the following form:

$$\begin{aligned} \psi(r,z,t) = \frac{1}{\sqrt{\pi}}\Bigg\{ & \sqrt{\frac{2Q'+V_0}{2Q'}}\cosh\left(k_i - \mathrm{i}\frac{\pi}{D}\right)z - \sqrt{\frac{2Q'-V_0}{2Q'}}\sinh\left(k_i - \mathrm{i}\frac{\pi}{D}\right)z \\ & + \frac{\sqrt{2}V_g^2\left[3(k_i - \mathrm{i}\pi/D)(z - V_g t)\sqrt{Q'}\right]^{-1}}{\left[\sqrt{2Q'-V_0}\cosh(k_i - \mathrm{i}\pi/D)z - \sqrt{2Q'+V_0}\sinh(k_i - \mathrm{i}\pi/D)z\right]}\Bigg\} \\ & \times \exp\left\{-\frac{r^2}{2} - \mathrm{i}\mu t - \frac{2\mathrm{i}V_g^3\left[3(k_i - \mathrm{i}\pi/D)^2(z - V_g t)\right]^{-1}}{\left[2Q'\cosh 2(k_i - \mathrm{i}\pi/D)z - V_0 - \sqrt{4(Q')^2 - V_0^2}\sinh 2(k_i - \mathrm{i}\pi/D)z\right]}\right\} \end{aligned}$$

$$+0\left(\varepsilon^2\right). \tag{26}$$

From expression (23), one can see that the Bloch wave state exists only when $Q' \geq 0.5V_0$ so that it has no linear counterpart.[23] This shows that there are some 'extra' nonlinear Bloch waves when the nonlinearity is strong enough. We conclude that soliton disappears when $Q' < 0.5V_0$. In the case of $Q' = 0.5V_0$, expression (26) can be rewritten as

$$\psi(r,z,t) = \frac{1}{\sqrt{\pi}}\left[\sqrt{2}\cosh\left(k_i - \mathrm{i}\frac{\pi}{D}\right)z - \frac{V_{\mathrm{g}}^2\,\mathrm{csch}\left(k_i - \mathrm{i}\pi/D\right)z}{3\sqrt{2}\left(k_{\mathrm{i}} - \mathrm{i}\pi/D\right)Q'\left(z - V_{\mathrm{g}}t\right)}\right] \times \exp\left\{-\frac{r^2}{2} - \mathrm{i}\mu t - \frac{2\mathrm{i}V_{\mathrm{g}}^3\left[3\left(k_i - \mathrm{i}\pi/D\right)^2\left(z - V_{\mathrm{g}}t\right)\right]^{-1}}{\left[2Q'\,\mathrm{sech}\,2\left(k_i - \mathrm{i}\pi/D\right)z - V_0\right]}\right\}. \tag{27}$$

As a typical example, we investigate the soliton properties of condensate ^{87}Rb as shown in Fig.1, because the ^{87}Rb atoms have been used to produce the condensate in recent BEC experiments in lattices. For ^{87}Rb, $m = 1.4 \times 10^{-25}$ kg, $\omega_\perp = 2\pi \times 320$ Hz, the value of lattice constant d varies between 0.4 and 1.6 μm, while the radial size and length of condensate are 17 μm and 300 μm respectively.[38] Here we assume that $d = 1\,\mu$m (i.e. the period of the lattice is $2D = 2a_0^{-1}d \approx 4$), normalized potential depth $V_0 = 0.06$, and decay rate $k_{\mathrm{i}} = 0.01$, unless otherwise stated. In our numerical calculation, we normalize the wave function. When $Q' = 0.03$, there appear envelope oscillations of the real and the imaginary parts of wavefunction [see Fig.1]. One can find that the wavefunction satisfies the parity relations $\mathrm{Im}\,\psi(r,z) = -\mathrm{Im}\,\psi(r,-z)$ and $\mathrm{Re}\,\psi(r,z) = \mathrm{Re}\,\psi(r,-z)$. This means that there exist instability phenomena for nonlinear Bloch states close to the edges of the Brillouin zone. In the presence of a repulsive interatomic interaction ($a_{\mathrm{s}} > 0$), the energy of this state will be shifted upward into the gap where the state cannot exist. Then one can expect the state to become unstable against small spatial modulations (modulational instability) so that new excitations must take place. With the strength of interaction Q' increasing from 0.03 to 0.6, the parity relation between the real and the imaginary parts of wavefunction hardly changes. When Q' further increases, however, V_0 can be neglected. Equation (26) is simplified into

$$\psi(r,z,t) = \frac{1}{\sqrt{\pi}}\left[\cosh\left(k_i - \mathrm{i}\frac{\pi}{D}\right)z - \sinh\left(k_i - \mathrm{i}\frac{\pi}{D}\right)z + \frac{V_{\mathrm{g}}^2}{3\left(k_i - \mathrm{i}\pi/D\right)\left(z - V_{\mathrm{g}}t\right)\exp\left[-\left(k_i - \mathrm{i}\pi/D\right)z\right]}\right] \times \exp\left\{-\frac{r^2}{2} - \mathrm{i}\mu t - \frac{2\mathrm{i}V_{\mathrm{g}}^3\left(k_i - \mathrm{i}\pi/D\right)^{-2}}{3\left(z - V_{\mathrm{g}}t\right)\left\{2Q'\exp\left[-2\left(k_i - \mathrm{i}\pi/D\right)z\right] - V_0\right\}}\right\}. \tag{28}$$

The curves of the real and the imaginary parts of wavefunction for $Q' = 1.0$, are shown in Fig.2. It is clear that there appears a stable envelop distribution of the same parity relation between the real and the imaginary parts of the nonlinear Bloch states on the edge of the Brillouin zone. The reason is that for the interaction strength much higher than the depth of optical lattice, the main effect is expected to come from the interatomic interaction, while modulational effect of the optical lattices cannot play a crucial role in determining the existence of solitons. We also check the stability of our analytical solution (28) by using the density of wavefunction varying with time, which is shown in Fig.3. One can see that the density of wave function has a maximum envelop amplitude, which corresponds to BEC droplets in experiment. The envelop amplitude stability propagates from $t = 0$ to $t = 1$. Based on the discussion above, $Q' = 0.5V_0$ which is the border curve of solitons appearance, while $Q' \approx 20V_0$ which is the border curve of soliton stability. In Fig.4, we depict roughly the parametric condition of interaction strength and the depth of optical

lattices for solitons to occur. We hope that our present study can offer an interesting reference to the relevant experimental observation in the future.

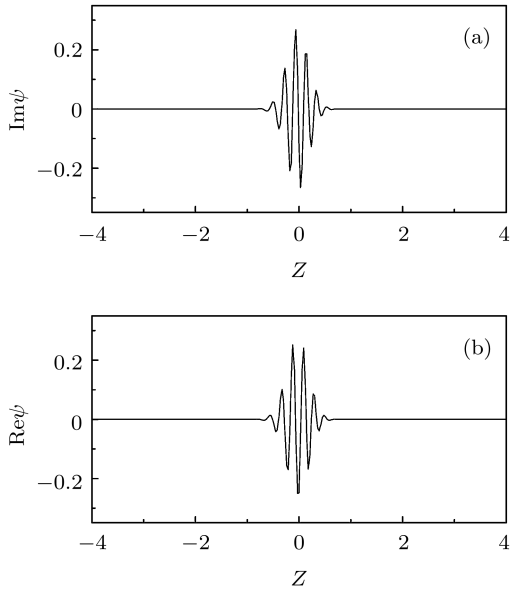


Fig.2. The distribution curves of the imaginary (a) and the real (b) parts of wavefunction of the condensates along the z axis for $V_0 = 0.06$, $Q' = 1.0$ (in the same units as those in Fig.1).

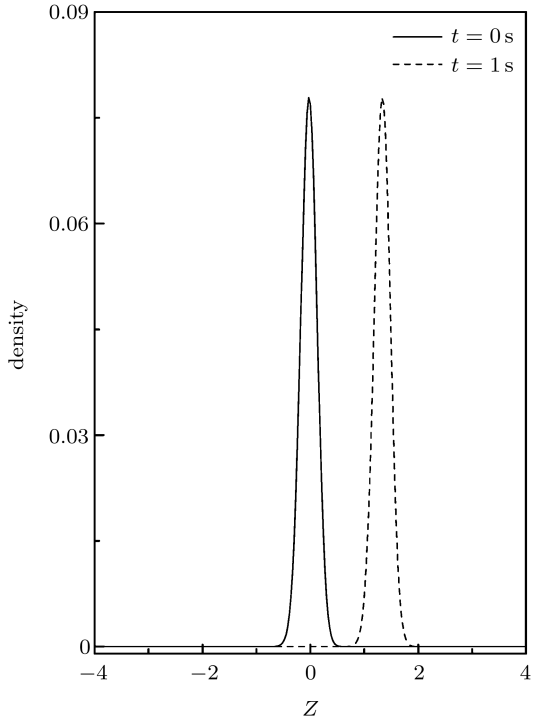


Fig.3. Time evolution of soliton density for the condensates trapped in optical lattices. the parameters used here are the same as in Fig.2.

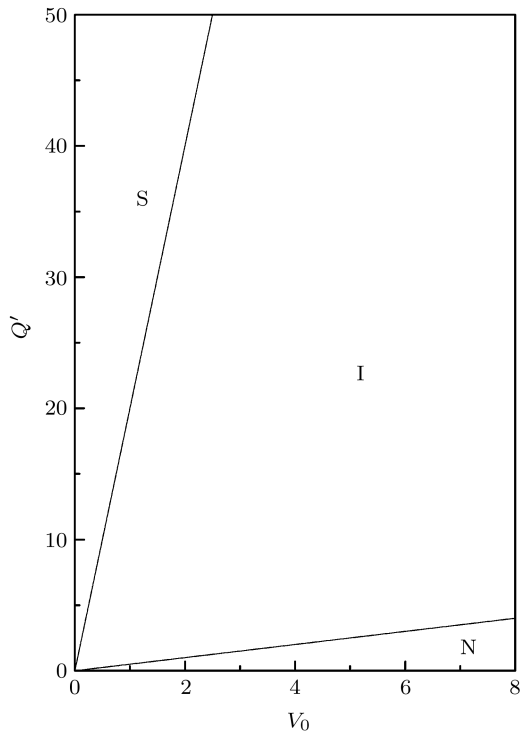


Fig.4. Regions of the parameters for the soliton to occur for the condensates trapped in optical lattices. S and I denote the regions of stable and unstable states respectively, and N represents the non-soliton region.

5. Conclusions

We have developed a systematic analytical approach to the study on the dynamic properties of the linear and the nonlinear excitations for quasi-1D BEC trapped in optical lattices. We derive analytically a novel linear dispersion relation with a periodicity which arises from the periodicity of the optical lattices. In addition, we obtain algebraic soliton solutions of wavefunction of the condensates. By analysing these soliton solutions, we find that the interaction strength has an important effect on soliton dynamic properties of the condensates. In the case of interaction strength less than half the depth of optical lattice the solitons disappear, however, under the interaction strength not less than half the depth of lattices solitons appear. And the stability of the solitons is determined by both the strength of interatomic interaction and the depth of optical lattice. For a fixed depth of lattices, the instability of solitons appears in the case in which the interaction strength is on the order of or slightly higher than the depth of lattices, however, the stable state exists when the interaction strength is much higher than the depth of lattice.

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