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Preparation of cluster state in large detuned cavity*

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An experimentally realizable physical scheme for preparing multiqubit cluster states from a large detuned atom–cavity interaction is proposed. The scheme is free of any type of measurement and insensitive to the cavity decay, and the cavity field is only virtually excited so that there is no information exchanging between atom and cavity. The time required for the gate operations is very short, which is important for decoherence. We also discuss the experimental feasibility of our scheme.

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Quantum mechanics provides novel features for information processing and extends its capabilities beyond the classical limit. Application of the superposition principle to a composite system leads to the essential concept of entanglement, which is one of the most striking features of quantum mechanics. Two or more quantum systems are entangled when it is impossible to describe their physical properties by means of a direct product of their respective density operators. This novel feature also provides a possibility of manipulating entanglement nonlocality. Thus, the entanglement is comprehensively applied to many aspects of quantum information theory such as quantum dense coding (QDC),[1] quantum key distribution (QKD),[2] quantum secret sharing (QSS),[3] quantum teleportation,[4] quantum algorithm,[5] and so on. So the generation of entangled state via various kinds of technologies attracts a great many researchers.

Recently, Briegel and Raussendorf[6] introduced a new class of N-qubit entangled states, the cluster states, from which it is sufficient to simulate any arbitrary quantum logic operation with only local single-qubit measurements. When $N = 2$ and 3, the cluster states are trivial as they are locally equivalent to Bell and GHZ states, respectively. For $N > 3$, the states exhibit a stronger entanglement property, which is harder to destroy by local operations than the corresponding GHZ state. The cluster states are also an essential resource for one-way quantum computing which opens a door for constructing reliable quantum computers by single-qubit measurements[7] and realizing quantum communication.[8] Up to now, a number of applications using cluster states in quantum computation have been proposed.[9] Thus the preparation of the cluster states has become a new research topic and schemes for the generation of the cluster states have been proposed in the context of trapped ions,[10] cavity QED[11] and the superconducting qubit.[12]

In this paper, we propose an alternative scheme for generating N-qubit cluster states by using two-qubit quantum phase gate (QPG) operation from two identical atoms interacting with the quantized field and respective classical microwave pulses. Three-level atoms are trapped in a high-Q optical cavity. The present scheme works in the large detuning regime and is also insensitive to the cavity decay. In addi-

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tion, no information exchanges between the atoms and the cavity mode since the cavity field is only virtually excited.

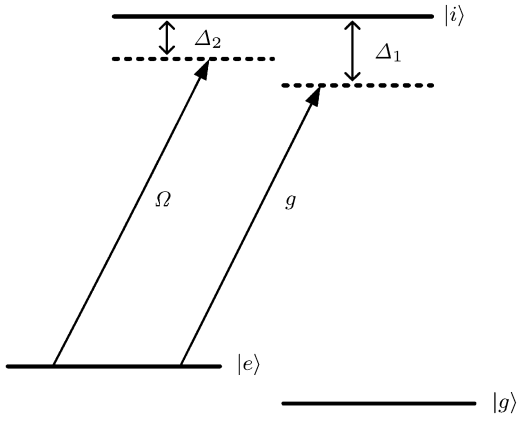


Fig.1. Relevant atomic energy-level configuration and the corresponding transitions in our scheme. $|e\rangle$ and $|g\rangle$ are stable ground states, and $|i\rangle$ is an excited state. The transition $|e_j\rangle \leftrightarrow |i_j\rangle$ is off-resonantly driven by a weak external field and a cavity mode.

We first introduce the Hamiltonian of the cavity QED system. We consider two identical three-level atoms simultaneously interacting with a single-mode cavity and driven by a classical field as shown in Fig.1. The atom transition $|i\rangle \leftrightarrow |e\rangle$ is coupled respectively with the single quantized cavity mode and classical field with coupling constants g and Ω, and the detunings of atom–cavity and atom–classical field interaction being Δ_1 and Δ_2, respectively. The atomic ground level $|g\rangle$ is not affected by the classical and cavity fields. Initially, the cavity is prepared in a vacuum state. Under the large detuning condition ($\Delta_1 \gg g$) and the action of weak external driving field, the full effective Hamiltonian for the dynamics of the system can be written as[13]

$$H = -\frac{g^2}{\Delta_1} S_+ S_- + \Omega(S_+ \mathrm{e}^{-\mathrm{i}\Delta_2 t} + S_- \mathrm{e}^{\mathrm{i}\Delta_2 t}), \qquad (1)$$

where $S_+ = \sum_{j=1,2} |i\rangle_j\langle e|$ and $S_- = \sum_{j=1,2} |e\rangle_j\langle i|$. With respect to two-qubit phase gate, adjust the parameters Ω and Δ_2 such as the following conditions are satisfied:

$$\frac{g^2}{\Delta_1} \gg \sqrt{2}\Omega \quad \text{and} \quad \Delta_2 = -2\frac{g^2}{\Delta_1}. \qquad (2)$$

It is easy to find that the effective operation of the system depends on the number of atoms participating in the gate.[13] Then we have the reduced effective Hamiltonian

$$H_{\text{eff}} = \sqrt{2}\Omega|\varphi_{2,1}\rangle_j\langle\varphi_{2,0}| + \text{H.c.} \qquad (3)$$

for such an initial state that all atoms are in the state $|g\rangle$, where the state $|\varphi_{m,n}\rangle$ describes n atoms transferring into the state $|i\rangle$ from $|e\rangle$. This interaction offers a square-root improvement of resources compared with the performance of a multiqubit gate with auxiliary qubits.[13] For other initial states, the effective interaction to the first order is dispersive as indicated from expression (2). When we choose the interaction time $t = \pi/\sqrt{2}\Omega$, the initial state $|e\rangle_1|e\rangle_2$ will acquire an $\exp(\mathrm{i}\pi)$ phase factor, while other states in computational bases remain unchanged as shown in the following:

$$|g\rangle_1|e\rangle_2 \to |g\rangle_1|e\rangle_2, \qquad (4a)$$
$$|e\rangle_1|g\rangle_2 \to |e\rangle_1|g\rangle_2, \qquad (4b)$$
$$|g\rangle_1|g\rangle_2 \to |g\rangle_1|g\rangle_2, \qquad (4c)$$
$$|e\rangle_1|e\rangle_2 \to -|e\rangle_1|e\rangle_2. \qquad (4d)$$

In this way, we have realized a two-qubit controlled phase gate. Obviously, quantum logic gate operations have been performed in the subspace spanned by $|e\rangle$ and $|g\rangle$ which can be used to store quantum information. As the upper excited state does not populate in the operation, the decoherence due to energy relaxation of the atomic levels is minimized.

Now, we consider the generation of the cluster states with the above gate operation. Firstly, we consider the case of 2-qubit cluster states. Initially, atoms 1 and 2 are prepared in the state $1/\sqrt{2}(|e\rangle + |g\rangle)$ and the cavity in vacuum state $|0\rangle_c$. Thus the state of the quantum system is

$$|\psi\rangle = \frac{1}{2}\big(|e\rangle_1 + |g\rangle_1\big)\big(|e\rangle_2 + |g\rangle_2\big)|0\rangle_c. \qquad (5)$$

Now we implement the process of the above 2-qubit quantum phase gate operation on atoms 1 and 2 with the assistance of the cavity. After the gate operation the state of the quantum system evolves to

$$|\psi\rangle = \frac{1}{2}(|e\rangle_1\sigma_z^2 + |g\rangle_1)(|e\rangle_2 + |g\rangle_2), \qquad (6)$$

where $\sigma_z^2 = |g\rangle_2\langle g| - |e\rangle_2\langle e|$, and we have omitted the state of the cavity which is now in the vacuum state. Thus, we have realized a standard two-qubit cluster

state.

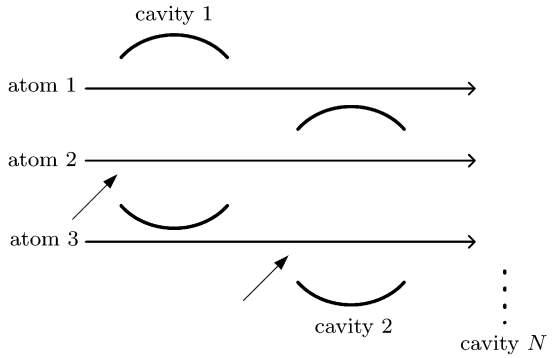


Fig.2. Schematic illustration of the processing. Firstly, atoms 1 and 2 cross cavity 1. Secondly, atoms 2 and 3 cross cavity 2. By analogy, we generalize the N-qubit cluster states. Every cavity is the same in our scheme. The short arrow represents the weak external driving field. The long arrow denotes the atomic moving direction.

Next, we consider N-qubit cluster states ($N \geq 3$), of which the process is shown in Fig.2. The state of the system is initially prepared in the state of

$$|\psi\rangle = \frac{1}{2^{N/2}} \bigotimes_{j=1}^{N} (|e\rangle_j + |g\rangle_j)\,|0\rangle_c\,. \tag{7}$$

Firstly, let atoms 1 and 2 simultaneously cross a high-Q cavity. They will undergo the evolution of Eq.(4) without interaction with any other atom. This leads Eq.(7) to

$$\begin{aligned}|\psi\rangle =& \frac{1}{2^{N/2}}(|e\rangle_1\sigma_z^2 + |g\rangle_1) \\ & \bigotimes_{j=2}^{N}(|e\rangle_j + |g\rangle_j)|0\rangle_c. \end{aligned}\tag{8}$$

Then, in the same way, we let atoms 2 and 3 undergo the above process, which leads Eq.(8) to

$$\begin{aligned}|\psi\rangle =& \frac{1}{2^{N/2}}(|e\rangle_1\,\sigma_z^2 + |g\rangle_1)(|e\rangle_2\,\sigma_z^3 \\ & + |g\rangle_2) \bigotimes_{j=3}^{N}(|e\rangle_j + |g\rangle_j)\,|0\rangle_c\,. \end{aligned}\tag{9}$$

For atom j ($j < N$), we repeat the above procedures for generating a bipartite cluster state on the atoms j and $j+1$ and the cavity (these selective interactions can be embodied via number of atoms) and end the process when $j = N$. Then we obtain N-qubit cluster states

$$|\psi\rangle = \frac{1}{2^{N/2}} \bigotimes_{j=1}^{N} (|e\rangle_j\,\sigma_z^{j+1} + |g\rangle_j), \tag{10}$$

where $\sigma_z^{j+1} \equiv 1$ and the state of the cavity mode has been omitted. The above states are the N qubit cluster states.

Before concluding, we give a brief discussion about the experimental feasibility of our scheme. Among varieties of systems being explored for hardware implementations for the preparation of all kinds of multipartite states, cavity QED system is the most favoured system because it manifests advantage when subjected to the coherent manipulations. The atoms can be detected in a selective and sensitive way by field ionization. The time constants involved are long enough to realize controlled and complex sequences. The interaction time can be perfectly controlled by a velocity selector.[14] The strong-coupling conditions are readily fulfilled.[14] In our scheme we consider caesium atoms[15] or calcium ions[16] trapped in optical cavity, whose feasibility in the quantum information context has been widely demonstrated in experiment. The radiative time is about $T_r = 3 \times 10^{-2}\,\mathrm{s}$, and the coupling constant is $g = 2\pi \times 24\,\mathrm{GHz}$.[14] Within the current technology, a quality factor of a cavity, Q, can reach 3×10^8, thus the reached atom–cavity interaction time $T < 10^{-4}\,\mathrm{s}$. Then the time required to complete the whole procedure is on the order of $10^{-3}\,\mathrm{s}$. Obviously, this time is much shorter than the atomic radiative time T_r. Meanwhile it is noted that the atomic state evolution is independent of the cavity state. So, the current scheme may be realizable by using the current cavity technology.

In summary, we have proposed a simple scheme for generalizing N-qubit cluster states in cavity QED. Unlike any other existing scheme, our scheme has the following advantages: (1) The atomic state evolution is independent of the cavity state. It can avoid the quantum information exchanging between atoms and cavity mode, and it also means that the present scheme is insensitive to the cavity decay. (2) Quantum logic gate operations have been performed in the subspace spanned by $|e\rangle$ and $|g\rangle$, therefore, the decoherence due to energy relaxation of the atomic upper levels is minimized. (3) The coherence time of the atomic ground levels $|e\rangle$ and $|g\rangle$ is long so that these states can be used to store quantum information. On the other hand, our present scheme suggested here is simple and highly feasible experimentally under current technology.

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