Vol 17 No 4, April 2008
1674-1056/2008/17(04)/1156-09

Chinese Physics B

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New exact periodic solutions to (2+1)-dimensional dispersive long wave equations*

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(Received 24 June 2007; revised manuscript received 19 October 2007)

In this paper, we make use of the auxiliary equation and the expanded mapping methods to find the new exact periodic solutions for (2+1)-dimensional dispersive long wave equations in mathematical physics, which are expressed by Jacobi elliptic functions, and obtain some new solitary wave solutions ($m \to 1$). This method can also be used to explore new periodic wave solutions for other nonlinear evolution equations.

Keywords: auxiliary equation method, expanded mapping method, (2+1)-dimensional dispersive long wave equations, periodic wave solutions

PACC: 0340K, 0290

1. Introduction

In recent years, noticeable progress has been made in constructing the exact solutions of nonlinear evolution equations. When exact solutions exist, the phenomena modelled by these nonlinear evolution equations can be better understood with the help of these solutions, and they are also useful for analysing the stability of these solutions and checking the numerical analysis for these nonlinear evolution equations. Several powerful methods have been proposed to construct the exact solutions for nonlinear partial differential equations, such as homogeneous balance method,[1−3] the tanh function method,[4−6] sine-cosine method,[7] Jacobi elliptic function expansion method,[8,9] and auxiliary equation method.[10,11] Very recently, the mapping method [12−14] has been developed to obtain the periodic solutions of nonlinear evolution equations without much extra effort and repetitive calculations.

This paper is concerned with the (2+1)-dimensional dispersive long wave equations and gives some new periodic wave solutions with the help of the mapping method. Long wave in shallow water is a subject of broad interest and has a long, colourful history. In this paper, we consider the (2+1)-dimensional dispersive long wave equations in the following form:

$$u_{yt} + v_{xx} + u_x u_y + u u_{xy} = 0, \tag{1a}$$

$$v_t + u_x + v u_x + u v_x + u_{xxy} = 0. \tag{1b}$$

Equations (1a) and (1b) are obtained first by Boiti *et al*[15] and have Kac–Moody–Virasoro type[16] and general symmetry algebras with some functions.[17] Some kinds of solutions to the (2+1)-dimensional dispersive long wave equations were given in Refs.[18,19]. A good understanding of the solutions for Eqs.(1a) and (1b) is very helpful to coastal and civil engineers in applying the nonlinear water model to coastal harbor design. Here we will extend the mapping method and seek for some new periodic wave solutions.

2. Method

Consider a given nonlinear evolution equation

$$N\left(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots\right) = 0. \tag{2}$$

We seek after the solutions of Eq.(2) by taking

$$u(x,t) = u(\xi), \quad v(x,t) = v(\xi),$$
$$\xi = kx + ly + ct, \tag{3}$$

where k, l and c are wave parameters to be determined later.

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Substituting expressions (3) into Eq.(2) yields an ordinary differential equation

$$F(u, u', u'', \ldots) = 0, \tag{4}$$

where prime ($'$) means '$\frac{\mathrm{d}}{\mathrm{d}\xi}$'. We assume that Eq.(4) has the solutions in the following form:

$$u(\xi) = \sum_{i=0}^{n} a_i f^i(\xi) + \sum_{i=1}^{n} b_i f^{-i}(\xi) + \sum_{i=2}^{n} c_i f^{i-2}(\xi) f'(\xi) + \sum_{i=1}^{n} d_i f^{-i}(\xi) f'(\xi), \tag{5}$$

where a_n, b_n, c_n, and d_n are constants to be determined later and positive integer n can be determined by considering the homogeneous balance between governing nonlinear terms and the highest order derivatives of u in Eq.(4), and $f(\xi)$ expresses the solutions of the following auxiliary ordinary differential equation

$$[f'(\xi)]^2 = pf^4(\xi) + qf^2(\xi) + r, \tag{6}$$

with p, q and r being real parameters to be chosen and determined by $f(\xi)$. Equation (6) is an elliptic equation which has the solutions constructed by Jacobi elliptic function. Furthermore, these include hyperbolic function when $m \to 1$, and trigonometric function when $m \to 0$. Relations between values of (p, q, r) and corresponding $f(\xi)$ in Eq.(6) are obtained as the following mapping method:

$f(\xi)$	p	q	r,
$\mathrm{sn}\xi$; $\mathrm{cd}\xi = \frac{\mathrm{cn}\xi}{\mathrm{dn}\xi}$	m^2	$-(m^2+1)$	1,
$\mathrm{cn}\xi$	$-m^2$	$2m^2-1$	$1-m^2$,
$\mathrm{dn}\xi$	-1	$2-m^2$	m^2-1,
$\mathrm{ns}\xi = \frac{1}{\mathrm{sn}\xi}$; $\mathrm{dc}\xi = \frac{\mathrm{dn}\xi}{\mathrm{cn}\xi}$	1	$-(m^2+1)$	m^2,
$\mathrm{nc}\xi = \frac{1}{\mathrm{cn}\xi}$	$1-m^2$	$2m^2-1$	$-m^2$,
$\mathrm{nd}\xi = \frac{1}{\mathrm{dn}\xi}$	m^2-1	$2-m^2$	-1,
$\mathrm{cs}\xi = \frac{\mathrm{cn}\xi}{\mathrm{sn}\xi}$	1	$2-m^2$	$1-m^2$,
$\mathrm{sc}\xi = \frac{\mathrm{sn}\xi}{\mathrm{cn}\xi}$	$1-m^2$	$2-m^2$	1,
$\mathrm{sd}\xi = \frac{\mathrm{sn}\xi}{\mathrm{dn}\xi}$	$m^2(m^2-1)$	$2m^2-1$	1,
$\mathrm{ds}\xi = \frac{\mathrm{dn}\xi}{\mathrm{sn}\xi}$	1	$2m^2-1$	$m^2(m^2-1)$,
$m\mathrm{cn}\xi \pm \mathrm{dn}\xi$	$-\frac{1}{4}$	$\frac{1+m^2}{2}$	$-\frac{(1-m^2)^2}{4}$,
$\mathrm{ns}\xi \pm \mathrm{cs}\xi$, $\frac{\mathrm{cn}\xi}{\sqrt{1-m^2}\mathrm{sn}\xi \pm \mathrm{dn}\xi}$	$\frac{1}{4}$	$\frac{1-2m^2}{2}$	$\frac{1}{4}$,
$\mathrm{nc}\xi \pm \mathrm{sc}\xi$	$\frac{1-m^2}{4}$	$\frac{1+m^2}{2}$	$\frac{1-m^2}{4}$,
$\mathrm{ns}\xi \pm \mathrm{ds}\xi$	$\frac{1}{4}$	$\frac{1-m^2}{2}$	$\frac{m^4}{4}$,
$\mathrm{sn}\xi \pm \mathrm{icn}\xi$; $\frac{\mathrm{dn}\xi}{\sqrt{1-m^2}\mathrm{sn}\xi \pm \mathrm{cn}\xi}$	$\frac{m^2}{4}$	$\frac{m^2-2}{2}$	$\frac{m^2}{4}$,
$m\mathrm{sn}\xi \pm \mathrm{idn}\xi$; $\frac{\mathrm{sn}\xi}{1 \pm \mathrm{cn}\xi}$	$\frac{1}{4}$	$\frac{1-2m^2}{2}$	$\frac{1}{4}$,

$\dfrac{\mathrm{dn}\xi}{\sqrt{\dfrac{m^2-1}{m^2}}\pm\mathrm{cn}\xi}$	$\dfrac{1}{4m^2}$	$\dfrac{1-2m^2}{2}$	$\dfrac{m^2}{4}$,
$\dfrac{\mathrm{sn}\xi}{1\pm\mathrm{dn}\xi}$	$\dfrac{m^2}{4}$	$\dfrac{m^2-2}{2}$	$\dfrac{1}{4}$,
$\dfrac{\mathrm{dn}\xi}{1\pm m\mathrm{sn}\xi}$	$\dfrac{m^2-1}{4}$	$\dfrac{1+m^2}{2}$	$\dfrac{m^2-1}{4}$,
$\dfrac{\mathrm{cn}\xi}{1\pm\mathrm{sn}\xi}$	$\dfrac{1-m^2}{4}$	$\dfrac{1+m^2}{2}$	$\dfrac{1-m^2}{4}$,
$\dfrac{\mathrm{sn}\xi}{\mathrm{cn}\xi\pm\mathrm{dn}\xi}$	$\dfrac{(1-m^2)^2}{4}$	$\dfrac{1+m^2}{2}$	$\dfrac{1}{4}$,
$\dfrac{\mathrm{cn}\xi}{\sqrt{1-m^2}\pm\mathrm{dn}\xi}$	$\dfrac{m^4}{4}$	$\dfrac{m^2-2}{2}$	$\dfrac{1}{4}$,
$\dfrac{\mathrm{cn}\xi\mathrm{dn}\xi}{\mathrm{sn}\xi}$	1	$2(m^2+1)$	$(1-m^2)^2$,
$\dfrac{\mathrm{sn}\xi}{\mathrm{cn}\xi\mathrm{dn}\xi}$	$(1-m^2)^2$	$2(m^2+1)$	1,
$\dfrac{\mathrm{sn}\xi\mathrm{dn}\xi}{\mathrm{cn}\xi}$	1	$2(1-2m^2)$	1,
$\dfrac{\mathrm{cn}\xi}{\mathrm{sn}\xi\mathrm{dn}\xi}$	1	$2(1-2m^2)$	1,
$\dfrac{\mathrm{sn}\xi\mathrm{cn}\xi}{\mathrm{dn}\xi}$	m^4	$2(m^2-2)$	1,
$\dfrac{\mathrm{dn}\xi}{\mathrm{sn}\xi\mathrm{cn}\xi}$	1	$2(m^2-2)$	m^4,

where $\mathrm{i}^2=-1$. Substituting Eq.(5) into Eq.(4), and using Eq.(6), the left-hand of Eq.(4) can be converted into a polynomial about $f^i(\xi)$ ($i=\ldots,-2,-1,0,1,2,\ldots$). Setting the coefficients of the different powers of $f^i(\xi)$ to be zero yields a set of algebraic equations for a_0, a_1, $\cdots$, a_n, b_1, $\cdots$, b_n and ξ. Solving the algebraic equations, we have a_0, a_1, …, a_n, b_1, …, b_n expressed by p, q, r. Substituting these results into Eq.(5) and using the foregoing mapping, we can obtain the periodic wave solutions of Eq.(4).

3. Application of the method

Substituting expression (3) into Eqs.(1a) and (1b), integrating Eq.(1a) twice and integrating Eq.(1b) once, and setting the integration constant to be zero, yield

$$lcu+\frac{1}{2}klu^2+k^2v=0, \tag{7a}$$

$$ku+kuv+k^2lu''+cv=0. \tag{7b}$$

Considering homogeneous balance, the solutions of Eqs.(7a) and (7b) can be assumed to be

$$u(\xi)=A_0+A_1f(\xi)+B_1\frac{1}{f(\xi)}+D_1\frac{f'(\xi)}{f(\xi)}, \tag{8a}$$

$$\begin{aligned}v(\xi)=&\,a_0+a_1f(\xi)+b_1\frac{1}{f(\xi)}+d_1\frac{f'(\xi)}{f(\xi)}\\&+a_2f^2(\xi)+b_2\frac{1}{f^2(\xi)}\\&+c_2f'(\xi)+d_2\frac{f'(\xi)}{f^2(\xi)}.\end{aligned} \tag{8b}$$

Then substituting Eqs.(8a) and (8b) into Eqs.(7a) and (7b) with Eq.(6) and setting the coefficients of

the polynomial to be zero, we can obtain the algebraic equations. Solving the algebraic equations, we can determine the coefficients

case 1

$$A_0=-\frac{c}{k},\ A_1=\pm 2k\sqrt{p},\ B_1=0,$$
$$D_1=0,\ a_0=\frac{c^2l}{2k^3},\ a_1=0,\ a_2=-2klp,$$
$$b_1=0,\ b_2=0,\ c_2=0,\ d_1=0,\ d_2=0; \tag{9}$$

case 2

$$A_0=-\frac{c}{k},\ A_1=0,\ B_1=\pm 2k\sqrt{r},\ D_1=0,$$
$$a_0=\frac{c^2l}{2k^3},\ a_1=0,\ a_2=0,\ b_1=0,$$
$$b_2=-2klr,\ c_2=0,\ d_1=0,\ d_2=0; \tag{10}$$

case 3

$$A_0=-\frac{c}{k},\ A_1=\pm 2k\sqrt{p},\ B_1=\pm 2k\sqrt{r},$$
$$D_1=0,\ a_0=\frac{c^2l}{2k^3}\pm 4kl\sqrt{p}\sqrt{r},\ a_1=0,$$
$$a_2=-2klp,\ b_1=0,\ b_2=-2klr,\ c_2=0,$$
$$d_1=0,\ d_2=0; \tag{11}$$

case 4

$$A_0=\pm\sqrt{2c}\sqrt[4]{q},\ A_1=0,\ B_1=0,$$
$$D_1=\pm\frac{\sqrt{2c}}{\sqrt[4]{q}},\ a_0=0,\ a_1=0,\ a_2=\pm\frac{\sqrt{2c}lp}{\sqrt[4]{q}},$$
$$b_1=0,\ b_2=\pm\frac{\sqrt{2c}lr}{\sqrt[4]{q}},\ c_2=0,\ d_1=0,$$
$$d_2=0,\ k=\pm\frac{\sqrt{2c}}{2\sqrt[4]{q}}; \tag{12}$$

case 5

$$A_0=\pm\sqrt{c}\sqrt[4]{(q\pm 6\sqrt{p}\sqrt{r})},$$
$$A_1=\pm\frac{\sqrt{c}\sqrt{p}}{\sqrt[4]{(q\pm 6\sqrt{p}\sqrt{r})}},$$
$$B_1=\pm\frac{\sqrt{c}\sqrt{r}}{\sqrt[4]{(q\pm 6\sqrt{p}\sqrt{r})}},$$
$$D_1=\pm\frac{\sqrt{c}}{\sqrt[4]{(q\pm 6\sqrt{p}\sqrt{r})}},$$
$$a_0=\pm\frac{2\sqrt{c}l\sqrt{p}\sqrt{r}}{\sqrt[4]{(q\pm 6\sqrt{p}\sqrt{r})}},\ a_1=0,$$
$$a_2=\pm\frac{\sqrt{c}lp}{\sqrt[4]{(q\pm 6\sqrt{p}\sqrt{r})}},\ b_1=0,$$
$$b_2=\pm\frac{\sqrt{c}lr}{\sqrt[4]{(q\pm 6\sqrt{p}\sqrt{r})}},$$
$$c_2=\pm\frac{\sqrt{c}l\sqrt{p}}{\sqrt[4]{(q\pm 6\sqrt{p}\sqrt{r})}},$$
$$d_1=0,\ d_2=\pm\frac{\sqrt{c}l\sqrt{r}}{\sqrt[4]{(q\pm 6\sqrt{p}\sqrt{r})}},$$
$$k=\pm\frac{\sqrt{c}}{\sqrt[4]{(q\pm 6\sqrt{p}\sqrt{r})}}. \tag{13}$$

We can see that all of the coefficients of $f'(\xi)$ are zero in cases 1–3, so the simplified form of Eq.(5) can be described as follows:

$$u(\xi)=\sum_{i=0}^{n}a_i f^i(\xi)+\sum_{i=1}^{n}b_i f^{-i}(\xi),$$

and its application can be found in Refs.[20–22].

Substituting expression (9) into Eqs.(8a) and (8b), the first concentration formulas of travelling wave solution of Eqs.(1a) and (1b) are

$$u(\xi)=-\frac{c}{k}\pm 2k\sqrt{p}f(\xi), \tag{14a}$$
$$v(\xi)=\frac{c^2l}{2k^3}-2klpf^2(\xi), \tag{14b}$$

where $\xi=kx+ly+ct$, $k\neq 0$, and $l\neq 0$.

Substituting expression (10) into Eqs.(8a) and (8b), the second concentration formulas of travelling wave solution of Eqs.(1a) and (1b) are

$$u(\xi)=-\frac{c}{k}\pm 2k\sqrt{r}\frac{1}{f(\xi)}, \tag{15a}$$
$$v(\xi)=\frac{c^2l}{2k^3}-2klr\frac{1}{f^2(\xi)}, \tag{15b}$$

where $\xi=kx+ly+ct$, $k\neq 0$, and $l\neq 0$.

Substituting expression (11) into Eqs.(8a) and (8b), the third concentration formulas of travelling wave solution of Eqs.(1a) and (1b) are

$$u(\xi)=-\frac{c}{k}\pm 2k\sqrt{p}f(\xi)\pm 2k\sqrt{r}\frac{1}{f(\xi)}, \tag{16a}$$
$$v(\xi)=\frac{c^2l}{2k^3}\pm 4kl\sqrt{p}\sqrt{r}-2klpf^2(\xi)-2klr\frac{1}{f^2(\xi)}, \tag{16b}$$

where $\xi=kx+ly+ct$, $k\neq 0$, and $l\neq 0$.

Substituting expression (12) into Eqs.(8a) and (8b), the fourth concentration formulas of travelling wave solution of Eqs.(1a) and (1b) are

$$u(\xi) = \pm\sqrt{2c}\sqrt[4]{q} \pm \frac{\sqrt{2c}}{\sqrt[4]{q}}\frac{f'(\xi)}{f(\xi)}, \tag{17a}$$

$$v(\xi) = \pm\frac{\sqrt{2c}lp}{\sqrt[4]{q}}f^2(\xi) \pm \frac{\sqrt{2c}lr}{\sqrt[4]{q}}\frac{1}{f^2(\xi)}, \tag{17b}$$

where $\xi = kx + ly + ct$, $k = \pm\dfrac{\sqrt{2c}}{2\sqrt[4]{q}}$, and $l \neq 0$.

Substituting expression (13) into Eqs.(8a) and (8b), the fifth concentration formulas of travelling wave solution of Eqs.(1a) and (1b) are

$$u(\xi) = \pm\sqrt{c}\sqrt[4]{(q \pm 6\sqrt{p}\sqrt{r})} \pm \frac{\sqrt{c}\sqrt{p}}{\sqrt[4]{(q \pm 6\sqrt{p}\sqrt{r})}}f(\xi) \pm \frac{\sqrt{c}\sqrt{r}}{\sqrt[4]{(q \pm 6\sqrt{p}\sqrt{r})}}\frac{1}{f(\xi)} \pm \frac{\sqrt{c}}{\sqrt[4]{(q \pm 6\sqrt{p}\sqrt{r})}}\frac{f'(\xi)}{f(\xi)}, \tag{18a}$$

$$v(\xi) = \pm \frac{2\sqrt{c}l\sqrt{p}\sqrt{r}}{\sqrt[4]{(q \pm 6\sqrt{p}\sqrt{r})}} \pm \frac{\sqrt{c}lp}{\sqrt[4]{(q \pm 6\sqrt{p}\sqrt{r})}}f^2(\xi) \pm \frac{\sqrt{c}lr}{\sqrt[4]{(q \pm 6\sqrt{p}\sqrt{r})}}\frac{1}{f^2(\xi)} \pm \frac{\sqrt{c}l\sqrt{p}}{\sqrt[4]{(q \pm 6\sqrt{p}\sqrt{r})}}f'(\xi)$$
$$\pm \frac{\sqrt{c}l\sqrt{r}}{\sqrt[4]{(q \pm 6\sqrt{p}\sqrt{r})}}\frac{f'(\xi)}{f^2(\xi)}, \tag{18b}$$

where $\xi = kx + ly + ct$,$k = \pm\dfrac{\sqrt{c}}{\sqrt[4]{(q \pm 6\sqrt{p}\sqrt{r})}}$, and $l \neq 0$.

With the aid of mapping and the concentration formulas (14a)–(18b) obtained previously, we can obtain the periodic wave solutions expressed by the Jacobi functions. Some similar forms of the solutions to expressions (14–16) can be found in Refs.[21, 22], but the solutions to Eqs.(17a)–(18b) have not been found from the previous studies.

3.1. The first group solutions

Choosing $f(\xi) = \dfrac{\mathrm{sn}\xi}{1 \pm \mathrm{dn}\xi}$, then $p = \dfrac{m^2}{4}$, $q = \dfrac{m^2 - 2}{2}$, and $r = \dfrac{1}{4}$, and inserting them into Eqs.(14a) and (14b), we can obtain the following periodic wave solutions of Eqs.(1a) and (1b) associated with case (1):

$$u_1(\xi) = -\frac{c}{k} \pm mk\frac{\mathrm{sn}\xi}{1 + \varepsilon\mathrm{dn}\xi}, \tag{19a}$$

$$v_1(\xi) = \frac{c^2 l}{2k^3} - \frac{klm^2}{2}\left(\frac{\mathrm{sn}\xi}{1 + \varepsilon\mathrm{dn}\xi}\right)^2, \tag{19b}$$

where $\varepsilon^2 = 1$, $k \neq 0$, and $l \neq 0$. The plots of $u_1(\xi)$ and $v_1(\xi)$ are shown in Figs.1 and 2 at $k = 1$, $l = 1$, $c = 1$, $t = 0$, and $m = 0.75$, and they take '+' in each sign '±', and '1' in each 'ε'.

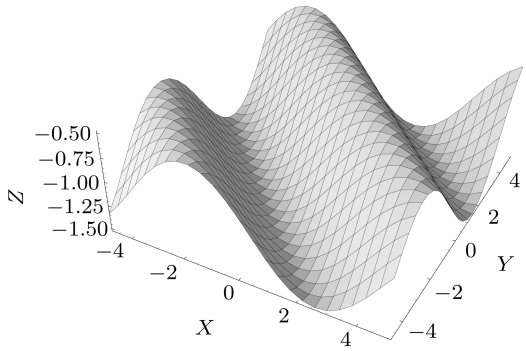


Fig.1. $u_1(\xi)$, the periodic wave solution of Eq.(19a).

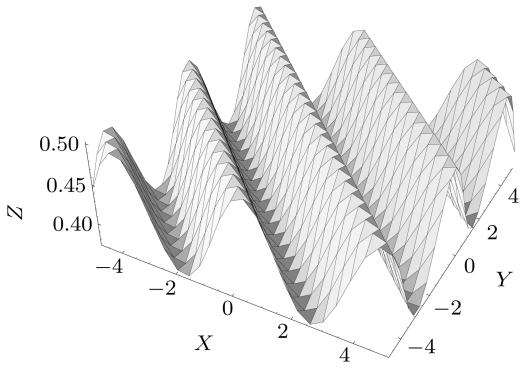


Fig.2. $v_1(\xi)$, the periodic wave solution of Eq.(19b).

When $m \to 1$, we have the bell-type solitary solutions as follows:

$$u_{1a}(\xi) = -\frac{c}{k} \pm k\frac{\tanh\xi}{1 + \varepsilon\mathrm{sech}\xi}, \tag{20a}$$

$$v_{1b}(\xi) = \frac{c^2 l}{2k^3} - \frac{kl}{2}\left(\frac{\tanh\xi}{1 + \varepsilon\mathrm{sech}\xi}\right)^2; \tag{20b}$$

the plots of $u_{1a}(\xi)$ and $v_{1b}(\xi)$ are shown in Figs.3 and 4 at $k=1,\ l=1,\ c=1,\ t=0$, and $m=1$, and they take '+' in each sign '±', and '1' in each 'ε'.

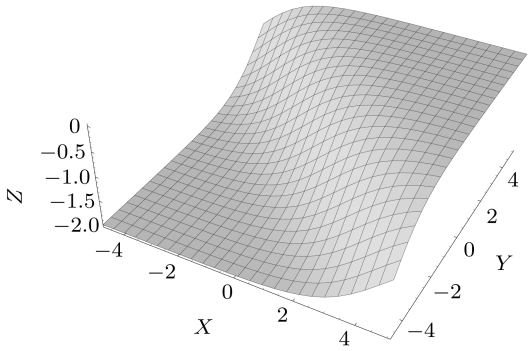


Fig.3. $u_{1a}(\xi)$, the bell-type solitary solution of Eq.(20a).

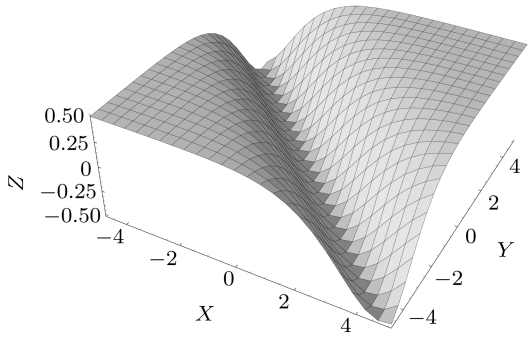


Fig.4. $v_{1b}(\xi)$, the bell-type solitary solution of Eq.(20b).

If inserting the same choice into expressions (16a) and (16b), we can obtain the following periodic wave solutions associated with case (3):

$$u_2(\xi) = -\frac{c}{k} \pm mk\frac{\mathrm{sn}\xi}{1+\varepsilon\mathrm{dn}\xi} \pm k\frac{1+\varepsilon\mathrm{dn}\xi}{\mathrm{sn}\xi}, \quad (21a)$$

$$v_2(\xi) = \frac{c^2 l}{2k^3} \pm klm - \frac{klm^2}{2}\left(\frac{\mathrm{sn}\xi}{1+\varepsilon\mathrm{dn}\xi}\right)^2 - \frac{kl}{2}\left(\frac{1+\varepsilon\mathrm{dn}\xi}{\mathrm{sn}\xi}\right)^2, \quad (21b)$$

where $\varepsilon^2=1$, $k\neq 0$, and $l\neq 0$. When $m\to 1$, we have the bell-type solitary solutions as follows:

$$u_{2a}(\xi) = -\frac{c}{k} \pm k\frac{\tanh\xi}{1+\varepsilon\mathrm{sech}\xi} \pm k\frac{1+\varepsilon\mathrm{sech}\xi}{\tanh\xi}, \quad (22a)$$

$$v_{2b}(\xi) = \frac{c^2 l}{2k^3} \pm kl - \frac{kl}{2}\left(\frac{\tanh\xi}{1+\varepsilon\mathrm{sech}\xi}\right)^2 - \frac{kl}{2}\left(\frac{1+\varepsilon\mathrm{sech}\xi}{\tanh\xi}\right)^2; \quad (22b)$$

the plots of $u_{2a}(\xi)$ and $v_{2b}(\xi)$ are shown in Figs.5 and 6 at $k=1,\ l=1,\ c=1,\ t=0$, and $m=1$, and they take '+' in each sign '±', and '1' in each 'ε'.

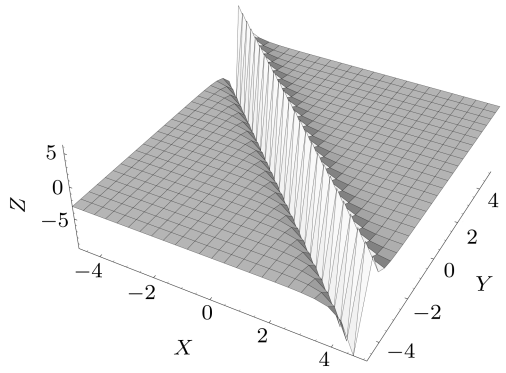


Fig.5. $u_{2a}(\xi)$, the bell-type solitary solution of Eq.(22a).

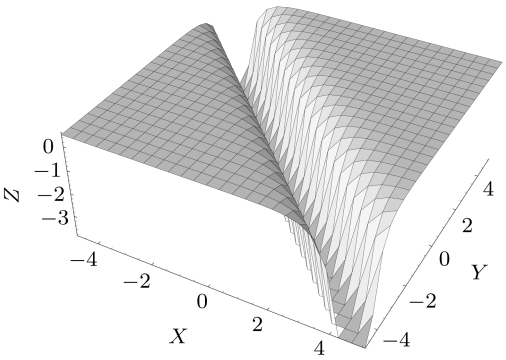


Fig.6. $v_{2b}(\xi)$, the bell-type solitary solution of Eq.(22b).

If inserting the same choice into expressions (18a) and (18b), we can obtain the following periodic wave solutions associated with case (5):

$$\begin{aligned} u_3(\xi) = {} & \pm\sqrt{c}\sqrt[4]{\frac{m^2+3\varepsilon m-2}{2}} \\ & \pm \frac{m\sqrt{c}}{\sqrt[4]{8m^2+24\varepsilon m-16}}\frac{\mathrm{sn}\xi}{1+\varepsilon\mathrm{dn}\xi} \\ & \pm \frac{\sqrt{c}}{\sqrt[4]{8m^2+24\varepsilon m-16}}\frac{1+\varepsilon\mathrm{dn}\xi}{\mathrm{sn}\xi} \\ & \pm \frac{\sqrt[4]{2}\sqrt{c}}{\sqrt[4]{m^2+3\varepsilon m-2}}\frac{\mathrm{cn}\xi}{\mathrm{sn}\xi}, \end{aligned} \quad (23a)$$

$$\begin{aligned} v_3(\xi) = {} & \pm\frac{\sqrt{c}lm}{\sqrt[4]{8m^2+24\varepsilon m-16}} \\ & \pm \frac{\sqrt{c}lm^2}{\sqrt[4]{64m^2+192\varepsilon m-128}}\left(\frac{\mathrm{sn}\xi}{1+\varepsilon\mathrm{dn}\xi}\right)^2 \\ & \pm \frac{\sqrt{c}l}{\sqrt[4]{64m^2+192\varepsilon m-128}}\left(\frac{1+\varepsilon\mathrm{dn}\xi}{\mathrm{sn}\xi}\right)^2 \\ & \pm \frac{\sqrt{c}lm}{\sqrt[4]{8m^2+24\varepsilon m-16}}\frac{\mathrm{cn}\xi}{1+\varepsilon\mathrm{dn}\xi} \\ & \pm \frac{\sqrt{c}l}{\sqrt[4]{8m^2+24\varepsilon m-16}}\frac{\mathrm{cn}\xi(1+\varepsilon\mathrm{dn}\xi)}{sn^2\xi}, \end{aligned} \quad (23b)$$

where $\varepsilon^2 = 1$, $\xi = kx+ly+ct$, $k = \pm\dfrac{\sqrt[4]{2}\sqrt{c}}{\sqrt[4]{m^2+3\varepsilon m-2}}$, and $l \neq 0$; the plots of $u_3(\xi)$ and $v_3(\xi)$ are shown in Figs.7 and 8 at $k = 1$, $l = 1$, $c = 1$, $t = 0$, and $m = 0.75$, and they take '+' in each sign '±', and '1' in each 'ε'.

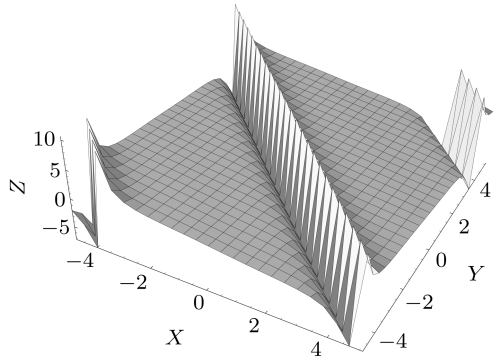


Fig.7. $u_3(\xi)$, the periodic wave solution of Eq.(23a).

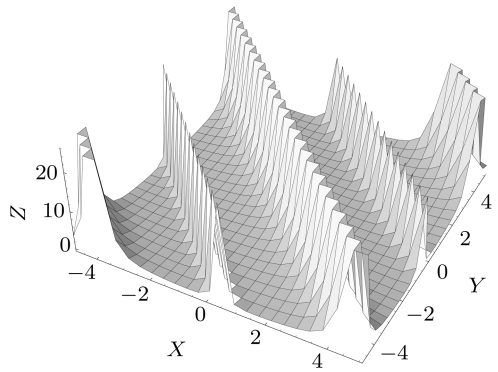


Fig.8. $v_3(\xi)$, the periodic wave solution of Eq.(23b).

When $m \to 1$, we have the bell-type solitary solutions as follows:

$$u_{3a}(\xi) = \pm\sqrt{c}\sqrt[4]{\frac{3\varepsilon-1}{2}} \pm \frac{\sqrt{c}}{\sqrt[4]{24\varepsilon-8}}\frac{\tanh\xi}{1+\varepsilon\mathrm{sech}\xi} \pm \frac{\sqrt{c}}{\sqrt[4]{24\varepsilon-8}}\frac{1+\varepsilon\mathrm{sech}\xi}{\tanh\xi} \pm \frac{\sqrt[4]{2}\sqrt{c}}{\sqrt[4]{3\varepsilon-1}}\frac{\mathrm{sech}\xi}{\tanh\xi}, \tag{24a}$$

$$v_{3b}(\xi) = \pm\frac{\sqrt{cl}}{\sqrt[4]{24\varepsilon-8}} \pm \frac{\sqrt{cl}}{\sqrt[4]{192\varepsilon-64}}\left(\frac{\tanh\xi}{1+\varepsilon\mathrm{sech}\xi}\right)^2 \pm \frac{\sqrt{cl}}{\sqrt[4]{192\varepsilon-64}}\left(\frac{1+\varepsilon\mathrm{sech}\xi}{\tanh\xi}\right)^2 \pm \frac{\sqrt{cl}}{\sqrt[4]{24\varepsilon-8}}\frac{\mathrm{sech}\xi}{1+\varepsilon\mathrm{sech}\xi} \pm \frac{\sqrt{cl}}{\sqrt[4]{24\varepsilon-8}}\frac{\mathrm{sech}\xi(1+\varepsilon\mathrm{sech}\xi)}{\tanh^2\xi}. \tag{24b}$$

3.2. The second group solutions

Choosing $f(\xi) = \dfrac{\mathrm{sn}\xi}{\mathrm{cn}\xi\mathrm{dn}\xi}$, then $p = (1-m^2)^2$, $q = 2(m^2+1)$, and $r = 1$, inserting them into expressions (15a) and (15b), we can obtain the following periodic wave solutions of Eqs.(1a) and (1b) associated with case (2):

$$u_4(\xi) = -\frac{c}{k} \pm 2k\frac{\mathrm{cn}\xi\mathrm{dn}\xi}{\mathrm{sn}\xi}, \tag{25a}$$

$$v_4(\xi) = \frac{c^2l}{2k^3} - 2kl\left(-\frac{\mathrm{cn}\xi\mathrm{dn}\xi}{\mathrm{sn}\xi}\right)^2 \tag{25b}$$

where $k \neq 0$, and $l \neq 0$, when $m \to 1$, we have the bell-type solitary solutions as follows:

$$u_{4a}(\xi) = -\frac{c}{k} \pm 2k\frac{\mathrm{sech}^2\xi}{\tanh\xi}, \tag{26a}$$

$$v_{4b}(\xi) = \frac{c^2l}{2k^3} - 2kl\left(\frac{\mathrm{sech}^2\xi}{\tanh\xi}\right)^2 \tag{26b}$$

If inserting the same choice into expressions (17a) and (17b), we can obtain the following periodic wave solutions associated with case (4):

$$u_5(\xi) = \pm\sqrt{2c}\sqrt[4]{2(m^2+1)} \pm \frac{\sqrt{2c}}{\sqrt[4]{2(m^2+1)}} \times \frac{\mathrm{cn}^2\xi\mathrm{dn}^2\xi + m^2\mathrm{sn}^2\xi cn^2\xi + \mathrm{sn}^2\xi\mathrm{dn}^2\xi}{\mathrm{sn}\xi\mathrm{cn}\xi\mathrm{dn}\xi}, \tag{27a}$$

$$v_5(\xi) = \pm\frac{\sqrt{2c}l(1-m^2)^2}{\sqrt[4]{2(m^2+1)}}\left(\frac{\mathrm{sn}\xi}{\mathrm{cn}\xi\mathrm{dn}\xi}\right)^2 \pm \frac{\sqrt{2c}l}{\sqrt[4]{2(m^2+1)}}\left(\frac{\mathrm{cn}\xi\mathrm{dn}\xi}{\mathrm{sn}\xi}\right)^2, \tag{27b}$$

where $\xi = kx+ly+ct$, $k = \pm\dfrac{\sqrt{2c}}{2\sqrt[4]{2(m^2+1)}}$, and $l \neq 0$; the plots of $u_5(\xi)$ and $v_5(\xi)$ are shown in Figs.9 and 10 at $k = 1$, $l = 1$, $c = 1$, $t = 0$, and $m = 0.75$, and they take '+' in each sign '±', and '1' in each 'ε'.

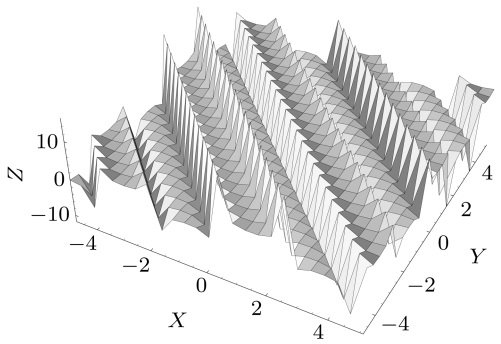


Fig.9. $u_5(\xi)$, the periodic wave solution of Eq.(27a).

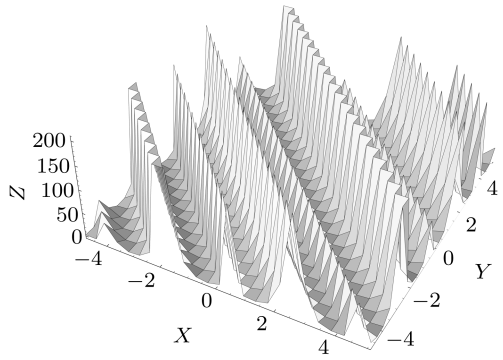


Fig.10. $v_5(\xi)$, the periodic wave solution of Eq.(27b).

When $m \to 1$, we have the bell-type solitary solutions as follows:

$$u_{5a}(\xi) = \pm 2\sqrt{c} \pm \sqrt{c}\frac{\operatorname{sech}^2\xi + (1+m^2)\tanh^2\xi}{\tanh\xi}, \tag{28a}$$

$$v_{5b}(\xi) = \pm\sqrt{c}l\left(\frac{\operatorname{sech}^2\xi}{\tanh\xi}\right)^2. \tag{28b}$$

3.3. The third group solutions

Choosing $f(\xi) = \dfrac{\mathrm{sn}\xi}{\mathrm{dn}\xi}$, then $p = m^2(m^2-1)$, $q = 2m^2-1$, and $r = 1$, inserting them into expressions (17a) and (17b), we can obtain the following periodic wave solutions of Eqs.(1a) and (1b) associated with case (4):

$$u_6(\xi) = \pm\sqrt{2c}\sqrt[4]{2m^2-1} \pm \frac{\sqrt{2c}}{\sqrt[4]{2m^2-1}}\frac{\mathrm{cn}\xi}{\mathrm{sn}\xi}, \tag{29a}$$

$$v_6(\xi) = \pm\frac{\sqrt{2c}lm^2(m^2-1)}{\sqrt[4]{2m^2-1}}\left(\frac{\mathrm{sn}\xi}{\mathrm{dn}\xi}\right)^2 \pm \frac{\sqrt{2c}l}{\sqrt[4]{2m^2-1}}\left(\frac{\mathrm{dn}\xi}{\mathrm{sn}\xi}\right)^2, \tag{29b}$$

where $\xi = kx + ly + ct$, $k = \pm\dfrac{\sqrt{2c}}{2\sqrt[4]{2m^2-1}}$, and $l \neq 0$, and when $m \to 1$, we have the bell-type solitary solutions as follows:

$$u_{6a}(\xi) = \pm\sqrt{2c} \pm \sqrt{2c}\frac{\operatorname{sech}\xi}{\tanh\xi}, \tag{30a}$$

$$v_{6b}(\xi) = \sqrt{2c}l\left(\frac{\operatorname{sech}\xi}{\tanh\xi}\right)^2, \tag{30b}$$

the plots of $u_{6a}(\xi)$ and $v_{6b}(\xi)$ are shown in Figs.11 and 12 at $k=1,\ l=1,\ c=1,\ t=0$, and $m=1$, and they take '+' in each sign '±', and '1' in each 'ε'.

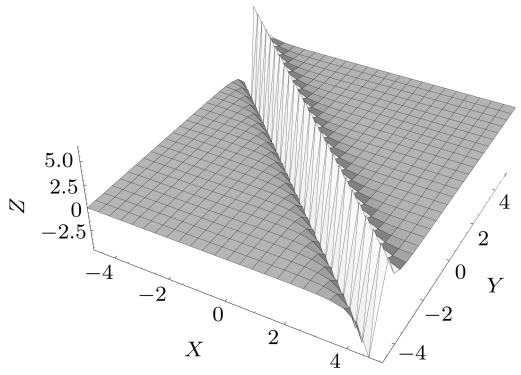


Fig.11. $u_{6a}(\xi)$, the bell-type solitary solution of Eq.(30a).

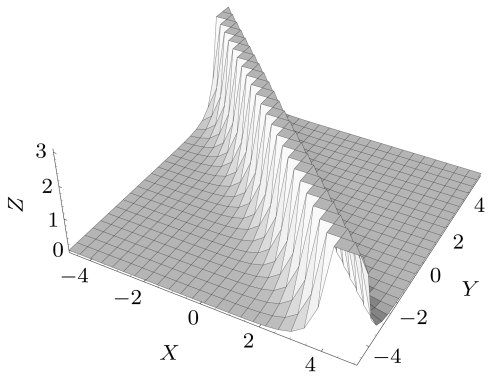


Fig.12. $v_{6b}(\xi)$, the bell-type solitary solution of Eq.(30b).

If inserting the same choice into expressions (18a) and (18b),we can obtain the following periodic wave solutions associated with case (5):

$$u_7(\xi) = \pm\sqrt{c}\sqrt[4]{(2m^2-1\pm 6\sqrt{m^2(m^2-1)})} \pm \frac{\sqrt{c}\sqrt{m^2(m^2-1)}}{\sqrt[4]{(2m^2-1\pm 6\sqrt{m^2(m^2-1)})}}\frac{\mathrm{sn}\xi}{\mathrm{dn}\xi} \pm \frac{\sqrt{c}}{\sqrt[4]{(2m^2-1\pm 6\sqrt{m^2(m^2-1)})}}\frac{\mathrm{dn}\xi}{\mathrm{sn}\xi} \pm \frac{\sqrt{c}}{\sqrt[4]{(2m^2-1\pm 6\sqrt{m^2(m^2-1)})}}\frac{\mathrm{cn}\xi}{\mathrm{sn}\xi}, \tag{31a}$$

$$v_7(\xi) = \pm\frac{2\sqrt{c}l\sqrt{m^2(m^2-1)}}{\sqrt[4]{(2m^2-1\pm 6\sqrt{m^2(m^2-1)})}} \pm \frac{\sqrt{c}lm^2(m^2-1)}{\sqrt[4]{(2m^2-1\pm 6\sqrt{m^2(m^2-1)})}}\left(\frac{\mathrm{sn}\xi}{\mathrm{dn}\xi}\right)^2 \pm \frac{\sqrt{c}l}{\sqrt[4]{(2m^2-1\pm 6\sqrt{m^2(m^2-1)})}}\left(\frac{\mathrm{dn}\xi}{\mathrm{sn}\xi}\right)^2 \pm \frac{\sqrt{c}l\sqrt{m^2(m^2-1)}}{\sqrt[4]{(2m^2-1\pm 6\sqrt{m^2(m^2-1)})}}\frac{\mathrm{cn}\xi}{\mathrm{dn}^2\xi} \pm \frac{\sqrt{c}l}{\sqrt[4]{(2m^2-1\pm 6\sqrt{m^2(m^2-1)})}}\frac{\mathrm{cn}\xi}{\mathrm{sn}^2\xi}, \tag{31b}$$

where $\xi = kx + ly + ct$, $k = \pm\frac{\sqrt{c}}{\sqrt[4]{(2m^2-1\pm 6\sqrt{m^2(m^2-1)})}}$, and $l \neq 0$, and when $m \to 1$, we have the bell-type solitary solutions as follows:

$$u_{7a}(\xi) = \pm\sqrt{c} \pm \sqrt{c}\frac{\mathrm{sech}\xi}{\tanh\xi} \pm \sqrt{c}\frac{\mathrm{sech}\xi}{\tanh\xi}, \tag{32a}$$

$$v_{7b}(\xi) = \pm\sqrt{c}l\left(\frac{\mathrm{sech}\xi}{\tanh\xi}\right)^2 \pm \sqrt{c}l\frac{\mathrm{sech}\xi}{\tanh^2\xi}. \tag{32b}$$

4. Conclusion

By the use of extended mapping method and the auxiliary equation (6), we have constructed a wealth of new periodic wave solutions and solitary wave solutions for the (2+1)-dimensional dispersive long wave equations. And with the help of new function transformation (5), some complex periodic wave solutions and solitary wave solutions are obtained, especially the solutions of Eqs.(17a), (17b), (18a) and (18b) have not been found from the previous studies. When $m \to 1$ and $m \to 0$, the solutions reduce to hyperbolic and trigonometric function solutions respectively. The method is promising and can be a useful guidance to solving a broad class of nonlinear problems in the theory of solitary waves.

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